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ABSTRACT

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What is the $(\varepsilon'/\varepsilon)_{\text{exp}}$ Telling Us?

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Abstract

Nature might be kinder than previously thought as far as $\varepsilon'/\varepsilon$ is concerned. We show that the recently obtained experimental value for $\varepsilon'/\varepsilon$ does not require sizeable $1/N$ and isospin-breaking corrections. We propose to display the theoretical results for $\varepsilon'/\varepsilon$ in a $(P^{1/2}, P^{3/2})$ plane in which the experimental result is represented by a $(\varepsilon'/\varepsilon)_{\text{exp}}$–path. This should allow to exhibit transparently the role of $1/N$ and isospin-breaking corrections in different calculations of $\varepsilon'/\varepsilon$. From now on theorists are allowed to walk only along this $(\varepsilon'/\varepsilon)_{\text{exp}}$–path.
1 Introduction

The totally unexpected observation [1] of a sizeable CP-violation in the $K^0 - \bar{K}^0$ oscillations immediately triggered theoretical speculations about a new superweak interaction [2] obeying the strict $|\Delta S| = 2$ selection rule. The large value of the associated $\varepsilon$-parameter was then justified by the huge amplification due to the tiny $K_L - K_S$ mass difference. Following this rather simple picture, it was absolutely unlikely that CP-violation would show up somewhere else in weak processes.

Almost exactly 37 years later, we know that superweak models have been definitely ruled out by the new generation of high-precision experiments on the $|\Delta S| = 1$ neutral $K$-decays. Indeed, the most recent measurements of the associated $\varepsilon'$-parameter that allows us to distinguish between $\pi^+\pi^-$ and $\pi^0\pi^0$ final states in $K_L$ decays give

$$\text{Re}(\varepsilon'/\varepsilon) = \begin{cases} 
(15.3 \pm 2.6) \cdot 10^{-4} & \text{(NA48) [3]}, \\
(20.7 \pm 2.8) \cdot 10^{-4} & \text{(KTeV) [4]}.
\end{cases}$$

Combining these results with earlier measurements by NA31 collaboration at CERN ($(23.0 \pm 6.5) \cdot 10^{-4}$) [5] and by the E731 experiment at Fermilab ($(7.4 \pm 5.9) \cdot 10^{-4}$) [6] gives the grand average

$$\text{Re}(\varepsilon'/\varepsilon) = (17.2 \pm 1.8) \cdot 10^{-4}.$$  \hspace{1cm} (2)

The Standard Model for electroweak and strong gauge interactions accommodates, in principle, both $\varepsilon$ and $\varepsilon'$-parameters in terms of a single CP-violating phase. Rather early theoretical attempts [7] have predicted $\varepsilon'/\varepsilon$ between $10^{-2}$ and $10^{-4}$. During the last decade a considerable progress in calculating $\varepsilon'/\varepsilon$ has been done by several groups. These papers are reviewed in [8] where all relevant references can be found. The short distance contributions to $\varepsilon'/\varepsilon$ are fully under control [9] but the presence of considerable long distance hadronic uncertainties precludes a precise value of $\varepsilon'/\varepsilon$ in the Standard Model at present. Consequently, while theorists were able to predict the sign and the order of magnitude of $\varepsilon'/\varepsilon$, the range

$$(\varepsilon'/\varepsilon)_{\text{th}} = (5 \text{ to } 30) \cdot 10^{-4}$$  \hspace{1cm} (3)
shows that the present status of \((\varepsilon'/\varepsilon)_{\text{th}}\) cannot match the experimental one.

