"Another View of the J-Curve"

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ABSTRACT

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Another View of the J-Curve

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Discussion Paper 2005-29

Département des Sciences Économiques
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ANOTHER VIEW OF THE J-CURVE\textsuperscript{1}

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Abstract

We use a two-good dynamic optimizing small open economy model to provide a new explanation of the J-Curve phenomenon in terms of habit persistence in consumption and sluggishness in capital adjustment. The results differ markedly depending on the permanence or temporary nature of the relative price change. A short-lived terms of trade worsening may lead to a once-for-all decrease in the marginal utility of wealth and to higher steady-state values of the habitual standard of living, the real expense, and the net foreign assets through the combination of intertemporal speculation, inertia, and hysteresis effects. Investment and real expense follow non-monotonic transitional paths and current account dynamics are driven by new forces. In accordance with recent empirical results, investment is procyclical, trade balance deteriorates initially, net foreign assets adjustment exhibits a J-Curve, and the current account surplus phase is associated with a fall in real income.

Keywords: Current account; Habit Formation; Temporary shock; J-Curve.
JEL Classification: F41, E22, E21, F32.

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Département des Sciences Économiques de l'Université catholique de Louvain
1 Introduction

The recent appreciation of the Euro and large swings in raw materials and energy prices have once again underscored the need for a clear understanding of the dynamic relationship between the terms of trade and the current account position. As matter of fact, it is widely recognized that terms of trade shocks are one of the main driving forces behind GDP fluctuations and the external asset position in developing and newly developed countries, particularly in small open economies that are dependant on export income generated from a few products.\(^1\) Moreover, the fall in the rate of economic growth due to the decrease in export revenues may be magnified by a slowdown in the investment process. At the same time, painful decreases in real income induced by adverse relative price perturbations make previous consumption spending levels unsustainable and eventually translate into large losses in welfare. To the extent that consumption and investment decisions are made optimally, great imbalances in the current account should also be optimal and the stock of foreign assets may display an overshooting phenomenon.

In view of these potential dangers of adverse relative price movements, there is now a large body of literature that theoretically examines the implications of terms of trade shocks on the current account. During the 1980s most models took the approach of abstracting from capital accumulation in infinite horizon or two-period models and found that the effect depends on whether the terms of trade shock is permanent or temporary, the specification of preferences, the size of intra-temporal and inter-temporal substitution effects induced by relative price changes (see e.g. Obstfeld [1982], [1983], Ostry [1988], Svensson and Razin [1983]). Still abstracting accumulating capital, a number of subsequent studies, such as Sen [1990], Karayalçın [1995], Mansoorian [1993], [1998], also introduced real money balances in the utility function, assumed heterogeneity between individuals, or introduced a habit-forming behavior. Importantly, however, all of these contributions share the common characteristic of considering a small open endowment economy and of focusing only on an unanticipated permanent terms of trade shock.\(^2\) This shortcoming was recently remedied by Sen and Turnovsky [1989] and Servën [1999], by developing an intertemporal maximizing framework to analyze transitory terms of trade perturbations by considering capital adjustment costs.\(^3\) Both studies find a prominent role for investment fluctuations in the determination of current account but a little role for consumption, which is primarily due to their specification of time separable preferences.

While clearly there has been considerable advancement in the area, the results derived from the aforementioned literature still fall considerably short of observed empirical regularities and thus there is still further need for a theory that can be reconciled with the evidence. In this regard, it is arguable that, first, a model must be sufficiently flexible to reproduce the non-monotonic transitional path of the current account following a terms of trade deterioration as featured by the data. This is, for example, a shortcoming of Mansoorian [1993] and Obstfeld [1982] who emphasize the implications of a habit-forming behavior and recursive preferences on the external asset position adjustment, respectively. While their introduction of time non-separable preferences restores sluggishness into the consumption-side, in their models the current account can only adjust monotonically because of abstracting from capital accumulation. More importantly, they focus exclusively on the effects of permanent perturbations which severely
limits their predictions. Secondly, empirical evidence also suggests that allowing for sluggishness in consumption in modelling the movements of the current account substantially improves the performance of the econometric structural models. Thus, although Sen and Turnovsky [1989] and Servén [1999] allow for capital accumulation facing adjustment costs, their analysis suffers from considering time separable preferences. Hence, the implied dynamics of the current account are mainly driven by investment fluctuations and are not determined by a more realistic interaction between saving and investment flows as empirical evidence suggests. Moreover, abstracting from consumption habits may lead to misleading conclusions regarding the long-term effects on macroeconomic aggregates of an unanticipated temporary terms of trade worsening.

The current paper attempts to address these shortcomings by allowing for both consumption and investment dynamics in a single theoretical framework. As such it relates to the existent literature adopting a continuous time intertemporal-optimizing framework in order to explore the implications of time non-separable preferences and investigate the possibility of a non-monotonic adjustment of the net foreign asset position (e. g. Karayalçin [1994], Ikeda and Gombi [1998], Mansoorian [1998]). Our formalization may be viewed as an extension of Ikeda and Gombi model by considering that consumption and investment have an import content. This two-good extension allows us to derive new results that match the empirical facts and provide new insight into current account dynamics. At the opposite of the formerly cited authors, we provide a systematic analysis of the J-Curve phenomenon by focusing on relative price perturbations affecting both consumption-side and production-side. Beyond [i] the degree of habit persistence in consumption, [ii] the installation costs of capital, and [iii] the time length of the shock, we find new crucial factors as [iv] the domestic contents of consumption and investment expenditure, [v] the long-run intertemporal elasticity of substitution (under time non-separable preferences) upon which depends the J-shape response of the net foreign assets following an adverse terms of trade shock. Finally, unlike the standard approach described by Junz and Rhomberg [1973] and Meade [1988] according to which the current account non-monotonic adjustment relies upon the size of import and export demand elasticities, we revisit the dynamic link between the relative price of exports and the current account within a two-good small open economy model incorporating sound micro-theoretic foundations.

Our main results are as follows. First, we show that after an unanticipated terms of trade worsening, the current account originally deteriorates since the negative saving flow due to consumption habits outweighs the negative investment flow if capital installation costs are sufficiently high. The current account then adjusts non-monotonically if the speed of adjustment of habits is higher than the speed of adjustment of the capital stock. In accordance with empirical results, saving and investment co-vary, real expense and real investment are procyclical, and the real income declines during the external account surplus phase. Second, following a short-lived transitory terms of trade deterioration, real expense and investment dynamics display a non-monotonic adjustment. This implies in turn that the J-shape response of the net foreign assets is no longer constrained by the speeds of adjustment of habits and capital stock. Instead, we demonstrate that the J-Curve phenomenon relies upon the combination of four effects: a smoothing effect originating from the temporary nature of the perturbation, an intertemporal speculation effect induced by the transitory fall of consumption-based and investment-based real
interest rates,\(^6\) an *inertia effect* stemming from consumption habit, and an *hysteresis* effect due to the dependence of steady state on initial conditions.\(^7\) During the first phase, the negative saving flow coupled with the positive investment flow deteriorates the current account. Once terms of trade are restored back to their initial level, saving and investment flows are reversed, the current account turns to be positive, and real income falls. Finally, the small open economy reaches the new steady-state with a higher stock of foreign assets. In words, we provide a modified Marshall-Lerner condition according to which a terms of trade worsening leads to a definitive improvement of the net foreign asset position.

The paper is organized as follows. In section 2, we present the framework of a two-good model of a small open economy, facing given terms of trade and world interest rate. In section 3, we analyze the equilibrium dynamics and the steady-state of the model. Section 4 explores in detail the effects of a permanent worsening of the terms of trade. In section 5, a consistent solution method for analyzing temporary shocks is applied to an adverse transitory relative price perturbation. Consumption, stock of habits, investment, and current account dynamics are studied and the results are compared to the effects of a permanent terms of trade deterioration. Conclusions and a short outlook on further research are contained in section 6.\(^8\)

## 2 The Framework

Consider a small open economy that is populated by a constant number of identical households and firms that have perfect foresight and live forever. We normalize, without loss of generality, the number of households to one. There are four types of goods. The representative firm is completely specialized in the production of a final good that can be consumed domestically or exported. This good can also be transformed, at some cost, in capital. The domestic good is an imperfect substitute for an imported good which can be used for consumption or investment. The country is small in world good and capital markets and faces given terms of trade (price of the domestic good in terms of the foreign good), \(p\), and world interest rate, \(r^*\).

### 2.1 Structure of the Economy

**Households**

At each instant the representative household consumes domestic goods and foreign goods denoted by \(d\) and \(f\). The measure of utility of consumption at \(t\), \(c(t)\), is given by the relation:

\[
c(t) = c(d(t), f(t)),
\]

where \(c(\cdot)\) is a positive, increasing, concave and linearly homogeneous aggregator function. The representative household maximizes the objective function

\[
U[C(0)] = \int_0^\infty u[c(d(t), f(t)), s(t)] \exp(-\delta t) dt,
\]

where \(u(\cdot)\) is a positive, increasing, concave and linearly homogeneous aggregator function. The representative household maximizes the objective function

\[
U[C(0)] = \int_0^\infty u[c(d(t), f(t)), s(t)] \exp(-\delta t) dt,
\]

\[
2\]
where $\delta$ is the consumer’s discount rate, and $s(t)$ a distributed lag on past real expenditure as (see Ryder and Heal [1973]),

$$s(t) = \sigma \int_{-\infty}^{t} c(d(\tau), f(\tau)) \exp \left(-\sigma (t - \tau)\right) d\tau. \quad (3)$$

From (3), the dynamic equation of the habit stock is given by

$$\dot{s}(t) = \sigma [c(t) - s(t)]. \quad (4)$$

Following Ryder and Heal [1973], the instantaneous utility function is assumed to be:

- [H1] increasing in current real expenditure, $u_1 > 0$;
- [H2] non-increasing in past real expense, $u_2 \leq 0$;
- [H3] increasing in a uniformly maintained real expense level, $u_1 (c, c) + u_2 (c, c) > 0$, which guarantees non-satiation;
- [H4] strictly concave in $c$ and $s$, $u_{12} (c, s) < 0$, $u_{22} < 0$;
- and concave in $(c, s)$, $u_{11} (c, s) u_{22} (c, s) - [u_{12} (c, s)]^2 \geq 0$;
- [H5] $\lim_{c \to 0} u_1 (c, s) = \infty$ and $\lim_{c \to 0} [u_1 (c, c) + u_2 (c, c)] = \infty$.

