"Design and control of fault-tolerant permanent magnet drives"

Baudart, François

Abstract
Permanent magnet drives are more and more widely used in high performance actuation systems due to their high efficiency associated to the control possibilities nowadays given by digital and power electronics. However, in aerospace applications, their spread is limited by some risks of failure. The first part of the thesis validates the fact that, with a proper choice of the motor technology and power electronics architecture, the main risk of failure is the loss of feeding of one phase of the motor. Hence the reliability can be substantially improved by making the drive able to palliate such a fault. A thorough analysis of the selected motor architecture (i.e. the segment motor with tooth-coil technology) is also performed. The second part investigates the motor torque control. A general method, based on a Lagrangian formulation, is introduced for determining the optimal currents allowing generating the needed torque, both in normal operation mode and after the loss of one phase. Th...

Document type: Thèse (Dissertation)

Référence bibliographique

Baudart, François. Design and control of fault-tolerant permanent magnet drives. Prom. : Dehez, Bruno ; Matagne, Ernest
DESIGN AND CONTROL OF FAULT-TOLERANT PERMANENT MAGNET DRIVES

PhD research
by
BAUDART François

Examining committee:

<table>
<thead>
<tr>
<th></th>
<th>Institution</th>
</tr>
</thead>
</table>
| Pr. Grégoire Winckelmans | UCL
| Pr. Bruno Dehez         | UCL
| Pr. Ernest Matagne      | UCL
| Pr. Francis Labrique    | UCL
| Dr. Paul Alexandre      | S.A.B.C.A.
| Pr. Christophe Espanet   | UFC (France)
| Pr. Eric Monmasson      | UTC (France)

Chairman
Supervisor
Supervisor

Final version
presented on 22th Augustus 2012
“And yet it moves…”

supposedly said by Galileo Galilei
In the whole thesis, except for these thanks, I will speak as "we", not "I", because I consider that this thesis would not have been possible without the help and support from a number of people I would like to thank before anything else.

First of all, I am deeply grateful to my advisors: Prof. Bruno Dehez, for his boundless and contagious energy and his inquisitive and vivacious mind, Prof. Ernest Matagne, for his tremendous genius and his ability to overcome problems of staggering complexity, and Prof. Francis Labrique, for his regular guidance, encouragement, criticism, faith and sharp intellect. During the four years of my PhD thesis, they have been models for me, professionally and humanely.

I am deeply grateful to Dr. Paul Alexandre from S.A.B.C.A. He gave me the opportunity to enrich the thesis by bringing the space application case. From S.A.B.C.A., I also want to thank Dan Telteu, Jean-Marie Zabus and Gwenaelle Messager, for the close partnership along the whole thesis.

I would like to express my gratitude to Pr. Christophe Espanet of the Université de Franche-Comté, Pr. Eric Monmasson of the Université de Cergy-Pontoise and Pr. Grégoire Winckelmans, for the time spent in the evaluation of my work, and for the advice they have provided me, both in content and in form, to raise the quality of the manuscript.

I would like to thank all my colleagues of the mechatronics division for the great time and the good atmosphere prevailing at the office. To name only one, I want to thank Jonathan Denies, accomplice of every day, with which galvanizing discussions have given rise to many ideas, and that allowed me to use the optimization tools developed as part of his thesis to add a fine stone to building mine. Special thanks to my
chief, Paul Fisette. As a true bandmaster, he is always very attentive to each of his subordinates, and gives the lab a harmonious tone in which it is a pleasure to play its part.

I also owe many thanks to the member of the LACTION lab, Thierry Daras, André Lengelé and Paul Sente, for the unfaible technical support they provided me, from the construction of the prototype to the installation of the experimental testbench. Many thanks also to Marie-Christine Vandingenen, Antoinette Dupont and Ghislaine Gobert for the quality of their administrative support.

I pay homage to the outstanding work done by Quentin Galloy and Laetitia de Viron through their master thesis, and Guillaume Ricour through its internship. They brought valuable contributions to the thesis.

I also wish to express my gratitude to some people from Switzerland, who welcomed me during my stay at the EPFL: the members of LAI lab and their director, Pr. Yves Perriard. They gave me access to their areas of expertise, and the opportunity to participate in the warm atmosphere of their lab. I particularly thank Omar Scaglione, Camille Dubois and Bastien Durel. I could not ask for better people for making this trip so enriching.

And of course, I cannot forget the many people I met, professionally or not, during this thesis, as well as my close friends for the good times together, and my family, parents, brother and sister for their unconditional love. These people are the foundation that made possible the completion of this work. I owe them a lot, and thank them with all my heart.

And eventually, thank you, dear reader, for taking the time to read these lines. I hope you will find the knowledge you seek in the following pages.
Abstract

Permanent magnet drives are more and more widely used in high performance actuation systems due to their high efficiency associated to the control possibilities nowadays given by digital and power electronics. However, in aerospace applications, their spread is limited by some risks of failure.

The first part of the thesis validates the fact that, with a proper choice of the motor technology and power electronics architecture, the main risk of failure is the loss of feeding of one phase of the motor. Hence the reliability can be substantially improved by making the drive able to palliate such a fault. A thorough analysis of the selected motor architecture (i.e. the segment motor with tooth-coil technology) is also performed.

The second part investigates the motor torque control. A general method, based on a Lagrangian formulation, is introduced for determining the optimal currents allowing generating the needed torque, both in normal operation mode and after the loss of one phase. Then, two new controller architectures for controlling the motor currents in the rotor frame are developed, aiming for a minimal reconfiguration when moving into fault-tolerant operation mode.