Though really expected this time, the non-vanishing value of a second CP-violating parameter has once again been determined by our experimental colleagues. However, one should not forget the tremendous efforts made by theorists to calculate \(\varepsilon'/\varepsilon\) in the Cabibbo-Kobayashi-Maskawa paradigm \([10]\) of the Standard Model. Simultaneously, one should not give up the hope that one day theorists will be able to calculate \(\varepsilon'/\varepsilon\) precisely. It is therefore important to have a transparent presentation of different theoretical estimates of \(\varepsilon'/\varepsilon\) in order to be able to identify the patterns of various contributions. On the other hand, having for the first time the definite precise number for \((\varepsilon'/\varepsilon)_{\text{exp}}\) it is crucial to learn what Nature is trying to tell us about theory. In this note, we intend to make first steps in both directions.

## 2 Basic Formulae

The standard parametrization for the hadronic \(K\)-decays into two pions:

\[
A(K^0 \rightarrow \pi^+\pi^-) = A_0 e^{i\delta} + \frac{1}{\sqrt{2}} A_2
\]

\[
A(K^0 \rightarrow \pi^0\pi^0) = A_0 e^{i\delta} - \sqrt{2} A_2
\]

\[
A(K^+ \rightarrow \pi^+\pi^0) = \frac{3}{2} A_2
\]

contains the necessary ingredients to produce non-vanishing asymmetries. For illustration consider

\[
a_{CP} \equiv \frac{\Gamma(K^0 \rightarrow \pi^+\pi^-) - \Gamma(\bar{K}^0 \rightarrow \pi^+\pi^-)}{\Gamma(K^0 \rightarrow \pi^+\pi^-) + \Gamma(\bar{K}^0 \rightarrow \pi^+\pi^-)} = \frac{\sqrt{2}\sin \delta}{(1 + \sqrt{2}\cos \delta + \omega^2/2)} \text{Im} \left( \frac{A_2}{A_0} \right)
\]

where

\[
\omega \equiv \frac{\text{Re } A_2}{\text{Re } A_0}.
\]

In order that \(a_{CP}\) is non-vanishing the two partial isospin amplitudes \(A_0\) and \(A_2\) must have a relative CP-conserving phase (extracted from \(\pi\pi\) scattering) which
turns out to be roughly equal to the phase of the $\varepsilon$-parameter:

$$\delta \approx \phi_\varepsilon \approx \pi/4 \quad (7)$$

and a relative CP-violating phase

$$\text{Im} \left( \frac{A_2}{A_0} \right) \neq 0. \quad (8)$$

These phases are nicely factorized in the physical parameter measuring direct CP-violation in hadronic $K$-decays

$$\varepsilon' = \frac{i}{\sqrt{2}} e^{-i\delta} \text{Im} \left( \frac{A_2}{A_0} \right) \quad (9)$$

if one defines

$$\eta_{+-} \equiv \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} \equiv \varepsilon + \frac{\varepsilon'}{1 + \frac{\omega}{\sqrt{2}} e^{-i\delta}} \quad (10)$$

$$\eta_{00} \equiv \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} \equiv \varepsilon - \frac{2\varepsilon'}{1 - \sqrt{2}\omega e^{-i\delta}}.$$

This allows to measure $\text{Re}(\varepsilon'/\varepsilon)$ through

$$\text{Re}(\varepsilon'/\varepsilon) = \frac{1}{6} (1 - \frac{\omega}{\sqrt{2}} \cos \delta) \left( 1 - \left| \frac{\eta_{00}}{\eta_{+-}} \right|^2 \right) \quad (11)$$

where we have kept the small $O(\omega)$ correction usually dropped by experimentalists but kept by theorists in the evaluation of $\varepsilon'$ using (9). Notice that the coincidence displayed in (7) implies an almost real $\varepsilon'/\varepsilon$ so that, already at this level, Nature is kind to us.