Since $c(.)$ is homothetic, the household’s maximization problem can be decomposed into two stages (see Frenkel and Razin [1987], chapter 6). At the first stage, the household minimizes the cost, $z_c(t) = p(t) d(t) + f(t)$, for a given level of subutility, $c(t)$, where $p(t)$ is the relative price of the domestic good. For any chosen $c(t)$, the optimal basket $(d(t), f(t))$ is a solution to

$$p_c (p(t)) c(t) = \min_{\{d(t), f(t)\}} \{p(t) d(t) + f(t) : c(d(t), f(t)) \geq c(t)\}. \quad (5)$$

The assumption that the subutility function $c(.)$ is linear homogeneous implies that the total expense in consumption goods can be expressed as $z_c(t) = p_c (p(t)) c(t)$, with $p_c (p(t))$ is the unit cost function dual (or consumption-based price index) to $c$. Intra-temporal allocations between domestic goods and imports follow from Sheppard’s Lemma (or the envelope theorem) applied to (5):

$$d(t) = p'_c (p(t)) c(t), \quad f(t) = \left[p_c (p(t)) - p(t) p'_c (p(t))\right] c(t), \quad (6)$$

with (see Deaton and Muellbauer [1980]),

$$p_c (p) > 0, \quad p'_c (p) > 0, \quad p''_c (p) < 0. \quad (7)$$

In the second stage, consumers choose their real expense, $c$, and rates of accumulation of consumption “experience” and traded bonds to maximize (2) subject to (3) and the flow budget constraint,

$$\dot{b}(t) = r^* b(t) + [D(t) + w(t)] - p_c (p(t)) c(t), \quad (8)$$

and initial conditions $s(0) = s_0$, $b(0) = b_0$. Households’ income consists of interest earnings, $r^* b(t)$, dividend payments on equity holdings, $D(t)$; moreover, households inelastically supply one unit of labor services and receive the wage, $w(t)$, per unit of time. The real stock of foreign assets held by the household, $b(t)$, is denominated in terms of the imported good since we assume that external borrowing and lending are measured in units of the foreign good.

**Firms**

4
Perfectly competitive firms produce output, $Y$, from labor, $l$, and capital, $k$, by means of a constant returns to scale production function, which is assumed to have the usual neoclassical properties of positive and diminishing marginal products. Like Abel and Blanchard [1983], the installation cost function $\psi(I(t)/k(t))$, is assumed to have the following properties:

$$\psi(0) = 0, \quad \psi'(.) > 0, \quad 2\psi'(.) + \frac{I}{k}\psi''(.) > 0. \quad (9)$$

Following Gavin [1992] and Servèn [1999], we assume that domestic and imported goods are converted in an investment good according to a linearly homogeneous technology:

$$J(t) = J(J_D(t), J_F(t)), \quad (10)$$

where $J_D$ and $J_F$ denote domestic and foreign inputs combined into the investment process. Since $J(.)$ is homogeneous of degree one, the investment decision can be done in two stages like consumption decision. The solution to the atemporal investment allocation problem can be written in the form

$$p_I(p(t))J(t) = \min_{\{J_D(t), J_F(t)\}} \{p(t)J_D(t) + J_F(t) : J(D(t), F(t)) \geq J(t)\}, \quad (11)$$

where the exact investment price index is a function of terms of trade and has the following properties

$$p_I(p) > 0, \quad p'_I(p) > 0, \quad p''_I(p) < 0. \quad (12)$$

From Sheppard’s lemma, we obtain investment demand for the domestic and imported goods:

$$J_D(t) = p'_I(p(t))J(t), \quad J_F(t) = \left[p_I(p(t)) - p(t)p'_I(p(t))\right]J(t). \quad (13)$$

In the second stage, the representative firm maximizes the present value of anticipated future cash flow:

$$\max_{\{I(t), l(t)\}} \int_0^\infty D(t)e^{-r^*t}dt = \max_{\{I(t), l(t)\}} \int_0^\infty \left\{pF(k, l) - wl - p_I(p) I \left[1 + \psi\left(\frac{I}{k}\right)\right]\right\}e^{-r^*t}dt, \quad (14a)$$

subject to

$$\dot{k}(t) = I(t), \quad (14b)$$

and the initial condition

$$k(0) = k_0. \quad (14c)$$

2.2 Macroeconomic Equilibrium

To obtain the macroeconomic equilibrium, we first derive the optimality conditions for households and firms and combine these with the accumulation equations. This leads to the set of equations

$$\max_{\{I(t), l(t)\}} \int_0^\infty D(t)e^{-r^*t}dt = \max_{\{I(t), l(t)\}} \int_0^\infty \left\{pF(k, l) - wl - p_I(p) I \left[1 + \psi\left(\frac{I}{k}\right)\right]\right\}e^{-r^*t}dt, \quad (14a)$$

subject to

$$\dot{k}(t) = I(t), \quad (14b)$$

and the initial condition

$$k(0) = k_0. \quad (14c)$$
\[ u_1(c, s) + \sigma \xi = p_c(p) \lambda, \]  
\[ pF_l(k, 1) = w, \]  
\[ q = p_I(p) \left[ 1 + \psi \left( \frac{I}{k} \right) + \left( \frac{I}{k} \right) \psi' \left( \frac{I}{k} \right) \right], \]  
\[ \dot{\lambda} = 0, \quad \text{i.e.} \quad \lambda = \bar{\lambda}, \]  
\[ \dot{\xi} = (\delta + \sigma) \xi - u_2(c, s), \]  
\[ \dot{q} = r^* q - \left[ pF_k(k, 1) + p_I(p) \left( \frac{I}{k} \right)^2 \psi' \left( \frac{I}{k} \right) \right], \]  
\[ \dot{b} = r^* b + pF(k, 1) - p_c(p) c - p_I(p) I \left[ 1 + \psi \left( \frac{I}{k} \right) \right], \]

and dynamic equations (4) and (14b), and the transversality conditions

\[ \lim_{t \to \infty} \lambda b \exp (-r^* t) = \lim_{t \to \infty} \xi s \exp (-r^* t) = \lim_{t \to \infty} qk \exp (-r^* t) = 0, \]

where \( \lambda, \xi, q \) are the co-state variables associated with dynamic equations (8), (3), and (14b).

The solution of the differential equation (15e) using (16) is given by

\[ \xi(t) = \int_t^\infty u_2(c(\tau), s(\tau)) e^{-(\delta + \sigma)(\tau - t)} d\tau. \]

The shadow price of habit stock is equal to the present discounted value of marginal disutility of consumption “experience”, \( u_2 \leq 0 \), which depreciates at the rate \( \sigma \).

Solving (15f) forward and ruling out “bubble trajectories”, we obtain

\[ q(t) = \int_t^\infty \left\{ pF_k[k(\tau), 1] + p_I(p) \left( \frac{I}{k} \right)^2 \psi'(\cdot) \right\} e^{-r^*(\tau - t)} d\tau. \]

According to (18), the shadow price of capital is equal to the present discounted value of the sum of the marginal product of capital and the reduction of the marginal cost induced by an increase in the capital stock for a given flow of investment, both expressed in the foreign good.

The first static efficiency condition (15a) requires that along an optimal path the sum of marginal current utility of real expense and its marginal contribution to the future felicity stream derived from a higher habitual standard of living is equal to the marginal utility of wealth in the form of internationally traded bonds measured in terms of the domestic good, \( p_c \lambda \). The second static efficiency condition (15b) establishes the usual equality between the marginal productivity of labor and the real wage. Equation (15c) equates the ratio of market price of installed capital to the replacement cost of capital, i.e. the Tobin’s \( q \), to the marginal cost investment.

With a constant rate of time preference and an exogenous world interest rate, we require that

\[ \delta = r^*, \]

in order to generate an interior solution. This standard assumption made in the literature implies that the marginal utility of wealth, \( \lambda \), must remain constant over time (see (15d)), and gives rise to the zero-root property (see Sen and Turnovsky [1990]).
Finally, the first transversality condition of (16) rules out the possibility of running up infinite
debt or credit and ensures that the nation remains intertemporally solvent.

3 Equilibrium Dynamics and the Steady-State

Equilibrium Dynamics

The static efficiency condition (15c) implies that the rate of investment is a function of Tobin’s $q$:

$$ I_k = \kappa \left( \frac{q}{p_I(p)} \right), \quad \kappa'(. > 0, \quad \kappa(1) = 0. \quad (20) $$

From (20), the rate of investment rises when the market price of capital is higher than investment replacement cost, that is to say the Tobin’s $q$, denoted by $\nu$, is greater than one.

Total differentiation of equation (15a), substitution of (4) and (15e), and elimination of $\xi$ using (15a) lead to the following dynamic equation of real expense:

$$ \dot{c} = \frac{1}{u_{11}} \left[ (\delta + \sigma) \left( u_1 - p_c(p) \bar{\lambda} \right) + \sigma u_2 - u_{12} \sigma (c - s) \right]. \quad (21) $$

Inserting the short run static solution (20) in (14b) and (15f), linearizing these with dynamic equations (3) and (21) around the steady-state, and denoting $\bar{x} = \bar{s}, \bar{\xi}, \bar{k}, \bar{q}, \bar{p}, \bar{c}$ the long-term values of $x = s, \xi, k, q, p, c$, we obtain in a matrix form

$$ \begin{pmatrix} \dot{s}, \dot{c}, \dot{k}, \dot{q} \end{pmatrix}^T = J \begin{pmatrix} s(t) - \bar{s}, c(t) - \bar{c}, k(t) - \bar{k}, q(t) - \bar{q} \end{pmatrix}^T, \quad (22) $$

where $J$ is given by

$$ J \equiv \begin{pmatrix} -\sigma & \sigma & 0 & 0 \\ u_{12} + \frac{\sigma}{\delta + 2\sigma} u_{22} & \delta + \sigma & 0 & 0 \\ 0 & 0 & 0 & \kappa'(1) \frac{k}{p_I(p)} \\ 0 & 0 & -\bar{p}F_{kk} & r^* \end{pmatrix}. \quad (23) $$

The Fisherian separation theorem implies that the matrix is block recursive. As we will see
below, the number of predetermined variables equals the number of negative eigenvalues and
the number of jump variables equals the number of positive eigenvalues, so there exists a unique
convergent path towards the steady-state.

The characteristic roots obtained from $J_{11}$ write as follows:

$$ \mu_i \equiv \frac{1}{2} \left\{ \delta \pm \sqrt{(\delta + 2\sigma)^2 + \frac{4\sigma(\delta + 2\sigma)}{u_{11}} \Gamma} \right\} \geq 0, \quad i = 1, 2, \quad (24) $$

where we let

$$ \Gamma = u_{12} + \frac{\sigma}{\delta + 2\sigma} u_{22} \leq 0. \quad (25) $$
The sign of $\Gamma$ depends on $u_{12}$. If the marginal utility of real expense is sufficiently increasing in stock of habits, the preferences of the representative consumer display “adjacent complementarity” and $\Gamma$ is positive (see Ryder and Heal [1973]). If $u_{12}$ has a negative or a low positive value, $\Gamma$ is negative and preferences are said to display “distant complementarity”.

We denote respectively $\mu_1 < 0$ and $\mu_2 > 0$ the stable and unstable real eigenvalues satisfying

$$\mu_1 < 0 < r^* < \mu_2, \quad (26)$$

under the condition (see Obstfeld [1992])

$$u_{12} + \frac{\sigma}{\delta + 2\sigma} u_{22} < - \left( \frac{\delta + \sigma}{\delta + 2\sigma} \right) u_{11}, \quad (27)$$

so that the long-run equilibrium is a saddle-point in $(s, c)$-space. While the habit stock evolves always gradually, the real expense, $c$, can jump instantaneously in response to a new information.