The third part deals with the optimal design of the motor for a given application: the thrust vector control of a launcher. For this application, the motor has to be designed according to a mission profile and the space environment. For that purpose, an optimization program based on a genetic algorithm is interfaced with an analytical model, coupling magnetic and thermal aspects. This analytical model has been validated by FEM analysis and tests on a prototype designed for that purpose.
Résumé

Les actionneurs électriques à aimants permanents sont de plus en plus utilisés dans les systèmes d’actionnement à hautes performances, grâce à leur grande efficacité et aux possibilités de contrôle qu’offrent à l’heure actuelle l’électronique digitale et l’électronique de puissance. Cependant, pour les applications aérospatiales, leur propagation est limitée par certains risques de panne.

La première partie de la thèse valide le fait que, par un choix judicieux du type de moteur et de l’électronique de puissance, le risque principal de défaillance est la perte d’alimentation d’une phase du moteur. La fiabilité peut dès lors être améliorée en rendant l’actionneur capable de pallier une telle faute. Une étude approfondie du type de moteur sélectionné (le moteur à phases segmentées) est aussi réalisée.

La seconde partie aborde le contrôle en couple du moteur. Une méthode générale basée sur une formulation Lagrangienne, est introduite afin de déterminer les courants optimaux permettant de développer le couple désiré, aussi bien en marche normale qu’après la perte d’une phase. Ensuite, deux nouvelles architectures de contrôle sont présentées, avec l’optique de minimiser la reconfiguration lors du passage en marche dégradée après perte d’une phase.

La troisième partie traite du dimensionnement optimal du moteur, pour une application donnée : le contrôle du vecteur de poussée d’un lanceur spatial. Pour cette application, le moteur a été conçu en tenant compte du profil de mission et de l’environnement spatial. Pour cela, un programme d’optimisation basé sur un algorithme génétique est interfacé avec un modèle analytique couplant aspects thermique et magnétique. Ce modèle analytique a été validé par une analyse éléments finis, ainsi que par des tests sur un prototype conçu dans ce but.
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CHAPTER 1

Context

1.1 The 'More Electric' trend

In aeronautical and aerospace applications, hydraulic actuators used for flight control and other secondary power systems tend to be replaced by electromechanical ones [1–8]. This is known as the 'More Electric' trend\(^1\) and has led in particular to important European Union programs as, for instance, POA (power optimised aircraft), MOET (More Open Electrical Technologies), DRESS (Distributed and Redundant Electro-mechanical nose wheel Steering System), ENFICA-FC (ENvironmentally Friendly Intercity Aircraft powered by Fuel Cells), and so on.

One of the main interest for moving from hydraulic actuators to electromechanical actuators lies in the mass minimization, which is a major design criteria in such applications. Indeed, taking into account the weight of the ancillary equipments linked to the actuators (reservoirs, pipes and hydraulic power generation for hydraulic ones, batteries, power electronics and wiring for electromechanical ones), electromechanical actuation systems are lighter than hydraulic ones and reach a higher efficiency [9].

\(^1\)Also referred more specifically as More Electric Aircraft (MEA) in the aeronautical field.
Another major design criterion for these applications is the reliability. From that point of view, electromechanical actuators are less efficient.

1.2 Reliability issues in electromechanical actuation systems

Even if, as far as we know, no precise statistics are available in the literature, the required reliability level for an actuation system in aeronautics or in aerospace is of order $10^{-10}$ catastrophic failure per hour of mission [10].

The reliability level of electromechanical actuators depends on the reliability of all the parts constituting them. A general schematic view of the parts constituting the electromechanical actuation system and the links between them is shown in Figure 1.1. A synchronous machine ensures the high dynamics needed for actuating the load via a mechanical transmission system. The motor is fed by a power electronics (in most of the case a PWM voltage sourced inverter) taking its power from a DC bus or a battery. An inner control loop regulates the torque developed by the motor by imposing the waveforms of the currents flowing in its phases on the basis of a torque reference and of feedback sensors (phase current sensors and a rotor position sensor). The torque reference comes from an outer loop integrating a position and/or a speed control depending on mission profile.

The digital electronics, the power electronics, the synchronous motor and the sensors are the four parts constituting the actuator. Based on what we can find in the literature, and considering all the possible fault that could appear (a switch of the power electronics undergoes a failure, two turns inside the motor goes into short-circuit, a current sensor is disconnected, ...), they have all a probability of failure which is of order $10^{-6}$ [4,11,12,33].² This is far from the required level of $10^{-10}$.

A solution frequently used to increase the reliability level consists in introducing redundancy in the system. There are three type of redundancy [13];

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²It is not easy to define probabilities of failure of parts constituting an actuator. As the technology differs from an application case to another, it is difficult to get enough data to compute precise statistics.
Reliability issues in electromechanical actuation systems

- Redundancy of the whole system: The actuator and the load are redounded. This is the simplest and safest way to improve the reliability. As the whole system is redounded, there is no link between the two actuation systems, and the reliability level will be of order $10^{-12}$. However, this solution is not always possible, as the load (for instance, an aircraft nose wheel) may not always be redounded.

- Redundancy of the actuator: Two actuators are placed in parallel and each one has its mechanical transmission connected to the same load. In normal operation, either one actuator is active and the other is in standby (passive redundancy) or the two actuators
are active and work in parallel (active redundancy). A clutch system allows to disconnect one actuator when a fault occurs. In the first case, a failure of the active actuator leads to its disconnection and to the connection of the passive one. In the second case, the faulted actuator is disconnected and the still healthy one continues alone the mission.

- Redundancy of the actuator without the mechanical transmission: Two motors with their own control and power electronics are placed in parallel and are associated to the same mechanical transmission via a clutch system [15, 16]. Either passive or active redundancy can still be applied. This type of redundancy is only possible if the mechanical transmission is designed to reach the required level of reliability without the need of using a redundancy.

In any case, if a total or a partial redundancy allows to reach the required reliability level, the mass increase is significant, as we need at least two motors and two power and control electronics (each one sized for the full power) and a clutch system allowing to disconnect the faulted one. In some applications, mainly aerospace applications, this mass increase is not possible. That is where the fault tolerance becomes a good alternative.