In the Standard Model, CP-violation only arises from the arbitrary quark mass matrices. A straightforward diagonalization shifts then the unique physical phase into the Cabibbo-Kobayashi-Maskawa (CKM) unitary mixing matrix $V$ associated with the $V - A$ hadronic charged current

$$J^{ab}_\mu = \bar{q}^a \gamma_\mu (1 - \gamma_5) q^b \equiv (\bar{q}^a q^b). \quad (12)$$
In this physical basis, we therefore have to start with the classical current-current 
$\Delta S = 1$ Hamiltonian

$$
\mathcal{H}^{\Delta S=1} = \sum_{q=u,c,t} \lambda_q J_{\mu}^q J_{\mu}^q \quad (\lambda_q \equiv V_{qs}^* V_{qd})
$$

$$
= \lambda_u [\bar{s}u(\bar{u}d) - (\bar{s}c)(\bar{c}d)]_{\Delta I=1/2,3/2}
+ \lambda_t [\bar{s}t(\bar{t}d) - (\bar{s}c)(\bar{c}d)]_{\Delta I=1/2}
$$

(13)

to estimate the $A_0$ and $A_2$ partial decay amplitudes.

The $\Delta I = 1/2, 3/2$ current-current operator involving only the light $u, d$ and $s$ quarks is just proportional to $\lambda_u$. A tree-level hadronization into $K$ and $\pi$ mesons fields would therefore imply $A_0 = \sqrt{2}A_2$, i.e. a vanishing $\varepsilon'$-parameter (see (9)). In
other words, a non-zero $\varepsilon'$-parameter is a pure quantum-loop effect in the Standard Model. Notice that these loop effects are also welcome to explain the empirical $\Delta I = 1/2$ rule :

$$
\omega_{\text{exp}} \approx \frac{1}{22} \ll \frac{1}{\sqrt{2}}.
$$

(14)

The quantum transmutation of the heavy $t\bar{t}$ and $c\bar{c}$ quark pairs into light $u\bar{u}$ and $d\bar{d}$ ones which, eventually, hadronize into final pion states allows now the pure
$\Delta I = 1/2$ current-current operator proportional to $\lambda_t$ to contribute to the $\Delta S = 1$ $K$-decays. In the most convenient CKM phase convention, we have

$$
\text{Im}\lambda_u = 0
$$

(15)
such that CP-violation only appears in the $A_0$ partial amplitude as long as isospin
is strictly respected in the “heavy-to-light” transmutation process. But in the Standard Model, neutral transmutations are possible through heavy quark annihilations
into gluons, $Z^0$ or photon that are represented by the so-called penguin diagrams.

While the latter electroweak contributions obviously break isospin symmetry, the former may also do so by producing first an off-shell iso-singlet mesonic state (mainly $\eta$ or $\eta'$) which then turns into an iso-triplet pion. These isospin-breaking (IB) effects respectively induced by the electric charge difference $\Delta e = e_u - e_d$ and the mass
splitting $\Delta m = m_u - m_d$ between the up and the down quarks are usually expected to show up at the percent level in weak decays. However, a CP-violating $\Delta I = 3/2$ amplitude turns out to be enhanced by the famous $\Delta I = 1/2$ rule factor $\omega^{-1}$ since

$$\text{Im} \left( \frac{A_2}{A_0} \right) = -\frac{\omega}{\text{Re} A_0} (\text{Im} A_0 - \frac{1}{\omega} \text{Im} A_2).$$  \hfill (16)$$

From these quite general considerations, one concludes that

$$\left( \frac{\varepsilon'}{\varepsilon} \right)_{th} = \text{Im} \lambda_t \left[ P^{1/2} - \frac{1}{\omega} P^{3/2} \right]$$  \hfill (17)$$

with $P^{1/2}$ and $P^{3/2}$, two separately measurable quantities defined with respect to the CKM phase convention defined in (15). Formally, $P^{1/2}$ and $P^{3/2}$ are given in terms of short distance Wilson coefficients $y_i$ and the corresponding hadronic matrix elements as follows

$$P^{1/2} = r \sum y_i \langle Q_i \rangle_0,$$  \hfill (18)$$

$$P^{3/2} = r \sum y_i \left[ \langle Q_i \rangle^{\Delta m}_2 + \omega^{\Delta m} \langle Q_i \rangle_0 \right]$$  \hfill (19)$$

where $r$ is a numerical constant and

$$\omega^{\Delta m} = \frac{(\text{Im} A_2)^{\Delta m}}{\text{Im} A_0}. \hfill (20)$$