Starting from an initial “consumption experience” $s(0) = s_0$, the stable dynamic time paths followed by $s$ and $c$ are given by

$$s(t) = \bar{s} + A_1 e^{\mu_1 t}, \quad (28a)$$
$$c(t) = \bar{c} + \left( \frac{\sigma + \mu_1}{\sigma} \right) A_1 e^{\mu_1 t}, \quad (28b)$$

where $A_1 = s_0 - \bar{s}$. When the stock of habits is expected to be higher, real expense and habitual standard of living co-vary in the same or in an opposite direction according to whether $(\sigma + \mu_1)$ is positive (adjacent complementarity, $\Gamma > 0$) or negative (distant complementarity, $\Gamma < 0$), i. e. according to whether the marginal current utility of real expense is sufficiently strongly increasing or decreasing (or weakly increasing) in future “consumption experience” with respect to the increase of marginal desutility of habits.

The two real characteristic roots obtained from $J_{22}$ write as follows:

$$\chi_i \equiv \frac{1}{2} \left\{ r^* - \sqrt{(r^*)^2 - 4 \frac{\kappa (1) k^p F_{kk}}{p_I (\bar{p})}} \right\} \geq 0, \quad i = 1, 2. \quad (29)$$

The stable and unstable eigenvalues satisfy

$$\chi_1 < 0 < r^* < \chi_2. \quad (30)$$

Hence the dynamics describe a saddle-point in $(k, q)$-space. The stable dynamic time paths followed by $k$ and $q$ are

$$k(t) = \bar{k} + B_1 e^{\chi_1 t}, \quad (31a)$$
$$q(t) = \bar{q} + \left( \frac{p_I (1) k}{\kappa (1) k} \right) B_1 e^{\chi_1 t}, \quad (31b)$$

where $B_1 = k_0 - \bar{k}$. The solution (31b) indicates that the stable branch is downward-sloping.

Substituting the short-run solution (20), linearizing the dynamic equation of the foreign asset stock (15g) in the neighborhood of the steady-state, using the fact that $pF_k (\bar{k}, 1) = r^* p_I (\bar{p})$
at the steady-state, substituting the general solutions of \( c, k, \) and \( q \), and finally invoking the transversality condition, one obtain the linearized version of the nation’s intertemporal budget constraint
\[
(b_0 - \bar{b}) = -p_I (k_0 - \bar{k}) - \frac{p_c (\sigma + \mu_1)}{\sigma (\mu_1 - r^*)} (s_0 - \bar{s}),
\]
where the initial stocks, \( k_0, b_0, s_0 \) are given. The stable solution for \( b(t) \) consistent with long-run solvency writes as follows
\[
(b(t) - \bar{b}) = -p_I (k(t) - \bar{k}) - \frac{p_c (\sigma + \mu_1)}{\sigma (\mu_1 - r^*)} (s(t) - \bar{s}).
\]
This equation describes the relationship between the stock of internationally traded bonds, the stock of physical capital, and the stock of habits along a stable path. Time derivatives of solutions (31a) and (33) and elimination of \( A_1 e^{\mu_1 t} \) and \( B_1 e^{\chi_1 t} \) allow to express the change of capital and foreign assets stocks in term of deviations from steady state as
\[
\begin{align*}
\dot{k}(t) &= \chi_1 (k(t) - \bar{k}), \\
\dot{b}(t) &= \mu_1 (b(t) - \bar{b}) + p_I (\mu_1 - \chi_1) (k(t) - \bar{k}),
\end{align*}
\]
where the multiplier \((\mu_1 - \chi_1)\) can be positive or negative according to whether the speed of adjustment of consumption habits, \(|\mu_1|\), is lower or greater than this of capital stock, \(|\chi_1|\). The new differential equation system has two stable roots \( \mu_1 < 0 \) and \( \chi_1 < 0 \); hence the steady-state is a stable node in \((k, b)\)-space (see figure 2).

**Steady-State**

The steady-state of the economy is obtained by setting \( \dot{c}, \dot{s}, \dot{k}, \dot{q}, \dot{b} = 0 \) and is defined by the following set of equations:
\[
\begin{align*}
&u_1 (\bar{c}, \bar{s}) + \frac{\sigma}{\delta + \sigma} u_2 (\bar{c}, \bar{s}) = p_c (\bar{p}) \bar{\lambda}, \\
&\bar{c} = \bar{s}, \\
&\bar{q} = p_I (\bar{p}), \\
&r^* p_I (\bar{p}) = \bar{p} F_k (\bar{k}, 1), \\
r^* \bar{b} + \bar{p} F (\bar{k}, 1) - p_c (\bar{p}) \bar{\epsilon} = 0,
\end{align*}
\]
and the intertemporal solvency condition
\[
(b_0 - \bar{b}) = -p_I (k_0 - \bar{k}) - \frac{p_c (\sigma + \mu_1)}{\sigma (\mu_1 - r^*)} (s_0 - \bar{s}).
\]
From (35a), the marginal utility of consumption along a constant path, i.e. the sum of marginal current utility of real expense \((u_1)\) and the capitalized value of marginal utility of “consumption experience” \((u_2/ (\delta + \sigma))\) adjusted by the parameter \( \sigma \) is equal to the marginal utility of wealth in terms of the domestic good, \( p_c \bar{\lambda} \). The equation (35b) requires that the real expense is equal to the habit stock at the steady-state. Equation (35c) asserts that long-run investment is zero when the market price of installed capital is equal to its replacement cost, i.e. when the Tobin’s \( q \) is equal to one. Equation (35d) indicates that in the long-run the marginal product of capital, \( \bar{p} F_k \), is equal to its user cost, \( r^* p_I \), both expressed in terms of the foreign good. Equation (35e) implies that in the steady-state equilibrium, the current account must be zero, that is,
the gross national product, \( r^*b + \bar{p}\bar{Y} \), must be equal to the total expenditure in consumption goods, \( p, (\bar{p}) \bar{c} \). Finally, the linearized version of the nation’s intertemporal budget constraint (35f) implies that the steady-state depends on the initial stocks \( k_0, b_0, \) and \( s_0 \). This dependency upon initial conditions comes from the assumptions of infinitely lived maximizing agents having a constant rate of discount and facing perfect capital markets and leads to hysteresis effects, that is, temporary terms of trade disturbances have permanent effects (see Sen and Turnovsky [1989], [1990]).

System (35) may be solved for the steady-state values by applying the two-step solution method described by Schubert [2002] and Schubert and Turnovsky [2002]. We first solve equations (35a) to (35e) as functions of marginal utility of wealth expressed in terms of the foreign good, \( \bar{\lambda} \), and of the terms of trade, \( p \). This yields to the following functions

\[
\bar{s} = \bar{c} = t (\bar{\lambda}, p), \quad t < 0, \quad t_p < 0, \quad (36a)
\]
\[
\bar{k} = u (p), \quad u_p > 0, \quad (36b)
\]
\[
\bar{q} = p_I (p), \quad p_I' > 0, \quad (36c)
\]
\[
\bar{b} = v (\bar{\lambda}, p), \quad v_p < 0, \quad (36d)
\]

In the second step, we insert these functions into the economy’s intertemporal budget constraint (eq (35f)), which may be solved for the equilibrium value of the marginal utility of wealth:

\[
\bar{\lambda} = g (s_0, k_0, b_0, p), \quad \lambda_s > 0, \quad \lambda_k < 0, \quad \lambda_b < 0, \quad \lambda_p < 0. \quad (37)
\]

Substituting then \( \bar{\lambda} \) into the other steady-state functions (36) gives the conventional steady-state values of the economy as functions of the terms of trade and the initial conditions, \( k_0, b_0 \) and \( s_0 \) (except for production-side variables, \( k \) and \( q \)).

## 4 A permanent Deterioration of Terms of Trade

We investigate the effects of a permanent decrease in the relative price of the domestic good (denoted by the subscript \( \text{perm} \)), \( p \), from \( p_0 \) to \( p_1 \), which occurs at time \( t = 0 \), where the economy is originally in steady state. Like Obstfeld [1983] and Sen and Turnovsky [1989], we assume that the small open economy is a net exporter of the domestic good at the steady-state, that is \((\bar{Y} - \bar{d}) > 0\). All agents perfectly understand the permanence of the terms of trade deterioration, but its occurrence at time \( t = 0 \) is unanticipated. Because of perfect foresight assumption, the transitional dynamics are affected by the expected long-run state of the economy.

### Steady-State Effects of an Unanticipated Permanent Terms of Trade Deterioration

The long-run effects after a permanent change in the relative price of the home good, consistent with long-run solvency, are obtained from the total differential of the equilibrium system
condition (27) is removed (see Becker and Murphy [1988]). Income which implies a decumulation of financial wealth and hence amplifies the long-term effects persistence in consumption is strong, consumers reduce initially to decrease their real expenditure. When preferences display adjacent complementarity, i.e. habit be explained in an intuitive way; a drop in

\[
\frac{d\bar{c}}{dp}_{\text{perm}} = \frac{d\bar{s}}{dp}_{\text{perm}} = \frac{\sigma (\mu_1 - r^*)}{p_c \mu_1 (\sigma + r^*)} (\bar{Y} - \bar{d}) > 0,
\]

(38a)

\[
\frac{dk}{dp}_{\text{perm}} = -\frac{F_k (1 - \alpha_I)}{p F_{kk}} > 0, \quad \frac{dq}{dp}_{\text{perm}} = q_1 (p) > 0,
\]

(38b)

\[
\frac{d\lambda}{dp}_{\text{perm}} = \frac{\sigma (\mu_1 - r^*)}{p_c \mu_1 (\sigma + r^*)} \left[ u_{11} + \frac{\delta + 2\sigma}{\delta + \sigma} \Gamma \right] - \frac{\alpha_c \tilde{\lambda}}{p} < 0,
\]

(38c)

\[
\frac{db}{dp}_{\text{perm}} = -\frac{(\sigma + \mu_1) (\bar{Y} - \bar{d})}{\mu_1 (\sigma + r^*)} + \frac{p F_k (1 - \alpha_I)}{p F_{kk}} \geq 0,
\]

(38d)

\[
\frac{d\bar{nx}}{dp}_{\text{perm}} = -r^* \frac{d\bar{b}}{dp}_{\text{perm}} = (\bar{Y} - \bar{d}) + p F_k \frac{dk}{dp}_{\text{perm}} - p_c \frac{d\bar{c}}{dp}_{\text{perm}},
\]

(38e)

where the net exports expressed in terms of the foreign good are denoted by \(nx\); they are defined as the difference between the domestic output and absorption, the latter being equal to the sum of consumption and investment expenditure, i.e. \(nx(t) = p F[k(t), 1] - p_c c(t) - p_I J(t)\).

A permanent decrease in \(p\) reduces the long-term values of real expense, \(\bar{c}\), and of habits, \(\bar{s}\), of the same amount (see (38a)). These changes are proportional to the loss\(^{16}\) in the purchasing power of exports in terms of consumption goods, \([(\bar{Y} - \bar{d}) / p_c] dp < 0\). The difference with the conventional time separable utility function is that the modification of \(c\) in the long-run is greater (in absolute value) the higher is habit persistence in consumption, i.e. the lower is \(|\mu_1|\). This can be explained in an intuitive way; a drop in \(p\) implies a fall in real income which incites households to decrease their real expenditure. When preferences display adjacent complementarity, i.e. habit persistence in consumption is strong, consumers reduce initially \(c\) but less than the drop in real income which implies a decumulation of financial wealth and hence amplifies the long-term effects of the negative income shock on consumption (compared to standard preferences). This process can be unstable if condition (27) is removed (see Becker and Murphy [1988]).