1.3 The fault-tolerant ability

The fault tolerance of a system is its ability to pursue its function despite a partial failure. The reliability level can hence be reached by making the actuation system able to alleviate all the type of faults that could appear with a probability higher than the required reliability level. If these faults are dealt before they propagate, then it is not necessary to consider more than one fault at a time. Indeed, the probability that two independent faults occurs in the same mission is of order $10^{-12}$, what is below the required reliability level.

An actuator able to alleviate all these faults is called "fault-tolerant", and runs in three different operation modes:

- **normal operation mode**: The whole actuator is healthy and running properly.
The application case: the thrust vector control of launchers

- **fault operation mode**: A fault occurs and the actuator is not running properly. This state will last the shortest time possible, just the time necessary to localize the faulted part of the actuator, and to trigger the reconfiguration.

- **fault-tolerant operation mode**: The actuator reconfigures itself to isolate the faulted part. The contribution of the remaining healthy part is increased in order to compensate the loss and to pursue the mission. It means that the actuator must be oversized to fulfill the mission despite a non-working part.

It appears that, in an electromechanical actuator, with a proper design of the mechanical parts (bearings, transmission system) [17], of the motor [18–20], and of the control electronics [21], the main hazard of fault comes from the power electronics and the sensors. The sensors can be easily reduded, without implying a significant mass increase, but the power electronics is more critical. However, by using a proper architecture for the power electronic converter, it is possible to limit a fault to the loss of feeding of some phases of the motor [14, 22–30]. The fault-tolerant ability can therefore be given to the actuator if it is able to face the loss of some phases of the motor, and to compensate the loss by modifying the current control of the still active phases.

This is also why polyphase motors are currently gaining interest in the aerospace application field [5, 6, 8, 14, 24, 25, 31, 32]. Indeed, besides the fact that running with a lost phase due to some failure requires a less important oversizing than classical 3-phase motors, the use of polyphase motors offer a greater versatility in the possibilities of enhancing the actuator of fault-tolerant abilities.

### 1.4 The application case: the thrust vector control of launchers

The application, which is part of the ‘More Electric’ trend, that we will follow along the thesis, is the actuation system used for controlling the thrust vector of European launchers [14].

The control of the thrust vector (TVC) allows to control the attitude of the launcher during its rise through the atmosphere. This control is performed by controlling the inclination of the nozzle via two actuators
attached to the skirt of the launcher (Figure 1.2). This system is found at every stage of European launchers such as the Ariane family or VEGA.

![Thrust vector control of a launcher](image)

**Figure 1.2: Thrust vector control of a launcher**

The core of the actuator is depicted in Figure 1.3. It consists in an permanent magnet electrical motor driving a mechanical transmission made of a ball-screw translating the rotation of the motor into a linear motion. The top of the nozzle is attached to the skirt via a spherical transmission (not represented in Figure 1.3). The linear motion of the screw of the first actuator will move the attitude of the nozzle in one direction, and the second actuator will move the nozzle in the perpendicular direction. The simultaneous controls of the two actuator allow to set the pitch and the yaw angle, and hence the trajectory of the launcher.

A specific feature of the TVC application is that the mission time is short (a few minutes to a few tens of minutes, depending on the stage). The main consequence is that the actuator must be designed as a function of a mission profile. It means that the actuator is not designed on the basis of a nominal torque and a nominal speed, but as a function of a maximum torque and a maximum speed *at every moment of the mission*.

Figure 1.4 shows an hypothetical example of mission profile, where a torque amplitude envelope and a speed amplitude envelope are defined as functions of the mission time. We can see four steps in that example:

---

3except for the first stage which requires more power and keeps hydraulic actuators for that reason.
The application case: the thrust vector control of launchers

Figure 1.3: Actuation system

- a first step where the speed is equal to zero: the nozzle is at rest and controlled at a constant attitude, waiting to be activated.

- a second step where the speed and the torque maximum values are simultaneously different from zero. The stage is activated and controls the attitude of the launchers. There is a peak value of the torque amplitude at the beginning of this step, to take into account the firing of the stage.

- a third step where the stage is desactivated and the nozzle is immobilized.

- a fourth step where the stage is reactivated.

The mission profile will have an important impact on the design, as the thermal transient behaviour will have to be considered\(^4\). Also, a flux

\(^4\)Design issues are more deeply discussed in part III of this thesis.
weakening operation mode of the motor is not possible, as the maximum torque value may be needed at the same time that the maximum speed value.

### 1.5 Switched reluctance motor vs permanent magnet motor

The motor type used in aerospace and aeronautical applications is in most of the case a permanent magnet synchronous motor, as it is the motor type with the highest efficiency and power density [36]. However the switched reluctance motor is also considered for aerospace and aeronautical applications, as it is intrinsically more reliable [34, 35], mainly for two reasons:

- Unlike classical permanent magnet motors, the windings of a switched reluctance machine are physically decoupled from each other, and they have a low magnetic coupling (i.e. low mutual inductances) between the phases.
- If two turns of a windings go into short-circuit, the rotation of the rotor will not induce a short-circuit current, as the rotor does not induce electromotive forces in the windings of the stator, unlike permanent magnet motors.
However, these two reasons may be questioned. Indeed:

- The permanent magnet motor that we consider for the application\(^5\), has physically decoupled phases, and a low magnetic coupling, as well as the switched reluctance motor.

- Interturn short-circuits in permanent magnet drives can be avoided with a proper design, in particular by increasing the thickness of the wire insulation. This increase of the thickness will undoubtedly cause a deterioration of the performances of the permanent magnet motor, because heat dissipation is impeded, but still, the permanent magnet drive would be lighter than the switched reluctance one designed for the same application. If interturn short-circuits should be considered anyway, short-circuit current can be limited [37, 38].