3 The $(\varepsilon'/\varepsilon)_{exp}$-Path

Having all these formulae at hand, we can ask ourselves what the result in (2) is telling us. The answer is simple. It allows us to walk only along a straight path in the $(P^{1/2}, P^{3/2})$ plane, as illustrated in Fig.1. The standard unitarity triangle analyses [11] give typically

$$\text{Im} \lambda_t = (1.2 \pm 0.2) 10^{-4} \hfill (21)$$

and, combined with (4), already allow us to draw a rather thin $(\varepsilon'/\varepsilon)_{exp}$-path in the $(P^{1/2}, P^{3/2})$ plane (see Fig.1). This path crosses the $P^{1/2}$-axis at $(P^{1/2})_0 = 14.3 \pm 2.8.$
We are of course still far away from such a precise calculation of \( P^{1/2} \) and \( P^{3/2} \). These two factors are dominated by the so-called strong \( Q_6 \) and electroweak \( Q_8 \) penguin operators. The short-distance Wilson coefficients \( y_6 \) and \( y_8 \) of these well-known density-density operators are under excellent control \([3]\). In particular, the \( \Delta I = 3/2 \) \( Z^0 \)-exchange contribution to \( \varepsilon'/\varepsilon \) exhibits a quadratic dependence on the top quark mass which makes it to compete with the \( \Delta I = 1/2 \) gluon-exchange one. Unfortunately, the resulting destructive interference between \( P^{1/2} \) and \( P^{3/2} \) strongly depends on the various hadronic matrix elements. Long-distance effects are therefore at the source of the large theoretical uncertainties illustrated by (1). Consequently, we advocate to adopt (temporarily) a different strategy to learn something from the new precise measurements of \( \varepsilon'/\varepsilon \). The proposed exposition of \( \varepsilon'/\varepsilon \) in the \( (P^{1/2}, P^{3/2}) \) plane turns out to be useful in this context.

Figure 1: \( \varepsilon'/\varepsilon \)\textsubscript{exp}-path in the \( (P^{1/2}, P^{3/2}) \) plane.
4 A simple observation

It is well-known that isospin-symmetry and large-$N$ limit represent two powerful approximations to study long-distance hadronic physics. Here, these well-defined approximations would allow us to neglect $P^{3/2}$ and to express the hadronic matrix elements of the surviving strong penguin operators responsible for $P^{1/2}$ in terms of measured form factors. Earlier attempts [12] to go beyond such a zero-order approximation provided us already with some insight about the sign of the $1/N$ and IB corrections to $\varepsilon'/\varepsilon$. Recent works including further $1/N$ [13] and IB [14] corrections confirm their tendency to increase $P^{1/2}$ and $P^{3/2}$ respectively. We illustrate these generic trends

$$
(\varepsilon'/\varepsilon)_{th} = (\varepsilon'/\varepsilon)_0 \{1 + O(1/N) - \frac{1}{\omega}O(IB)\}. 
$$

as $(1/N)$ and (IB) arrows in Fig. 1. A systematic calculation of all $1/N$ and IB corrections is not yet available, but a direct comparison between the measured value $(\varepsilon'/\varepsilon)_{exp}$ and the zero-order approximation $(\varepsilon'/\varepsilon)_0$ should already tell us something about their magnitudes within the Standard Model. Indeed, if the experimental value quoted in (2) is larger than the zero-order theoretical approximation, one needs $1/N$ corrections along the $P^{1/2}$ axis:

$$
(\varepsilon'/\varepsilon)_{exp} > (\varepsilon'/\varepsilon)_0 \Rightarrow 1/N \text{ corrections.}
$$

On the other hand, an experimental value smaller than the zero-order approximation would be an indication for sizeable IB corrections along the $P^{3/2}$ axis:

$$
(\varepsilon'/\varepsilon)_{exp} < (\varepsilon'/\varepsilon)_0 \Rightarrow IB \text{ corrections.}
$$

And here comes the surprise! It turns out that $(\varepsilon'/\varepsilon)_0$ lies on the $(\varepsilon'/\varepsilon)_{exp}$-path in Fig. 1. It is the crossing of this path with the $P^{1/2}$ axis.