From expression (38c), the permanent decrease in \(p\) has two influences which work in the same direction on marginal utility of wealth expressed in terms of the foreign good. First, a fall in \(p\) lowers domestic real income which induces a drop in consumption. This income effect is proportional to \((\bar{Y} - \bar{d})\) (see the first term on the right hand side of eq (38c)). Second, a negative change in \(p\) favors the consumption of the domestically produced good, \(d\), and incites households to reduce the level of their consumption in the foreign good. This substitution effect is proportional to \(\alpha_c\), the share of domestic goods in consumption expenditure (see the second term on the right hand side of eq (38c)). These two adjustments lead to an increase in marginal utility of wealth expressed in terms of the foreign good.

When the expenditure in capital goods has an import content, a decrease in \(p\) leads to a drop in the marginal product of capital expressed in the foreign good greater in absolute value than the reduction of the user cost of capital. Adjustment in the long-run calls for a lower capital stock, \(\bar{k}\). At the new steady-state, the shadow price of installed capital is definitely reduced to the level \(\bar{q}_1 = p_I (p_1)\) and Tobin’s \(q\) is equal to unity.

The change in the foreign assets stock, \(\bar{b}\), is the result of two forces. First, in response to an adverse permanent terms of trade perturbation, financial wealth increases or decreases according
to whether households’ preferences display distant \(((\sigma + \mu_1)^{dist} < 0)\) or adjacent complementarity \(((\sigma + \mu_1)^{adj} > 0)\), i.e.

\[
\frac{d\tilde{a}}{dp}_{\text{perm}} = -\frac{p_c (\sigma + \mu_1) \, d\tilde{s}}{\sigma (\mu_1 - r^*) \, dp}_{\text{perm}} \lesssim 0 \quad \text{according to} \quad (\sigma + \mu_1) \lesssim 0,
\]

where the non-human wealth, \(a = b + qk\), is the sum of the value of internationally traded bonds and the value of domestic equities, both expressed in terms of the foreign good. Second, a drop in the relative price of domestic goods discourages investment which affects positively the stock of foreign bonds. Finally, we assume that under adjacent complementarity, the savings flow dominates the investment flow, i.e the long-run value of net foreign assets decreases following a permanent terms of trade deterioration. The Harberger-Laursen-Metzler (H-L-M hereafter) effect according to which a terms of trade worsening leads to a current account deficit depends on this assumption that can be expressed in a more formal way:\(^{17}\)

\[
d\bar{b}/dp\bigg|_{\text{perm}}^{\text{adj}} = -p_I \frac{d\tilde{k}}{dp}_{\text{perm}} - \frac{p_c (\sigma + \mu_1) \, d\tilde{s}}{\sigma (\mu_1 - r^*) \, dp}_{\text{perm}} > 0,
\]

where the notation \(\text{adj}\) means that we refer to the adjacent complementarity case. At the new steady-state, the current account must be zero, \(\bar{c}a = 0\); the trade balance must mirror the balance of net interest earnings, i.e. \(\bar{n}\bar{x} = -r^*\bar{b}\). Under adjacent complementarity, the economy must raise net exports to compensate the losses in interest earnings, i.e. \(\bar{n}\bar{x}_1 = -r^*\bar{b}_1 > \bar{n}\bar{x}_0 = -r^*\bar{b}_0\). This can be achieved through the decline in the real expense by an amount greater than the fall in the real income due to a lower relative price (deflator effect) and a smaller capital stock (see (38e)).

**Transitional Dynamics**

We assume that the economy is initially in an “old” steady-state. Due to the zero-root property, the marginal utility of wealth jumps instantaneously at time \(t = 0\) to its new steady-state value, \(\bar{\lambda}\), and remains constant from thereon. Whereas \(s_0\) is predetermined, real expense is free to jump at time \(t = 0\) to situate the economy on the stable branch:

\[
\frac{dc(0)}{dp}_{\text{perm}} = \left[1 - \frac{\sigma + \mu_1}{\sigma}\right] = \frac{\mu_1 \, dc}{dp}_{\text{perm}} \gtrless \frac{d\bar{c}}{dp}_{\text{perm}} \quad \text{according to} \quad (\sigma + \mu_1) \lesssim 0, \quad (40)
\]

where we differentiated the stable solution (28) evaluated at time \(t = 0\) with respect to \(p\). With conventional time separable preferences, a permanent deterioration of terms of trade leads to a decline in real expense equal to the drop in real income. The economy jumps immediately to the new steady-state and saving is not affected (see Obstfeld [1983], p. 139). When preferences of habit-forming consumers display adjacent complementarity, a negative income shock calls for a (decreasing) gradual response of \(c\). This sluggish consumption adjustment follows from an initial jump of real expense less important than its decline in the long-run (see eq (40)). Agents choose \(c(0)\) by assigning a positive weight to the initial stock of habits, \(s_0\), and a weight less than one to the annuity value of total wealth deflated by the consumption price index, \(r^*[b_0 + W(0)]/p_c\), i.e. the permanent income. Therefore, households will reduce initially their consumption but less than the drop in real income.

When the representative household tries to maintain his habits, he decumulates financial wealth after the permanent deterioration of terms of trade. The economic intuition behind this
result rests on the degree of habit persistence in consumption determined by $\mu_1$ which in turn depends on the value of $u_{12}$. With adjacent complementarity ($\Gamma > 0$), a consumption experience decrease in the long-run implies an initial rise in the rate of time preference, i.e.:

$$\frac{d\rho(0)}{dp} = -\frac{u_{11}(\delta + \sigma)}{pc\lambda}\left[\left(\frac{\sigma + \mu_1}{\sigma}\right)^{adj}\left(\delta + 2\sigma\left(\frac{\delta + \sigma}{u_{11}(\delta + \sigma)}\right)^{\Gamma}\right)\frac{d\bar{s}}{dp} < 0.\right]$$

(41)

where the expression in square brackets is negative. In words, the high positive value of the cross-partial derivative of the felicity function, $u_{12}$, implies that the marginal utility of current real expense is greater than the marginal utility of future real expense, so that consumption is always biased toward the present. Figure 1 depicts the adjustments of $c$ and $\rho$ following an unanticipated permanent deterioration of terms of trade. When intertemporal preferences display adjacent complementarity, the real expense and the stock of habits monotonically decrease toward their new long-run values, $\bar{c}_1 = \bar{s}_1$. In the same time, the rate of time preference falls toward the fixed discount rate, $\delta$, as $s$ converges to its new steady-state value.

Whenever investment expense has an import content, a fall in the home goods’ relative price leads to a greater decline in the marginal product of capital than in its real user cost. With $k_0$ being predetermined, the market price of capital jumps instantaneously from $\bar{q}_0$ down to $q(0)$. The depressing effect on investment from the decrease in $q$ is greater the larger is the gap between the short-run decline of the market price of capital, $q(0)$, and the new investment price index, $p_I(p_1) = \bar{q}_1$, i.e the higher is the import content of capital goods. Figure 3 depicts the decumulation of capital stock, $\dot{k}(t) < 0$, which slows down, $\ddot{k}(t) > 0$, as economy approaches the steady-state. The decline of $k$ raises its marginal productivity over time, and therefore its market price at a decreasing rate. Whenever one abstract from capital good imports, the marginal product of capital shifts by the same amount than the user cost and the domestic capital stock remains unaffected. If investment has an import content as we may expect in a
small open economy, an unanticipated permanent terms of trade deterioration induces a fall in the long-run value of the equipment goods’ stock.

Having discussed the dynamic transition of real expense and investment toward the new long-run equilibrium of the economy, we turn now to the current account and net exports adjustment following a worsening of the terms of trade. The transitional dynamics can be best described by the use of phase portraits for the stable manifold. As the stable manifold for the stock of foreign assets is two-dimensional, its speed of convergence is a weighted average of the speeds of adjustment of saving and investment. The flexibility provided by the additional eigenvalue allows the system to match the non-monotonic convergence of current account featured by the data. Because we are interested in providing a new explanation of the J-Curve phenomenon, we restrict the study by assuming that [i] the representative household’s preferences exhibit adjacent complementarity \((\sigma + \mu_1)_{adj} > 0\), [ii] the adjustment speed \(|\mu_1|\) of habits is higher than the adjustment speed \(|\chi_1|\) of capital, and [iii] inequality (39), according to which a permanent terms of trade worsening reduces the stock of foreign assets in the long-run, holds. Transitional dynamics can be disentangled in two phases by noting that there exists a date \(t = \tilde{t}\) (with \(ca(\tilde{t}) = 0\)) such that the stock of foreign assets overshoots along the stable trajectory (see figure 2).\(^{19}\) To summarize, we find

\[
\begin{align*}
ca(t) &= \dot{b}(t) \leq 0, & \text{for } t \leq \tilde{t}, \\
ca(t) &= \dot{b}(t) > 0, & \text{for } t > \tilde{t}.
\end{align*}
\]

Figure 2: An unanticipated permanent terms of trade worsening and net foreign assets adjustment in the \((k, b)\)-space: the J-Curve - \(\mu_1 < \chi_1\) - Adjacent complementarity.

An unanticipated permanent worsening of terms of trade leads to an immediate decumulation of foreign bonds and a deterioration in the trade balance, the initial level of \(b\) being
Figure 3: An unanticipated permanent terms of trade worsening: saving and investment dynamics

\[ \dot{b}(0) = ca(0) = \left[ prx_1 \frac{dk}{dp}_{\text{perm}} + \mu_1 \frac{p_c (\sigma + \mu_1) d\bar{s}}{\sigma (\mu_1 - r^*) dp}_{\text{perm}} \right] \frac{dp}{dp_{\text{perm}}} = -\frac{p_c}{(\mu_1 - r^*)} \dot{c}(0) - p_I \dot{k}(0) < 0, \]

(43)

where we differentiated (33) with respect to time, then evaluated the expression at time \( t = 0 \), and assumed a permanent fall in \( p \). The initial current account deficit is a consequence of the fact that the initial negative flow of saving, \( \dot{a}(0) = S(0) < 0 \), more than outweighs the influence of the initial negative flow of investment which gives rise to the H-L-M effect (see figure 3). The assumption of an habit-forming behavior implies a decline in real expense less than proportional to the reduction in permanent income, i. e. an “excess smoothness” of consumption, and therefore a fall in financial wealth. At the same time, high installation costs imply a slow adjustment speed of capital stock, and then a small initial decumulation of equipment goods. The greater is the habit persistence in consumption, the higher are the installation costs and the lower is the import content of investment, the strongest is the negative initial response of saving and the smallest is the positive impact of investment on current account.

During the first phase \( 0 < t \leq \tilde{t} \), consumption experience and physical capital decrease smoothly (see figure 2). Since the representative household reduces its financial wealth faster than the representative firm decumulates its capital stock, the non-human wealth \( a \) approaches its steady-state faster than \( k \). The fact that habit persistence effects dominate investment discouraging effects until date \( \tilde{t} \) gives rise to an overshooting of net foreign assets’ adjustment, i. e. \( b(\tilde{t}) < \bar{b}_1 < \bar{b}_0 \).