In addition to that, the highly non-linear magnetic behaviour of switched reluctance motors limits their performances in terms of precision and ease of control. Problems related to the minimization of torque pulsations and stator vibrations are far from resolved at this time, and requires a refined complex modelization that has to be taken into account in the control [39–42].

Compared to this, permanent magnet motors offer a great versatility of high precision controls, and are therefore more suitable for the development of reliable fault-tolerant reconfigurations. This is why this type of motor is considered in this thesis, and why the switched reluctance motor is discarded.

\(^5\)The motor is detailed in chapter 3.
References


References


References


Switched reluctance motor vs permanent magnet motor


CHAPTER 2

Content and contribution of the thesis

The topic of this thesis focuses on the study of an actuation system used for controlling the thrust vector control of next generation launchers, and especially on the design of fault-tolerant actuation systems using polyphase motors. The contribution is focused on the motor design and on the optimal control of the torque it must develop. The thesis is split into three parts.

Part I: Fault-tolerant motor and associated power converter architectures

The first part is based on a survey of the literature and validates the choice of one motor technology (i.e. the segment motor) and the need, for reliability reasons, to develop power electronics and control architectures allowing to withstand the loss of feeding of one phase resulting from a failure at the power electronics level or at the current sensors level[vii].

Chapter 3 performs a thorough analysis of segment motors. The specificities inherent to the structure of such motors are highlighted.

1 All the references mentioned in this chapter are our publications in the frame of this thesis.
In particular, the relationship between the number of phases and the number of pole pairs is formalized. Then the segment motor is compared with other type of permanent magnet synchronous motors, in order to underline its benefits and drawbacks impacting the reliability and the efficiency. In order to establish a solid base for the third part, devoted to design issues of such motor in space applications, some improvements of the basic structure and the hot spots for the design, such as the importance of the iron losses, are pointed out. Finally, we present the equivalent electrical circuit, necessary for introducing the second part, dedicated to control issues.

Chapter 4 gives an overview of the power electronic architectures that allow the development of fault tolerant abilities. We first present power architectures that are able to remain fully operational in case of an open circuit failure or a short circuit failure of one semiconductor. These power architectures do not require a reconfiguration of the control, and hence no fault tolerant abilities.

However, we also show that if we introduce the possibility of compensate the loss of one phase in open circuit by a reconfiguration of the control, then it is possible to consider power electronics that are twice less bulky than the full fault tolerant architectures. Indeed, firstly, these new family of power electronics does not need to be safe against open circuit failure (as this is managed by the reconfiguration of the control). And secondly, a short-circuit failure has not to be fully managed by the power electronics, but only transformed into an open circuit failure, such that it can be managed by the reconfiguration of the control.

Part II : Control issues

The second part investigates the control issues associated to the motor torque control including the possibility to maintain the torque at its reference value after the loss of feeding of one phase (i.e. to operate in a fault-tolerant mode).

In chapter 5, we first use a Lagrangian formulation for determining the optimal waveforms of the currents which must flow in the motor phases in order to obtain a given electromagnetic torque both in normal operation mode or after the loss of feeding of one phase\([v,ix,xiii]\). The proposed formulation is more general than those found in the literature and allows to solve the problem for motors with any number of
phases and under various constraints such as to impose to the currents to have a sum equal to zero or to have sinusoidal shapes in both modes of operation.

Chapter 6 then investigates how to ensure the control of the motor currents according to the reference values determined in the previous chapter[iv,vi,x-xii]. Two original controller architectures (published in [xi] and [xii]) are introduced. The two controllers are in the rotor frame. The first controller uses one generalized transformations of variables in order to control the phase currents with one multivariable PI controller. The second one uses one transformation for each phase of the machine, in order to control the currents independently, but with identical controllers.

The same philosophy of the fault-tolerant reconfiguration is used in both architectures: the core of the controller is not modified when the reconfiguration from the normal operation mode into a fault-tolerant one occurs. The advantages are that it minimizes the reconfiguration and makes easy the transition from normal to fault-tolerant operation mode. Besides, it allows to “plug” the reconfiguration strategies to any other controller developed for normal operation mode only, but for applications that could require fault-tolerant abilities.

Chapter 7 presents the implementation and the validation of the two controller architectures by simulations and experiments. The experimental results are obtained on a testbench using a motor that has been modified to be reconfigured as a 3-phase machine or a 6-phase machine at will.

Part III : Design issues

The third part of the thesis deals with the design of the motor with the aim of obtaining the best possible design for a given application, in our case the thrust vector control of a launcher. Indeed, for aerospace applications, the motor must not be designed in terms of steady state operation but in terms of the time evolution of its torque and speed during the mission it must perform.

In chapter 8, we introduce a multi-objective design strategy that fully decouples the optimization process, that makes the design evolve, and the evaluation process, that builds the design and analyses its performances on the basis of a analytical model. This model includes both
electromagnetic and thermal aspects. As it is aimed to be coupled with an optimization process, the model has to be fast, but still to be precise enough. Some points related to this model, as the computation of the cogging torque or of the rotor losses, have already given rise to some publications[viii,xiv].

The optimization strategy is based on genetic algorithms developed in the laboratory. For all the dimensional parameters that need to be optimized, the optimization algorithm proposes normalized values, and give them to the evaluation algorithm. The latter translates these normalized values into values for the dimensional parameters, builds the solution and establishes the magnetic and electrical models and the thermal behaviour as a function of the mission profile. Then, it evaluates the solution on the basis of evaluation functions that require to be minimized (mass, harmonic content of the EMFs, ...), and constraints that require to be satisfied (thermal limitation, maximum outer diameter, ...). These evaluations and constraints are given back to the optimization algorithm, and allows it to rank the solutions, to select the best ones, and to manipulate them in order to propose new normalized values for the dimensional parameters to the evaluation algorithm. If this process is repeated a sufficient number of times, it will eventually converge to a set of optimal solutions.