Indeed $(\varepsilon'/\varepsilon)_0$ can easily be estimated. In the large-N limit, the non-perturbative parameter $\hat{B}_K$ relevant for the usual analysis of the unitarity triangle equals $3/4$ [15]. This implies

$$
\text{Im} \lambda_t = (1.24 \pm 0.06) \cdot 10^{-4}
$$

\[25\]
to be compared with (21) that uses $\hat{B}_K = 0.85 \pm 0.15$. Moreover, in the large-N limit the hadronic matrix element of the strong penguin density-density operator $Q_6$ factorizes ($B_6 = 1$). A simple dependence on the inverse of the strange quark mass squared arises then to cancel the scale dependence of $y_6$ [10]. Taking the central values of the strange quark mass $m_s(2 GeV) = 110$ MeV and of the QCD coupling $\alpha_s(M_Z) = 0.119$ relevant for $y_6$, we obtain

$$ (\frac{\varepsilon'}{\varepsilon})_0 = (17.4 \pm 0.7) \times 10^{-4} \tag{26} $$

where the error results from the error in $\text{Im} \lambda_t$. In obtaining (26) we have taken also into account the contribution of the other ($Q_4$) surviving QCD penguin operator in the large-N limit. Without this contribution we would find $18.4 \pm 0.7$, still within the $(\varepsilon'/\varepsilon)_{\text{exp}}$-path. Clearly, as $\varepsilon_0$ is roughly proportional to $(\Lambda_{\text{MS}}^{(4)})^{0.8}/m_s^2$ with $\Lambda_{\text{MS}}^{(4)} = 340 \pm 40$ MeV and $m_s(2 GeV) = (110 \pm 20)$ MeV, improvements on these input parameters are mandatory.

Although this rather intriguing coincidence between (2) and (26) seems to indicate small $1/N$ and IB corrections, one cannot rule out a somewhat accidental conspiracy between sizeable corrections canceling each other

$$ \mathcal{O}(1/N) - \frac{1}{\omega} \mathcal{O}(IB) \approx 0. \tag{27} $$

The latter equation describes the walking along the $(\varepsilon'/\varepsilon)_{\text{exp}}$-path.

At this point, it is also worth noticing that CP-violation in the simplest extensions of the Standard Model, the models with minimal flavour-violation, might behave just like an IB correction along the $P^{3/2}$ axis. The reason is that the $Z^0$-penguin maximally violates the decoupling theorem. Consequently, it depends quadratically on the top quark mass and is also quite sensitive to new physics [17]. If such is the case, one will have a hard time to disentangle new sources of CP-violation beyond the Standard Model from ordinary IB corrections.

Finally the $(\varepsilon'/\varepsilon)_{\text{exp}}$-path can be shifted vertically in the $(P^{1/2}, P^{3/2})$ plane by new physics contributions to the quantities used for the determination of $\text{Im} \lambda_t$ but this is a different story.
5 Conclusion

Nature might be kinder than previously thought as far as $\varepsilon'/\varepsilon$ is concerned. Indeed, present data do not require sizeable $1/N$ and IB corrections. Improvements on the input parameters $\alpha_s(M_Z)$ and $m_s$ leading to our estimate of $(\varepsilon'/\varepsilon)_0$ are mandatory. We have proposed to display the theoretical results in a $(P^{1/2}, P^{3/2})$ plane in which the experimental result is represented by a $(\varepsilon'/\varepsilon)_{\text{exp}}$-path. This plot should allow to exhibit transparently the role of $1/N$ and isospin-breaking corrections in different theoretical results for $\varepsilon'/\varepsilon$.

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