At some point of time, \( t = \tilde{t} \), the trajectory cuts the demarcation line \( \dot{b}(t) = 0 \) (see figure 2).
From point \( B \), the deterioration in the net foreign asset position is then followed by a current account surplus. Over the period \( t > \tilde{t} \), investment influence becomes dominant over the habit effects. The rate at which saving falls turns to be lower than the rate at which investment decreases. This exercises a rise in the stock of foreign assets which converges in direction to its new steady-state level \( \tilde{b}_1 \) at point \( E_1 \). Turning to the trade balance adjustment, the effect of a decline in capital goods on the net exports operates through three channels: [i] the reduction of real output, [ii] the deceleration of the negative investment flow as \( k \) approaches its new steady-state value, and [iii] the rise in marginal installation costs for a given investment flow. During the second phase, net exports decline over time but remain above their steady-state level.

Finally, at the new steady-state, investment has ceased, capital and hence output have fallen. Levels of habits and real expense are lower than at the initial steady-state. As decumulation of foreign bonds outweighs their accumulation during the second phase, \( b \) is definitively reduced. Since current account must be balanced in the long-run, the losses in interest earnings from abroad must be compensated by an improvement of the trade balance. Short-term and long-term effects in the present formal setup following an unanticipated terms of trade worsening may provide some explanations of recent empirical results. First, in accordance with estimations of Leonard and Stockman [2002], we find that the second phase of current account adjustment is associated with a fall in the real income, \( p_1 \dot{Y}(t) = p_1 F_1 \dot{k}(t) < 0 \), which results from the decline in both the home goods’ relative price and the stock of physical capital. Second, investment and saving co-vary along the stable adjustment path, i.e. \( \dot{a}(t) < 0 \) and \( \dot{k}(t) < 0 \), as we depict on figure 3 (see Tesar [1991], Mendoza [1995]). Third, investment and real expense are procyclical, \( \dot{k}(t) < 0 \) and \( \dot{c}(t) < 0 \), and positively correlated with the terms of trade (see Mendoza [1995]).

We may summarize the main results of this section. Following an unanticipated permanent terms of trade worsening, the H-L-M effect holds if consumption displays a high inertia implying a negative saving flow stronger than the negative investment flow. Transitional dynamics being determined by long-term changes in perfect foresight models, the initial current deficit relies upon the long-term fall of net foreign assets and the Marshall-Lerner (M-L hereafter) condition fails to hold. Finally, the J-Curve pattern of external asset position is observed whenever the speed of adjustment of physical capital is lower than the speed of adjustment of habits, that is if capital installation costs are sufficiently high.

5 An Unanticipated Temporary Deterioration of Terms of Trade

The study of an unanticipated transitory terms of trade perturbation in the present formal setup allows to provide some new insights of the real expense and net foreign asset position transitional dynamics by introducing an habit-forming behavior as we shall see below. In addition, the analytical techniques developed in this paper differ considerably from previous studies in that they allow to determine formal solutions for investment and consumption blocks, and then for current account, without retaining functional forms. Finally, it permits to emphasize the relevant factors for investment and consumption decisions, and therefore current account dynamics, following an unanticipated temporary terms of trade worsening in an intertemporal
We suppose that terms of trade decrease unexpectedly at time $t = 0$ from the original level $p_0$ to level $p_1$ over the period $0 \leq t < T$, and they revert back at time $T$ permanently to their initial level $p_T = p_2 = p_0$. The temporary nature of the perturbation needs to consider two periods, say period 1 ($0 \leq t < T$) and period 2 ($t \geq T$). We have retained the two-step analytical procedure proposed by Schubert and Turnovsky [2002] and impose one single intertemporal solvency constraint which links the period 1 with period 2 through the new initial conditions. During the transition period 1, the economy accumulates capital, foreign assets, and habit stocks. Since this period is by its very nature unstable, it would lead the nation to violate its intertemporal budget constraint. By contrast, the adjustment process taking place in period 2 is stable and must satisfy the economy’s intertemporal budget constraint. At the same time, the zero-root property requires the equilibrium value of marginal utility of wealth to adjust once-and-for-all when the shock hit the economy. So $\lambda$ remains constant over the periods 1 and 2. The aim of the two-step method is to calculate the deviation of $\lambda$ such that the country satisfies one single and overall intertemporal budget constraint, given the new relevant initial conditions $k_T, b_T$, and $s_T$ prevailing when the shock ends and accumulated over the unstable period. The consistency of two-step procedure rests on its capacity to link the agents’ decisions over the two phases and calculate the initial jump of the marginal utility of wealth such that the single national intertemporal budget constraint holds.

5.1 Production-Side Dynamics

Although we adopted a formal setup for the investment-side closely related to the framework specified by Gavin [1992] and Servèn [1999] and our results are qualitatively the same, the solution method we apply to study temporary terms of trade disturbances effects yield to some new formal solutions for temporary shocks which allow for an analytical comparison with formal solutions obtained with permanent shocks and for a rigorous study of transitional dynamics.$^{22}$

Since steady-state values (36b) and (36c) of capital stock and its market price are only function of terms of trade and do not depend on marginal utility of wealth, a transitory home goods’ relative price disturbance has not permanent effects on production-side aggregates. Therefore, $k$ and $q$ revert back to their initial levels when the shock ends. The dynamic evolutions of $k$ and $q$ when the terms of trade disturbance is active are governed by a couple of equations which contain an explosive part, say terms with $e^{\mu_2t}$; that is why we say that period 1 is unstable. After an unanticipated temporary drop of $p$, the market price of equipment goods jumps instantaneously in the same direction as after a definitive reduction of $p$ but is softened by a scale down factor:

$$\left. \frac{dq(0)}{dp} \right|_{\text{temp}} = (1 - e^{-\chi_2T}) \left. \frac{dq(0)}{dp} \right|_{\text{perm}} > 0.$$  

(44)

The fall is moderated by the factor $0 < (1 - e^{-\chi_2T}) < 1$ which is an increasing function of the length $T$ of the shock and a decreasing function of capital installation costs.$^{23}$ As the Tobin’s $q, \nu$, is the ratio between the capital market price and the investment price index, investment flow may be positive or negative depending on the strength of the initial fall of $q$. From the
solutions prevailing in the case of a temporary terms of trade shock, it can be shown that the market price of capital may jump under or above the transitory lower level of the investment price index, $p_I(p_1) = \bar{q}_1$:

$$q(0) - \bar{q}_1 = \frac{p_I}{p} \left[ \begin{array}{c} \text{capital profitability effect} \\ \text{inter.temp. spec. effect} \end{array} \right] \begin{cases} r^* (1 - \alpha_I) & (1 - e^{-\chi^2 T}) \\ \chi^2 & (1 - e^{-\chi^2 T}) \alpha_I \end{cases} \left[ \begin{array}{c} (+) \\ (-) \end{array} \right] d\bar{p} \leq 0. \quad (45)$$

On the one hand, a drop in the home goods’ relative price discourages investment by lowering capital marginal productivity below its user cost, as long as investment expenditure has an import content. Since the negative shock is transitory, $q$ declines less compared to its reaction to a negative permanent shock: this is the transitory capital profitability effect. On the other hand, as the relevant real rate of interest is relatively low while firms expect a rise in home goods’ relative price, $r^I = r^* - \alpha_I \bar{p}/p < r^*$, there is an incentive to raise investment because the price of future real investment in terms of present real investment is transitorily higher. In words, firms perfectly understand the temporary nature of the perturbation and perfectly expect a terms of trade improvement at time $T$ which is reflected by a transitory lower real user cost of capital. This is the intertemporal speculation effect.

From (45), a transitory unfavorable terms of trade movement, $d\bar{p} = p_1 - p_0 < 0$, implies an initial jump of $q$ below or above its period 1’s steady-state value, i.e. $\nu(0) < 1$ or $\nu(0) > 1$, according to whether the first term or the second term in square brackets predominates. The former represents the capital profitability effect which is an increasing function of [1] the share of imported capital goods in investment expenditure, $(1 - \alpha_I)$, and [2] the shock’s length, $T$. The latter represents the intertemporal speculation effect which works in an opposite direction on investment. The shorter-lasting is the shock, the less the present value of future marginal product is affected and therefore the smaller is the fall in the market price of installed capital; the higher is the domestic content in investment expenditure, $\alpha_I$, the greater is the fall in the investment-based real interest rate $r^I$ below the world interest rate $r^*$. This last effect encourages the domestic firms to benefit from the transitorily low level of the investment price index.

There exists a length of the relative price disturbance $\hat{T}$ such that the initial value of the Tobin’s $q$, $\nu(0)$, is equal to unity. Whenever the negative terms of trade perturbation lasts only a short time period and/or the share of home goods in investment expenditure is important, the intertemporal speculation effect outweights the capital profitability effect. This implies that the second term of the expression (45) dominates the first term. Investment dynamics depicted on figure 5 indicate that the capital stock adjustment over time exhibits a non-monotonic behavior in transition to the steady-state equilibrium. The moderated initial jump of the capital market price implies a Tobin’s $q$ higher than unity and a positive flow of investment, $I(0) > 0$. Over the first phase, $0 < t < T$, the capital increases, $\dot{k}(t) > 0$, at a growing rate, $\ddot{k}(t) > 0$, that is the positive value of trajectory’s slope in the $(k, q)$-space becomes higher. Although the economy moves along an unstable and explosive path, all domestic firms perfectly anticipate that the negative terms of trade worsening will end in the near future and that the marginal product will rise at time $T$. Because the marginal productivity of capital is transitory low and the rise of the stock of capital goods intensifies it, the no-arbitrage condition calls for increases in the market
5.2 Consumption-Side Dynamics

We describe now the short-term and long-term effects of an unanticipated temporary terms of trade worsening on the real expense and the stock of habits. In a two-good model with conventional time separable preferences or recursive preferences, an unanticipated temporary terms of trade worsening leads to a fall in the marginal utility of wealth which is a scaled-down factor, of the response to a permanent shock, say \( d\lambda = \lambda_p (1 - e^{-r^*T}) dp \). This result is not longer obtained in a two-good model whenever consumers have habits as we can see from the equation below:

\[
\frac{d\lambda}{dp}_{\text{temp}} = (1 - e^{-r^*T}) \frac{d\lambda}{dp}_{\text{perm}} - \frac{p\lambda}{pc} \frac{dp}{\mu_1} \left( e^{-r^*T} - e^{-\mu_2 T} \right) \leq 0
\]

where \( p/c = \alpha_c/p \), and \( (e^{-r^*T} - e^{-\mu_2 T}) \) is positive because \( \mu_2 > r^* \). From expression (46), an adverse transitory relative price disturbance exerts on \( \lambda \) two possibly offsetting effects. The \textit{wealth} effect comes from the fall of the real income which reduces the present value of wealth. The first term on the right-hand side of (46) indicates that the change of the marginal utility of wealth equals the change after a permanent terms of trade shock, \( \lambda_p \), scaled-down by the term \( 0 < (1 - e^{-r^*T}) < 1 \). The \textit{intertemporal speculation} effect coupled with an \textit{inertia} effect work in an opposite direction of the \textit{wealth} effect. This phenomenon reflected by the second term, i.e. \( -\frac{\alpha_c}{p} \frac{\lambda}{\mu_1} (e^{-r^*T} - e^{-\mu_2 T}) > 0 \), may outweigh the \textit{wealth} effect if the shock’s persistence is not too high (i.e. a low value of \( T \)) and/or the share of domestic goods in consumption expenditure is important (i.e. a high value of \( \alpha_c \)) and/or the habit persistence in consumption is strong (i.e. a low value of \( |\mu_1| \)).