Chapter 9 is dedicated to the validation of the analytical model by comparison with a 2D FEM modelization and experimental tests. For that purpose, a 6-phase prototype has been built in a pre-design process. The validation shows that the model is precise enough to be used in an optimization process.

It is worthy to mention that an industrial partner active in the field of aerospace applications has accepted to participate to the supervision of this thesis in order to allow us to have a realistic approach of the constraints linked to the considered application.
Publications associated to the thesis


Publications associated to the thesis


Fault-tolerant motor and associated power converter architectures
Segment motor basics

3.1 Introduction

Surface mounted permanent magnet synchronous machines can be classified into two families, depending on whether or not they possess overlapping or non-overlapping windings. Classical 3-phase machines, like the one illustrated in Figure 3.1(c), are part of the first family. The second family uses a tooth-coil technology: each coil is wound around one tooth. Figures 3.1(a) and 3.1(b) give the two basic structures of machines using a tooth-coil technology: the segment and the non segment machines. In segment machines only one tooth every other two is wound and each coil fully fills the two slots surrounding the tooth. In non segment machines, each tooth is wound and each coil only fills the half of the slots surrounding the tooth around which it is wound.

The choice of using PM synchronous machines with a tooth coil technology or with a classical overlapping coil technology induces another major difference: the link between the number of slots per phase and the number of rotor pole pairs. For instance, the four pole classic machine illustrated in Figure 3.1(c) uses a double layer configuration of concentrated overlapping windings. Hence each phase fills two slots per pole pair and the total number of slots is equal to the number of poles times
Segment motor basics

(a) 6-phase segment motor
(b) 12-phase non-segment motor
(c) 3-phase classical motor

Figure 3.1: Basic structures of synchronous motors

the number of phases. With four poles, the number of slots is equal to twelve.

This is obviously not the case in machines using a tooth coil technology. With the same number of slots:

- the segment motor shown in Figure 3.1(a) has five pole pairs and six phases, each made of one coil filling two neighboring slots;

- the non segment motor shown in Figure 3.1(b) has twelve phases and five pole pairs, each one made of one coil filling two neighboring half slots.

In this chapter we will first present the basic rule governing the link
between the number of stator slots and the number of pole pairs in machines using a concentrated coil technology by limiting our analysis to the segment motors.\textsuperscript{1} We will after analyze the benefits and the drawbacks of segment motors in comparison with classical ones. We will eventually discuss some improvements and variants of the basic structure of segment motor and conclude the chapter by presenting the equivalent circuit which can be used for modeling these machines from the control point of view.

### 3.2 Description of the structure of segment motors

#### 3.2.1 Basic relation between the number of stator slots and the number of rotor pole pairs

In its simplest configuration, a segment motor has one coil per phase on the stator side. This coil is wound around a tooth and completely fills the two slots surrounding the tooth. Hence the number of teeth (and also slots) is equal to two times the number \( n \) of phases. By considering for the moment this simplest configuration on the stator side, the problem is to find the number \( p \) of rotor pole pairs, such that the pole pairs induce in the \( n \) phases a balanced set of EMFs\textsuperscript{2}, as in classical machines.

It is not mandatory that two successive coils constitute two successive phases from an electrical point of view, i.e. to have an angular mechanical phase shift between two successive phases equal to \( \pm 2\pi/n \). This shift may be equal to \( m2\pi/n \) with \( m \) an integer between 1 and \( n - 1 \). In order to get the same mechanical angular shift between each consecutive phases, \( m \) and \( n \) must be relatively prime, as shown in the examples given in Figures 3.2 and 3.3. For a six phase machine (Figure 3.2), \( m \) may take the values 1 or 5, yielding a mechanical shift between two successive phases of \( \pm \frac{2\pi}{6} \) or \( \pm \frac{5\pi}{6} \). The successive phases are noted \( A, B, C, D, E \) and \( F \). For a five phase machine (Figure 3.3), \( m \) may take

---

\textsuperscript{1}It should be noted that it is not possible to find in the literature a common terminology for referring to the type of machine we are considering. Some authors speak of machines with decoupled phases or machines with non overlapping concentrated windings, or simply of machines with concentrated windings. The last designation is ambiguous, as it is already in use in classic machines in opposition to spread windings.

\textsuperscript{2}electromotive forces
the values 1, 2, 3, 4, yielding a mechanical shift between two successive phases of $\frac{2\pi}{5}$, $\frac{2\pi}{5}$, $\frac{4\pi}{5}$ or $\frac{4\pi}{5}$. The successive phases are noted $A$, $B$, $C$, $D$ and $E$.

![Diagrams of phase arrangements for a 6-phase and a 5-phase motor](image)

**Figure 3.2**: 2 possibilities of phase arrangement for a 6-phase motor ($n = 6$)

**Figure 3.3**: 4 possibilities of phase arrangement for a 5-phase motor ($n = 5$)

It is enough to study the cases for which $m < n/2$. Indeed, with $k < n/2$, for $m = k$ and $m = n - k$ the angular shift between two successive phases is the same provided that, in the second case, the shift is taken in the counterclockwise direction instead of the clockwise direction.