These effects can be best commented from the investigation of the expression of real expense’s initial response given by:

\[
\frac{dc(0)}{dp}_{\text{temp}} = -\frac{\mu_1}{\sigma} \frac{dc}{dp}_{\text{perm}} \left( 1 - e^{-r^*T} \right) + \frac{\mu_2}{\sigma} \lambda_p (e^{-r^*T} - e^{-\mu_2 T}) \geq 0
\]

The first term on the right-hand side of (47) captures the \textit{wealth} effect dampened by the \textit{smoothing} effect. The latter is greater the shorter is the shock’s length reflected by the parameter \( T \), that is the higher is the deviation of current income from permanent income. Following an adverse terms of trade perturbation, the fall of \( c \) is moderated by the factor \( (1 - e^{-r^*T}) \) compared to a permanent decrease of \( p \). In words, since agents know that the decrease in \( p \) is only temporary, the present value of the necessary reduction in real expense to satisfy the intertemporal budget constraint is less than for an equal but permanent decline in \( p \). The second term on the right-hand side of (47) reflects the \textit{intertemporal speculation} effect coupled with the \textit{inertia} effect which work against the \textit{wealth} effect. Intuitively, since agents know that the decline in the relative price
Figure 4: An unanticipated temporary terms of trade worsening and real expense dynamics: the wealth, smoothing, intertemporal speculation, inertia, and hysteresis effects

$p$ and hence the advantage of a lower cost of consumption goods last only temporarily, they wish to benefit from it by consuming at higher rates.\textsuperscript{26} Under the conditions of a low value of $T$, a high share of domestic goods $\alpha_c$ in consumption expenditure, and a strong habit persistence, the domestic households may be encouraged to raise initially their real expense (i.e. $c(0) > \bar{c}_0$).

If the intertemporal speculation effect predominates, agents may raise in the short-run and more importantly in the long-run their real expense as they adjust smoothly their consumption. The latter conclusion differs markedly from Obstfeld’s result. Such dynamics emerge since households wish to maintain their new higher standard of living induced by a greater purchasing power in terms of consumption goods. The inertia effect makes possible the rising temporal path of real expense after a negative perturbation through the fall by a sufficient amount of the time preference rate below the consumption-based real interest rate. An habit-forming behavior departs from the behavior prevailing with time separable preferences since agents raise their consumption only gradually. As agents expect to be accustomed to the greater level of consumption experience, their financial wealth accumulation behavior tends to amplify the intertemporal speculation effect in the long run. The dynamics of real expense over the unstable (period 1) and stable (period 2) periods affect the accumulation of internationally traded bonds which in turn influence the once-for-all jump of the marginal utility of wealth. By assuming an habit-forming behavior, $\lambda$ may be permanently lower and $c$ definitively higher after a transitory decline in $p$ depending on the strength of the intertemporal speculation effect.

Since multiple cases may emerge and we are only interested in providing a new explanation to the $J$-Curve phenomenon, we shall restrict our discussion to adjacent complementarity preferences and a short-lived negative terms of trade perturbation. Figure 4 graphs real expense dynamics when the intertemporal speculation effect combined with the inertia effect outweigh
the wealth effect softened by the smoothing effect. This implies higher habits at the steady state of period 1. In the adjacent complementarity case, a higher consumption experience in the long-run implies that the marginal utility of real expense in the future is greater than in the present. The rising habitual standard effect leads to a growing path over time for real expense while the value of the time preference rate is transitorily below the real rate of interest, \( r^c = r^* - \alpha_c \dot{\rho}(t) / \rho(t) < r^* \). At some date, the value of the time preference rate becomes greater than \( r^c \) and the path of real expense is decreasing until date \( T \). The real expense dynamics over period 1 can be clarified from the following dynamic equation:

\[
\dot{c}(t) = -\frac{p_c \lambda}{u_{11}} \left( r^* - \alpha_c \frac{\dot{\rho}(t)}{\rho(t)} - \rho(t) \right) \geq 0, \quad 0 < t < T,
\]

where the gap between \( r^c \) and \( \rho(t) \) influences the consumption adjustment over the period \( 0 < t \leq T \). According to (48), the greatest is the fall of the time preference rate, the highest is the domestic content in consumption expenditure, and the most likely a positive change in real expense following its upward initial jump. Over the unstable period 1, the time preference rate increases as the stock of habits rises. At a date \( t = \hat{t} \), the time preference rate is equal to the real rate of interest and the real expense stops increasing, that is \( \dot{c}(\hat{t}) = 0 \) (see figure 4). As \( \rho(t) \) keeps on increasing since the consumption experience rises over the entire period 1, the time preference rate becomes higher than \( r^c \) which in turn leads to a decreasing temporal path of \( c \) until time \( T \). The economic intuition behind this result may be explained as follows. Since the period 1’s steady-state, \( (\bar{s}_1, \bar{c}_1) \), is not viable in solvency terms, the economy cannot raise real expense over the entire period 1 and must establish the level of consumption habits at time \( T \) to a level that satisfies the overall intertemporal budget constraint.

At time \( T \) when the negative perturbation is ended, there are no news effects since all agents perfectly anticipated that change in domestic relative price and the marginal utility of wealth remains constant in the neighborhood of time \( T \). Only the improvement in the terms of trade, that is \( p_1 - p_2 < 0 \), constitutes a disturbance effect and exercises an impact on the real expense. The long-run value of \( c \) falls because of the rise of the marginal utility of wealth measured in terms of the domestic good, \( p_c(p_2) \bar{\lambda} > p_c(p_1) \bar{\lambda} \) and establishes to the level \( \bar{c}_2 < \bar{c}_1 \). The expiration of the terms of trade disturbance leads to an abruptly rise in the consumption-based real interest rate, that is \( r^c(T^+) = r^* \), which remains constant during the convergence toward the new steady-state. When preferences display adjacent complementarity, the marginal utility of real expense is strongly increasing with the habitual standard of living \( (\Gamma > 0) \), and the expectation of a higher value of the habit stock in the final steady-state implies that the time preference rate is below its long-run value, \( \delta = r^* \). The explanation comes from the marginal utility of future real expense which is higher than the marginal utility of current real expense. This, in turn, encourages to forgo present for future real expense. The temporal path of \( c \) is monotonically increasing over period 2

\[
\dot{c} = -\frac{p_c \lambda}{u_{11}} (r^* - \rho(t)) > 0, \quad t \geq T.
\]

As the consumption experience increases, the time preference rate rises and converges toward the psychological time discount rate. Since the representative consumer smooths the change in real expense, this behavior makes possible the accumulation of financial wealth when the terms of trade are favorable (over period 2).
5.3 Current Account Dynamics: a J-Curve?

We show now that current account dynamics may exhibit a sequence of deficit-surplus after an unanticipated transitory terms of trade worsening. The specification of time non-separable preferences induces a departure from the conventional time separable utility function as the temporal path of real expense is no longer flat over respectively periods 1 and 2. Instead of the recursive preferences specification, transitory terms of trade shocks have permanent effects through the hysteresis effects and the stock of foreign assets may reach a permanently higher level. It differs also from the conclusions obtained within a one-good small open economy model with habits since real expense and investment may rise after a transitory negative disturbance, driven by the intertemporal speculation effect. The fact that our dynamic system can generate more flexible dynamic paths and may lead to a multiplicity of possible adjustments comes at price and requires to restrict the scope of possibilities. Although it would be interesting to investigate all of them, we set some assumptions for reasons of space and in order to emphasize the new transitional paths, and to provide a new explanation of the J-Curve adjustment of the current account in accordance with the recent empirical results obtained by Leonard and Stockman [2002].

Since the relative price perturbation has no permanent effects on $k$, and therefore on the real domestic product, a fall in $\lambda$ implies a permanent rise in real expense which in turn must be exactly outweighed by higher interest revenues in order to guarantee the equilibrium of the current account at the new steady-state. Formally, in the new long-run equilibrium, the gross national product must be equal to the absorption, that is

$$p_2F(\bar{k}_2, 1) + r^*b_2 = p_c(p_2)\bar{c}_2,$$

where $p_2 = p_0$ and $\bar{k}_2 = \bar{k}_0$. When $\bar{c}_2 > \bar{c}_0$, the long-run foreign assets stock must be higher, $\bar{b}_2 > \bar{b}_0$, in order to allow the small open economy to reach the steady-state.

Formally, after tedious computations, the initial response of the current account can be calculated as

$$\frac{dca(0)}{dp} \bigg|_{temp} = \frac{dS(0)}{dp} \bigg|_{perm} \left(1 - e^{-r^*T}\right) - p_I \frac{dI(0)}{dp} \bigg|_{perm} \left(1 - e^{-\chi_2T}\right) + \frac{\kappa'(1)\bar{k}}{p}e^{-\chi_2T}\alpha_I$$

$$- \frac{\kappa'(1)\bar{k}}{p}e^{-\chi_2T}\alpha_I$$

The sign of the expression (51) depends on five key factors: [i] the length of the shock (T), [ii] the import content of investment and consumption expenditure ($\alpha_I$ and $\alpha_c$), [iii] the marginal installation costs reflected by $\kappa'$ (and measured by $\chi_2$), [iv] the degree of habit persistence in consumption, ($|\mu_1|$), and [v] the long-run intertemporal elasticity of substitution (under time non-separable preferences) as it is shown below. The first two terms on the right-hand side of (51) indicate that saving and investment responses to a temporary shock are moderated by the factors $(1 - e^{-r^*T})$ and $(1 - e^{-\chi_2T})$ compared to their reaction to a permanent perturbation. The shorter-lasting is the adverse terms of trade perturbation, the less are the fall in present values
Figure 5: An unanticipated temporary terms of trade worsening and the current account dynamics: the J-Curve phenomenon
of real income and of the capital revenues, the lower are the responses of saving and investment, and therefore their initial impact on the current account. Since $\chi_2 > r^*$, the dampening factor is smaller for investment than for saving, i.e. $(1 - e^{-\chi_2 T}) > (1 - e^{-r^* T})$. The third and fourth terms represent the intertemporal speculation effect that affects respectively investment and consumption decisions. The higher are the shares of domestic goods in investment and consumption expenditure, and the less persistent is the adverse terms of trade perturbation, the stronger is the real interest rate effect, and the more probably the current account deteriorates.