To get a balanced set of induced fluxes in the phases, the number of pole pairs must be chosen such that the electrical phase shift between two consecutive phases is $\pm\frac{2\pi}{n}$. In other words, if phase $i$ is aligned with a rotor pole, phase $i + \frac{n}{1}$ must have a mechanical phase shift of
Description of the structure of segment motors

$\pm \frac{2\pi}{np}$ with the rotor pole of same magnetization which is the closest to it, as shown in Figures 3.4(a) and 3.4(b). The sign of the phase shift defines the rotating direction of the rotor (clockwise or anti clockwise) for which the system of fluxes induced in the successive phases is a direct one. The pole pairs aligned with phase $i$ and the one aligned with phase $i + 1$ with a phase shift of $\pm \frac{2\pi}{np}$ can be separated by any number of pole pairs. We define this number $q$. The angular shift between the pole pairs associated to phases $i$ and $i + 1$ is then equal to $q \frac{2\pi}{p}$.

\[ m \frac{2\pi}{n} \]

\[ q \frac{2\pi}{p} \]

\[ \frac{2\pi}{np} \]

Figure 3.4: Angular shift between the pole pairs associated to phase $i$ and $i + 1$

The condition for obtaining a balanced set of induced fluxes in the phases of segment machines can therefore be written in its mathematical

---

As in a classic machine the system of fluxes induced is a direct one (+) for one direction of rotation of the rotor and an inverse one (-) for the other direction of rotation of the rotor.
form as:

\[ m \frac{2\pi}{n} = q \frac{2\pi}{p} \pm \frac{2\pi}{np} \text{ [rad]} \quad (3.1) \]

The mechanical angular shift between two successive phases \((i\text{ and } i+1)\) must match the angular step corresponding to \(q\) pole pairs plus or minus the positive or negative phase shift of \(\frac{2\pi}{np}\).

By multiplying both side by \(\frac{np}{2\pi}\), equation (3.1) becomes:

\[ mp = qn \pm 1 \quad (3.2) \]

The number of phases \(n\) and the number of pole pairs \(p\) must thus be chosen to fulfill the mathematical condition given by equation (3.2), with \(q\) a positive integer between 0 and \(p\) and \(m\) a positive integer between 1 and \(n/2\). The machine illustrated in Figures 3.1(a) matches the condition by setting \(m\) and \(q\) equal to 1. This setting always allows to satisfy the linking condition, whatever the number of phases \(n\) may be.

Table 3.1 lists the parameters \(n, p, m, q\) of the possible structures of motor fulfilling the linking condition, for 3-phase to 11-phase motors, with 1 to \(2n+1\) rotor pole pairs. A glance at the two first columns highlights the fact that the two parameters \(n\) and \(p\) must always be relatively prime in order to verify the linking condition. In fact, it can be shown that if \(n\) and \(p\) are relatively prime, there is one and only one solution for \(m\) and \(q\) satisfying equation (3.1) with \(m < n/2\) and \(q < p/2\). A short demonstration is detailed in appendix 3.8. The last column indicates if the system of induced EMFs is a direct one (‘+’) or an inverse one (‘-’) for a clockwise rotation of the rotor. Figures 3.5(a), 3.5(b) and 3.5(c) gives some examples of possible structures among those listed in Table 3.1.

### 3.2.2 Structure repetition and phase reversal

To complete the panel of solution, we must introduce the concepts of structure repetition and phase reversal.

The basic structure we have first described can be concentrated on \(\frac{1}{k_{th}}\) of the motor periphery and repeated \(k\) times. To illustrate this, Figure 3.6(a) shows a 3-phase 2-pole pairs structure. This structure is
Description of the structure of segment motors

Table 3.1: Table of the possible structures fulfilling the linking condition

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Figure 3.5: Three of the structures listed in Table 3.1
Description of the structure of segment motors

Figure 3.6: Repetition of a 3-phase motor with 2 pole pairs

Figure 3.7: Phase reversal of the repeated structure

then adapted to fill in one half of the stator periphery as shown in Figure 3.6(b) and is repeated in the other half (Figure 3.6(c)). This structure is a 3-phase machine, with two coils per phases which may be connected in series or in parallel (Figure 3.7).

In particular, a structure with an odd number \( n \) of phases can be repeated two times along the stator periphery. In that case, if the terminals of the \( n \) phases of the repeated structure are inverted, we get in the \( 2n \) windings a balanced set of fluxes which means that the machine can be seen as a \( 2n \) phases machine, without structure repetition. This is illustrated in Figure 3.9. By reversing the terminals of the windings
of the repeated structure, we add a phase shift of $180^\circ$ to the three repeated phases EMFs. Eventually we get a 6-phase 4-pole pair structure. Therefore, a solution with an odd number of phases $n$ and $p$ pole pairs gives a second solution with $2n$ phases and $2p$ rotor pole pairs. The trick of the phase reversal does not work with a machine with an even number of phases because to each phase already corresponds another phase with a phase shift of $180^\circ$.

Table 3.2 gives the set of possible solutions without structure repe-
Description of the structure of segment motors

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Table 3.2: Values of \( k \) as a function of values of \( n \) and \( p \)

The partition of machines with a number of phases ranging from 3 to 12 and a number of pole pairs ranging from 1 to 15.

- Entries set to '1' correspond to values of \( n \) and \( p \) that fulfill the basic rules given by equation (3.2) and already listed in Table 3.1.
- Entries set to '2' add the solutions coming from a structure repetition with a phase reversal.
- Entries set to '*' correspond to values of \( n \) and \( p \) for which a machine does not exist.

3.2.3 Synthesis

The link conditions between \( n \) and \( p \) imply that the number of phases and the number of rotor pole pairs can never be equal. Therefore the stator pole pitch of segment motors\(^4\) does not match the rotor pole pitch, unlike in classical synchronous motor.

- If \( n > p \), the stator pitch is smaller than the rotor one. When a magnet is aligned with a wound tooth, a part of the flux it produces flows in the two adjacent non-wound teeth.

\(^4\)In segment motors the stator pole pitch is equal to the angular distance between two slot openings, as each winding is wound around one teeth.
• If \( n < p \), the stator pitch is greater than the rotor one. Then a part of the field of the two adjacent magnets reduces the flux in the wound tooth.

In both cases, the flux induced by the magnets in the windings is reduced by a factor called the \textit{pitch factor}. Intuitively we understand that we must chose a number of pole pairs close to the number of phases in order to have a pitch factor close to 1. In the literature, most of the segment motors have a number of pole pairs equal to \( n + 1 \) or \( n - 1 \). The choice of one or the other solution depends on criteria such as iron losses, weight, electrical frequency, etc. These criteria, as well as the pitch factor, are discussed in the next section.