Concerning investment choices, the higher is $\kappa'(1)$, the lower are marginal installation costs and the more investment is responsive to a terms of trade shock (see the third term of (51)). Let us now turn to the consumption reaction to a transitory fall of the consumption-based real interest rate, $r^c$, below the world interest rate, $r^*$. With time separable preferences and a CRRA utility functional, the fourth term reduces to $\frac{de}{\sigma} - \frac{r^*}{\theta}$ where the intertemporal elasticity of substitution, denoted by $\eta_{add}$, is equal to $1/\theta$ (see Cardi [2004]).29 Allowing for habits implies that the representative agent cares not only about the level of current consumption but also to his consumption experience. When the utility function is supposed to be of the CRRA form as assumed by Carroll et al. [2000], the long-run intertemporal elasticity of substitution is no longer equal to $1/\theta$ but equal to $\eta_{hab} = 1/[(\gamma + \theta (1 - \gamma)]$ where $\gamma$ is the weight attached to accumulated consumption experience, $s$.30

The long-run intertemporal elasticity of substitution under time non-separable preferences is greater than this obtained under conventional preferences whenever $\theta > 1$ that is if $u_{12} > 0$ (see Alvarez-Cuadrado et al. [2004], Carroll et al. [2000]). The fourth term on the right-hand side of (51) can be rewritten as follows $\frac{de}{\sigma} - \frac{r^*}{\theta}$ where the intertemporal elasticity of substitution, denoted by $\eta_{add}$, is equal to $1/\theta$ (see Cardi [2004]). Servèn [1999]). The novel result is that consumption habits amplify the intertemporal speculation effect compared to the conventional time separable case by raising the long-run intertemporal elasticity of substitution above the inverse of the coefficient of relative risk aversion. In response to a temporary fall in the real cost of consumption following a transitory terms of trade worsening, habits make consumers more willing to reallocate expenditure towards the present and against the future. The dampening term $-e^{\mu_1 T}$ reflects the fact that habit-forming agents dislike large and rapid changes in consumption and prefer to smooth the variation of their real expense over time. The more increasing is the marginal utility of real real expense with respect to the stock of habits, the lower is the absolute value the characteristic root, $|\mu_1|$, and the smoother is the real expense’s reaction. If the shock is sufficiently short, it may be desirable for households to have a temporal decreasing path of their real expense immediately after the upward jump.31 Finally, the last and fifth term represents the impact of the consumption smoothing effect on the current account. This term is decreasing with the shock’s persistence. This can be explained as follows. A temporary shock results in a larger impact on current than on permanent income, and this gap is as greater as the perturbation is short-living. Therefore, the current account will be more affected the shorter-lasting is the perturbation.

The discussion on transitional dynamics rests on the assumption that the intertemporal speculation effect predominates, in particular one consider that the relative price change is short-living.
Starting off from an initial steady-state (period 0), the initial increase in real expense and investment expenditure leads to an increase in domestic absorption. In the same time, as the initial stock of physical is predetermined, the decline in the relative price of home goods induces a fall in the real income. Therefore, the trade balance deteriorates and the current account turns negative at time \( t = 0^+ \) (see figure 5). Over the period 1, the transition does not take place along a path converging toward a viable steady-state. This unsustainable long-run equilibrium \((\bar{b}_1, \bar{k}_1, \bar{s}_1)\) would ultimately lead the small open economy to intertemporal insolvency. At time \( t = \check{t} < T \), real expense begins its decreasing and investment keeps on increasing. As the growth of capital goods raises the real income and saving becomes less negative, the current account remains negative but may eventually improve. At time \( T \), the relative price of home goods is restored to its original level, \( p_2 = p_0 \). Since the switch of terms of trade was perfectly anticipated, no new information is diffused. The levels of economic aggregates remain unchanged in the neighborhood of time \( T \). The consumption and investment-based real rates of interest increase abruptly to the level of the world interest rate, that is \( r^c(T^+) = r^I(T^+) = r^* \), and remain constant as the economy converges toward the final steady-state. The rise in the marginal utility of wealth measured in terms of the domestic goods induces a fall in the real expense steady-state from the level \( \bar{c}_1 \) to the level \( \bar{c}_2 \). At the same time, the absence of hysteresis effects on production-side aggregates implies that the stock of physical capital must return to its initial level, \( \bar{k}_2 = \bar{k}_0 \). The real income reaches its maximum at time \( T^+ \), that is \( p_2 F[k(T^+), 1] \). The current account becomes unambiguously positive as the terms of trade improve. The economy switches back to a sustainable transition, consistent with intertemporal solvency, that satisfies the unique and overall intertemporal national budget constraint

\[
    b_T - \bar{b}_2 = -p_I (k_T - \bar{k}_2) - \frac{p_c (\sigma + \mu_1)}{\sigma (\mu_1 - r^*)} (s_T - \bar{s}_2),
\]

where the economy’s unstable period 1 transition is contained in stocks \( k_T, s_T, \) and \( b_T \) serving as new initial conditions. According to the intertemporal solvency condition (52), the stocks of habits and of capital goods move respectively in the same and in an opposite direction with the stock of foreign assets.

Since the rate of change in the stock of habits is continuous, that is \( \dot{s}(T^-) = \dot{s}(T^+) > 0 \), the consumption experience keeps on increasing. The adjustment calls for a growth of real expense as households’ preferences exhibit adjacent complementarity. The consumption and capital goods adjustments deteriorate the trade balance by respectively raising absorption and lowering real income. As the fall in \( k \) decreases in absolute value, that is \( \dot{I}(t) > 0 \), the net exports are declining without ambiguity over the stable period 2. Thanks to the inertia effect, the real expense rises slowly and progressively as the level of terms of trade is favorable, a stable transition calls for rising wealth through the accumulation of internationally traded bonds. Since the real income is above its long-run equilibrium value, and the investment is negative, and the real expense is below its steady-state, the current account remains positive along the stable path. Once the small open economy reaches the final steady-state, the stock of equipment goods is restored to its initial level and the market price of capital is equal to the original investment price index. Real expense and the habitual standard of living reach a higher level in the new long-run equilibrium. This result departs markedly from the conclusions of Obstfeld [1983] and Servën [1999] who assume time separable preferences or conclusions obtained by Ikeda and Gombi [1998] who restrict their
analysis to a one-good small open economy model.

The study of a temporary fall in the home goods’ relative price in the present setup allows for re-examining the H-L-M effect, the J-Curve, and the M-L condition. Following an unanticipated transitory terms of trade worsening, the shorter-lasting is the shock, the higher is habit persistence in consumption, and the more important are domestic contents of consumption and investment expenditure, the stronger is the intertemporal speculation effect affecting consumption and investment decisions, and the more likely a short-run current account deficit implying a H-L-M effect. At the opposite of a permanent disturbance, the J-Curve pattern displayed by net foreign assets does no longer rely upon the speeds of adjustment of capital and habit stocks. Instead, whenever the intertemporal speculation effect (coupled with the inertia effect for consumption) and the smoothing effect more than outweigh the wealth effect, the current account deteriorates first triggered by the positive investment flow and the negative saving flow. Once the shock ends, the current account turns to be positive as the signs of saving and investment flows are reversed. In accordance with Leonard and Stockman [2002] empirical results, the surplus phase of the current account is associated with a fall in real income. Finally, the M-L condition holds if the marginal utility of wealth jumps initially and definitively downward, that is if the intertemporal speculation effect driven by the long-run intertemporal elasticity of substitution (under time non-separable preferences) and coupled with an inertia effect predominates. Therefore, the stock of foreign assets reaches a higher level at the new steady-state after an adverse transitory terms of trade shock as the model generates an hysteresis phenomenon.

6 Conclusion

We have revisited the effects of adverse relative price disturbances on the external asset position by allowing for consumption and investment sluggishness originating from habit-forming behavior and capital adjustment costs. This intertemporal general equilibrium approach is particularly appealing since it does not restrict the analysis of the current account-relative price link by overemphasizing the savings reaction or by assigning a prominent role to investment fluctuations due to the specification of time separable preferences.

Our main results may be summarized as follows. First, it is formally shown that the initial current deficit (or H-L-M effect) and the non-monotonic adjustment of net foreign assets (or the J-Curve) depends on the relative strength of consumption habits and investment inertia after an unanticipated permanent terms of trade deterioration. The second main result is probably the most startling. Following an unexpected transitory fall in the home goods’ relative price, both three effects hold if the intertemporal speculation effect coupled with an inertia effect and a smoothing effect more than outweigh the wealth effect. Our interpretation of the J-Curve and the Marshall-Lerner condition in an intertemporal optimizing framework contrasts markedly with the standard view of these phenomena. The non-monotonic behavior of the current account and the long-term rise of the stock of foreign assets result from optimal consumption and investment decisions taken by infinitively-living households and firms instead of competitiveness aspects related to the price-elasticities of demand. Moreover, although we relaxed the time-separability
assumption for preferences, the model displays the zero-root property and generates an hysteresis phenomenon implying the possibility that net foreign assets reach a higher level at the new steady-state. Second, in accordance with Leonard and Stockman’s empirical results, the surplus phase of current account (second period of the J-Curve) is associated with a decline in the real income triggered by the investment decrease. Third, beyond the formalization of Krugman’s economic intuition about the J-Curve phenomenon, the present framework extends Obstfeld [1983] analysis by introducing an habit-forming behavior. It has been shown that real expense adjustment may exhibit a hump-shape response when the adverse transitory disturbance is at work and may eventually reach a higher level at the new steady-state if the shock is not too persistent.

Our model emphasizes [i] the potential importance of combining time non-separable preferences and a Tobin’s q investment theory to explore current account dynamics in a framework allowing for trade in consumption and capital goods and [ii] sheds light on the short-run and the long-run macroeconomic responses by differentiating the effects of relative price changes of different degree of persistence. We believe that the present model predictions may provide some interesting economic lessons regarding recent economic developments. In particular, we argue that this contribution yields valuable economic insights about the macroeconomic effects of an appreciation of the euro, in keeping that the predictions of the theoretical model depend on the degree of persistence of the terms of trade improvement. If the appreciation is long-living, we may expect a reversed J-Curve with a deficit phase (or second phase) associated with a positive investment flow and a rise in the real income, that is the opposite of the standard view (which predicts a fall in real income driven by declining exports). This conclusion holds after a short-lived appreciation of the euro but a long-term decrease in the real expense and in the stock of foreign assets should more probably arise instead of a long-run rise.

The new explanation of the J-Curve phenomenon suggested in this paper calls for new empirical studies which may focus on an extension of the structural model initiated by Glick and Rogoff [1995] to a two-good framework with habit-forming consumers. The introduction of habit persistence in consumption in the Glick and Rogoff discrete-time formalization has been performed by Gruber [2002] and leads to encouraging results. The work needs to be pursued by considering that the consumption and investment expenditures have an import content and by exploring the effects of relative price changes in an empirical way.

The complexity of the model elaborated in the present paper restricts the possible extensions. If consumption durable goods are introduced, the analytical tractability is no longer maintained and a numerical analysis is necessary. A future interesting study would be to incorporate some consumption durable goods by abstracting from an habit-forming behavior. We may expect that the adjustment would be close to the one obtained by assuming distant complementarity in preferences. Although after a terms of trade permanent worsening a sequence deficit-surplus is not possible, the current account may exhibit a J-Curve adjustment (under some conditions) after an unanticipated transitory terms of trade worsening. Finally, a two-country model will be also an interesting way to extend our model as it would endogenize the world interest rate as well as the relative price. Such a formalization has been recently performed by Gombi and Ikeda [2003] by abstracting from capital accumulation in a one-good framework.
Fischer [1993] and Easterly et al. [1993] show that terms of trade variations influence economic growth. According to panel data estimations of Loayza et al. [2000], terms of trade play a key role in explaining saving rates variations. Other empirical works show that a great part of real income, current account and net exports fluctuations can be attributed to terms of trade shocks, particularly for small open economies (see e. g. Fox et al. [2002], Otto [2003]). Using stochastic dynamic general equilibrium models with multi-sectors and recursive preferences, Kose [2002] and Mendoza [1995] show that a great part of national real income and net exports fluctuations may be attributed to relative price shocks.