### 3.3 Specific features and design issues of segment motors

As said before, the segment motor is a non-overlapping winding motor type. To highlight its advantages and drawbacks, it will be useful to compare it with the other main non overlapping windings motor type: the non-segment motor, shown in Figure 3.1(b); and with the classical motor (i.e. with two layer overlapping windings) shown in Figure 3.1(c).

The benefits and drawbacks of choosing \( n + 1 \) or \( n - 1 \) pole pairs will also be highlighted. The schematic diagrams of the segment motors used for the comparison are represented in Figure 3.10.

#### 3.3.1 Pitch factor

The flux induced by the magnets in the windings is reduced due to the fact that the stator pole pitch does not match the rotor pole pitch, unlike in the classical synchronous machines. The flux reduction is expressed by a factor named the \textit{pitch factor}. This factor has a direct impact on the torque production of the machine.

The value of this factor can be easily computed by assuming a sinusoidal distribution of the magnet field in the air gap. Taking the center of a north pole as origin for the position \( \zeta \) of a point along the air gap in the stator frame (Figure 3.11), the field produced by the magnets at the periphery of the stator in absence of slots can be expressed as:

\[
B_s(\theta_m, \zeta) = B \cos(p(\zeta - \theta_m))
\]  

(3.3)
Specific features and design issues of segment motors

Figure 3.10: 6-phase segment motor: (a) with 5 pole pairs and (b) with 7 pole pairs

Figure 3.11: $\theta_m$ the rotor position with respect to a reference slot and $\zeta$ the position of a point along the air gap

with $B$ the amplitude of the magnets field in the air gap and $\theta_m$ the mechanical angle between the reference magnet and the reference tooth.

With $r_{si}$ the stator inner radius and $L_m$ the motor length, the indu-
Segment motor basics

ced flux in one turn of the windings is given by:

\[ \phi_{\text{seg}} = r_{si} L_m \int_{-\frac{\pi}{2n}}^{\frac{\pi}{2n}} B_s(\theta_m, \zeta) d\zeta \]

\[ = 2r_{si} L_m B_p \sin \left( \frac{p \pi}{n} \right) \cos(p\theta_m) \]  

(3.4)

In the classical synchronous machine, the magnets field is integrated from \(-\frac{\pi}{2p}\) to \(\frac{\pi}{2p}\), and the flux in one turn of the windings is simply:

\[ \phi_{\text{clas}} = 2r_{si} L_m B_p \cos(p\theta_m) \]  

(3.5)

Hence by comparing equation (3.4) with equation (3.5), one may see that the flux of the segment motor is given by the same expression as the expression of the flux of the classical motor times the pitch factor, which is:

\[ k_p = \sin \left( \frac{p \pi}{n} \right) \]  

(3.6)

This factor is always less than 1. Table 3.3 gives the pitch factor for the solutions given in table 3.2. Pitch factors greater than 0.8 are grayed and highlight the fact that the pitch factor is high when the number of pole pairs \(p\) is close to number of phases \(n\). This is also the case for \(p\) close to \(3n\), \(5n\), ... but it implies a higher electrical frequency. It should be noted that:

- the efficiency increases with the number of phases;
- both solutions \(p = n + 1\) and \(p = n - 1\) always give the higher pitch factor and the lowest electrical frequency.

It should be noted that it is also possible to act on the pitch factor by using teeth of unequal width, for instance by making the wound teeth thicker and the non wound teeth thinner, or the opposite. This will be analyzed in section 3.4. It should also be noted that when the airgap field produced by the magnets is non sinusoidal and contains harmonics, the harmonic content of the flux across a phase is also dependent of the pitch factor and may be reduced by giving to the pitch factor a proper value.
3.3.2 Magnetic isolation

The segment motor is the only one with a physical separation of the windings. This configuration yields a low magnetic coupling between phases.

To get an idea of the value of the coupling, one may envisage two basic analytical models. The two models consider an equivalent smooth airgap machine where each phase is represented by a turn at the stator periphery which carries all the phase current. By approximating their relative permeability to 1, the magnets are replaced by air.

### 3.3.2.1 First analytical model

The first model assumes a radial magnetic strength (H) and an infinite iron permeability. Figure 3.12(a) gives the schematic diagram of this asymptotic model. This yields for the airgap field produced by one phase the distribution represented in Figure 3.12(b).

Outside the coil area the field generated by the current flowing into it is uniformly distributed along the airgap area. The following relationship is obtained between the airgap field $H_1$ inside the coil area and the airgap field $H_2$ outside this area:

$$H_2 = -\frac{1}{2n-1}H_1$$  \hspace{1cm} (3.7)

As the field $H_2$ is in front of the other phases, the ratio of the mutual
Segment motor basics

\[ H_1 \]
\[ H_2 \]
\[ \theta_{m0} \]
\[ \pi/n \]
\[ M \]
\[ L \]
\[ n \]
\[ \frac{M}{L} = \frac{H_2}{H_1} = -\frac{1}{2n-1} \] (3.8)

The greater the number of phases, the smaller the ratio. Furthermore with this model each phase has the same mutual inductance with all the other ones. This will be used in section 3.5 for building the equivalent lumped parameter model of segment motors.

3.3.2.2 Second analytical model

The second basic analytical model take into account the fact that due to the presence of the surface mounted permanent magnets the equivalent effective air gap is thick and that a significative part of the flux path does not cross the airgap. Considering the extreme case represented in Figure 3.13 where the flux of the coil goes into the two adjacent non-wound teeth, the mutual inductance is equal to 0.