Number of empirical studies have shown that terms of trade changes have a large temporary component (see e. g. Reinhart and Wickham [1994] for developing countries, Cashin and Mc Dermott [2002] for developed countries) and that the duration of relative price shocks varies widely across countries (see Cashin, Mc Dermott, Patillo [2004]). Surprisingly, the analysis of transitory terms of trade perturbations in intertemporal optimizing continuous time models in an analytical way are scarce, particularly when capital accumulation is considered. When such an investigation is performed, some papers are not very explicit in their solution methods, or contain an inconsistency as emphasized by Schubert and Turnovsky [2002], or report only a numerical analysis (see for example Glenn [1997]). The study of an unanticipated temporary terms of trade deterioration, using a consistent explicit solution procedure, coupled with the specification of time non-separable preferences allows to a new and interesting insight into the consumption-current account relation as we shall see below.

Sen and Turnovsky [1989] incorporates a labor-leisure arbitrage and assumes that investment has zero import content. Serven [1999] considers trade in both consumption and capital goods.

Based on the extension of the Glick and Rogoff [1995] approach, Gruber [2002] has recently shown that the introduction of and suggest to relax the time separability assumption in open economy formalizations.

Our analysis departs also from the Ikeda and Gombi’s analysis in the analytical method applied to study unanticipated transitory terms of trade shocks. This new solution procedure has been recently proposed by Schubert and Turnovsky [2002] which corrects an inconsistency in the preceding solution method initiated by Sen and Turnovsky [1990]. This technical point may seem unimportant but this is not the case. First, the solution procedure has the merit to clarify that when the zero-root property holds, the long-term changes of macroeconomic aggregates are completely determined by the change in the equilibrium value of the marginal utility of wealth. Second, they show that transitional dynamics of aggregates may differ markedly depending on the consistency of the method retained.

Obstfeld [1983] was the first to show the possibility for real expense to rise initially after an unanticipated transitory terms of trade worsening. Once the shock ends, real expense reaches definitively a lower steady-state level. One of the main findings of our contribution is that relaxing the time separability assumption in preferences may lead to opposite long-term responses of real expense. It is formally shown that real expense may be persistently higher at the new steady-state in the present formal setup. The combination of a strong habit persistence in consumption, an intertemporal speculation effect, and an hysteresis phenomenon plays a key role in generating the long-run change in real expense.

The hysteresis phenomenon has been discussed by Giavazzi and Wyplosz [1984] and Sen and Turnovsky [1990].

Almost all formal computations can be retrieved in a larger previous version of this paper available upon request.

The parameter $\sigma \geq 0$ determines the relative weight of real expenditure at different times. Performing the differentiation of (3) with respect to time, one obtains a motion equation of the stock of habits where $\sigma$ is the coefficient (or speed) of adjustment. For example, taking a value of $\sigma = 0.6$ in the line of empirical results of Constantinides [1990], then the time required to close 95% of the discrepancy between $s(t)$ and $c(t)$ by changes in $s(t)$ following a change in $c(t)$ is five years (because $e^{-0.6t} = 0.05$ for $t \approx 4.99$).
The unit cost dual function, \( p_c(\cdot) \), is defined as the minimum total expense in consumption goods, \( z_c \), such that \( c = c(d(t), f(t)) = 1 \), for a given level of terms of trade, \( p \). The minimized unit cost function depends on the terms of trade and is expressed in terms of the foreign good.

The dynamic equation of the foreign assets’ stock is obtained by the substitution of the dividend flow expression, \( D(t) \) (see (14a)).

We refer to \( J_{11} \) and \( J_{22} \) as the submatrices composed respectively by the two first and two last lines and columns of matrix \( J \).

When intertemporal preferences display distant complementarity, inequality (27) is always satisfied. This inequality is necessary to ensure that adjacent complementarity is not too strong and that the dynamic system exhibits a saddle point stability (see Becker and Murphy [1988]).

The left hand side of (35b) is the Volterra derivative applied to the functional (2) expressed in current value and obtained along a constant path (see Ryder and Heal [1973]).

The dependency on initial conditions can be avoided thanks to the assumptions of an endogenous psychological rate of discount (see e. g. Obstfeld [1982], Ikeda [?]), imperfect world capital markets (see e. g. Fisher and Terrell [2000]), finite life (see e. g. Matsuyama [1987]) or a continuum of infinitely lived agents with different birth dates (see e. g. Weil [1989]).

Following Sen and Turnovsky [1989] and Servèn [1999], we assume that the small open economy is a net exporter of the home good, and symmetrically a net importer of the foreign good at the steady-state. This assumption is formalized as follows:

\[
p (\bar{Y} - \bar{d}) = (\bar{f} - r^* \bar{b}) > 0.
\]

Recent empirical studies find strong support of the H-L-M effect (see e. g. Otto [2003]).

To see it, evaluate the stable solution (31b) at \( t = 0 \) letting \( \bar{d} \equiv \bar{k} - k_0 \), and substitute (38b) to obtain

\[
q(0) - \bar{q}_1 = \left( \frac{p I \chi}{\kappa (1)} \right) \frac{F_k (1 - \alpha I)}{p F_{kk}} dp.
\]

From the expression above, the gap between the short run shadow value of capital, \( q(0) \), and its new steady state value, \( \bar{q}_1 = p I (p_1) \), is greater, i. e. the decline in Tobin’s q is larger, the higher is the share of import goods in investment expenditure, \( 1 - \alpha I \).


Business cycle properties documented by Mendoza [1995] and Senhadji [1998] for developed countries and developing countries, more particularly the negative contemporaneous correlation between current account and the relative price of imports, suggest the presence of a H-L-M effect. It finds also a strong empirical support in Otto [2003] by using a structural vector autoregression model and rests on the strength of saving decline with respect to negative investment flow in our framework (see eq (43)).

Mansoorian [1993], [1998] studies exclusively the impact of a permanent terms of trade deterioration by introducing habit formation behavior in an economy without capital accumulation. Sen and Turnovsky [1989] analyze the response of economic aggregates after a temporary terms of trade disturbance by allowing for a labor-leisure choice and capital adjustment costs. They do not consider habit-forming consumers and capital good imports. The framework of Ikeda and Gombi [1998] is relatively close to ours but we allow for two consumption and capital goods. Moreover, the authors apply the solution procedure proposed by Sen and Turnovsky [1990] which contain an inconsistency (see Schubert [2002], chapter 2, and Schubert and Turnovsky [2002]). Finally, our
study differs from Servën’s analysis in retaining dependent intertemporal preferences and in the analytical method we have chosen to study an unanticipated transitory terms of trade shock.

We have retained a formalization of the supply-side analogue to these specified by Gavin [1992] and Servën [1999] who extend Abel and Blanchard’s model to an open economy by allowing international trade in goods and assets. Gavin [1992] studies temporary terms of trade disturbances only graphically and Servën [1999] applies the solution procedure proposed by Buiter [1984].

To clarify this point, remember that the dampening term \(1 - e^{-\chi_2 T}\) is an increasing function of the unstable eigenvalue of \(\chi_2 > 0\) which in turn raises with the derivative of the inverse function \(\kappa'(1)\) evaluated at the steady-state; the value of the latter is greater the smaller are capital installation costs. In conclusion, the term \(1 - e^{-\chi_2 T}\) is a decreasing function of adjustment costs.

See Cardi [2004] for a formal treatment through the application of the two-step analytical approach to the zero-root property case.

The calculation of impact effect on real expense follows from the stable solution of \(c\) prevailing at period 1. All analytical results for temporary terms of trade shocks are available from the author upon request.

As emphasized by Obstfeld [1983], following an unanticipated fall in \(p\) which lasts only a short time period, the transitory low value of the consumption index price, \(p_c(p_1) < p_c(p_2)\), makes an incentive for the households to benefit from the temporary decline of the consumption-based real rate of interest, \(r^c < r^*\).

See for example Servën [1999] who examines the impacts of permanent and transitory terms of trade shocks.

The subscripts “add” and “hab” refer to time separable and time non-separable preferences.

In the long-run, the stock of habits is equal to real expense, that is \(\bar{s} = \bar{c}\). This leads to a long-run intertemporal elasticity of substitution, denoted by \(\eta_{hab}\), equal to

\[
\eta_{hab} = -\frac{\bar{u}_{11} + \frac{\bar{u}_1}{\bar{u}_2}}{\bar{u}_{11} + \frac{\bar{u}_1}{\bar{u}_2}}\frac{\bar{c}}{\bar{c}},
\]

where the superscript \(^\bar{\cdot}\) means that variables are evaluated at the steady-state point. As in Carroll et al. [2000], if the utility is assumed to be of the CRRA form

\[
u(c, s) = \frac{1}{1 - \theta} \left(\frac{c}{s}\right)^{1-\theta},
\]

where \(\theta\) is the coefficient of relative risk aversion and the parameter \(\gamma \in [0, 1]\) the strength of habits, the long-run intertemporal elasticity of substitution reduces to

\[
\eta_{hab} = \frac{1}{\gamma + \theta (1 - \gamma)} > \eta_{add} = \frac{1}{\theta} \text{ if and only if } \theta > 1.
\]

As the cross-partial derivative of the felicity function, \(u_{12}\) is equal to \(-\gamma (1 - \theta) c^{-\theta} h^{-\gamma (1 - \theta) - 1}\), its sign is positive if and only if \(\theta > 1\). This in turn implies that the long-run intertemporal elasticity of substitution under an habit forming-behavior is greater than the intertemporal elasticity of substitution under time separable preferences. The reason for the discrepancy between the long-run horizon elasticities is remarkably explained by Carroll et al. [2000] (see p. 347). For a given rise in the real expense in the long run, the marginal utility of real consumption in the future rises by a lower amount than in the time separable case since a greater habitual standard of living at the new steady state results in a loss in utility, which in turn raises the intertemporal elasticity of substitution.

To see it more formally, differentiate the formal solution (period 1) with respect to time, evaluate at time
\[ t = 0, \text{ and differentiate with respect to } p: \]
\[
\frac{dc(0)}{dp} \bigg|_{\text{temp}} = \mu_1 \left( \frac{\sigma + \mu_1}{\sigma} \right) + \mu_2 \left( \frac{\sigma + \mu_2}{\sigma} \right),
\]
\[
= (\sigma + \mu_1) \frac{dc(0)}{dp} \bigg|_{\text{temp}} + \frac{\mu_1 \mu_2}{\sigma} t_p e^{-\mu_2 T},
\]
where the second expression indicates that households may raise their real expense immediately after the shock (\( c(0) > \bar{c}_0 \)) and decrease \( c \) thereafter (\( \dot{c}(0) < 0 \)); whenever intertemporal preferences display adjacent complementarity, the first and second term on the right hand-side have opposite signs. As the second term is decreasing with the shock's length, we may deduce that a very weak persistent shock may be associated with a temporal decreasing path of real expense.

\[ \text{References} \]