3.3.2.3 Synthesis

The two analytical models given above are limit cases. The real value of the ratio \( M/L \) lies somewhere between 0 and the value given by equation (3.8) depending among others on the thickness of the effective airgap (the
Specific features and design issues of segment motors

Figure 3.13: Field distribution according to the second model

physical airgap plus the thickness of the magnets). With a big effective airgap the second model is more realistic while it is the opposite when the effective airgap is small. Indeed, literature gives results between the two scenarios: [1] gives a $M/L$ ratio of $-3.5\%$ for a 4-phase motor and [2] a ratio of $-4.4\%$ for a 5-phase one while equation (3.8) estimate these values to $-14.3\%$ and $-11.1\%$ respectively.

This is not the case with non-segmented and classical motors. Those two types of motors mutual inductance are much higher, because there are no non-wound teeth. Even the part of the flux which does not cross the airgap goes through another phase.

### 3.3.3 Electrical isolation

Electrical discharge is one severe cause of failure in aeronautical applications. At high altitude the apparition of a corona effect is facilitated by the low pressure. The windings of a motor in mission in that environment can more easily reach the requisite electrical potential gradient to create the ionization of the air and electric arcs occurs. Those arcs can appear between two coils or between one coil and the motor frame. In addition to the electrical perturbation of the system, they can lead to an irreversible degradation of the coil insulation and to a short circuit, which is a severe failure.

In segment motor, the physical isolation of the windings reduces the
probability of an arcing over two distinct phases. The failure is limited in one phase at most. Besides, increasing the number of phases fractionates the power per phases and hence the electrical gradient.

3.3.4 Thermal isolation

We could also expect a thermal isolation of the phases. Even if the stator yoke has a high thermal conductivity a short circuit failure can lead to a sharp elevation of the temperature locally, as the physical isolation reduces heat exchanges between the windings. The source of heating and the faulted phase should therefore be more easily determined.

3.3.5 Slot fill factor

The slot fill factor expresses the ratio of the cross section of copper in a slot on the cross section of the slot itself. With the same winding technique, the segment motor has a higher slot fill factor than the non segment and the classical ones because each slot holds only one coil. Non-segment motors require a higher insulation because they divide the slots to hold coils of two phases. Classical motors using a double layer configuration, as shown in figure 3.10(a) need a top-bottom splitting of the slots. In [3] the following comparative values of the slot fill factor are given:

<table>
<thead>
<tr>
<th>Motor type</th>
<th>Slot fill factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>segmented</td>
<td>0.5</td>
</tr>
<tr>
<td>non-segmented</td>
<td>0.46</td>
</tr>
<tr>
<td>classical</td>
<td>0.35</td>
</tr>
</tbody>
</table>

The interest of increasing the slot fill factor lies in two main points:

- either an increase of the current which may flow into the slots at constant slot size and constant Joule’s losses, as the increase of the filling factor increases the copper surface or at constant copper surface and slot current (and hence also constant Joule’s losses) a reduction of the slot size and hence of the stator yoke. This will be further discussed in section 3.3.7;

- a higher heat dissipation, as the thermal conductivity increases with the reduction of the volume of impregnation resin inside the slot;
Another advantage of non-overlapping windings in regards with overlapping ones is that they can be preformed or prepressed to enhance the slot fill factor. Appendix 3.7.1 gives more details about the use of preformed and prepressed windings. Some authors indicate that they can reach slot fill factors of nearly 80% with these techniques. However they have an important impact on the manufacturing of the motor.

3.3.6 Winding heads

In segment machines (as well as in non segment ones), the winding heads are much shorter than in classical machines. Firstly the distance between the two slots hosting a winding is much smaller in segment machines. For instance, for the 12 slot machines shown in figure 3.1, the length of the winding heads for the 6-phase segment machine is corresponding to one sixth of the stator periphery instead of a fourth for the classic one. Secondly in classic machines each winding head must be shaped in order to cross the other ones, what increases significantly their length.

3.3.7 Joule’s losses

As said in section 3.3.1 the pitch factor $k_p$ reduces the flux induced in windings of segment motors in comparison with the flux of classical motors (see equation 3.6). The phase current in segment motors must be increased by a factor $1/k_p$ to reach the same torque. With the same copper surface per slot in both machines, this increases the current density in the windings by a factor $1/k_p$ in comparison with the classical motor.

But this does not mean that the Joule’s losses are increased by a factor $1/k_p^2$. Indeed as these Joule’s losses are proportional to the square of the current density time the total volume of copper, the increase in current density is counterbalanced by a reduction of the volume of copper due to shorter winding heads.

Furthermore, in segment motors the slot fill factor is higher than in classical ones (section 3.3.5). This means that for the same copper section per slot, slots are smaller in segment motors than in classic ones. On the contrary, for the same slot surface, the copper section will be greater in a segment motor. Assuming that it is possible to keep the same current density as in a classic motor by increasing the copper section by a factor $1/k_p$ without increasing the slot size above that of a classic motor, the Joule’s losses will be increased by a factor $1/k_p$ instead
of \(1/k_p^2\) for a segment motor with the same copper section as a classic one. Indeed an increase of the copper section per slot by a factor \(1/k_p\) means a proportional increase of the total volume of copper and hence of the losses. Nevertheless the total volume of copper in a segment motor, with a copper section per slot \(1/k_p\) greater than in a classic one, may remain smaller than that of the classic motor as the increase in section may be compensated by the shortening of the winding heads.

Depending on the dimensions of the motor Joule’s losses can be higher or smaller in a segment motor. An elongated motor will preferably be a classical one while a discoid one would reach less Joule’s losses if it is a segment one.

### 3.3.8 Yoke thickness

For the same number of stator slots, segment and non segment motors have a greater number of pole pairs than classical ones, as it is shown for instance in Figure 3.1. For motors with 12 slots, the segment motor (Figure 3.1(a)) has 5 pole pairs while the classical one (Figure 3.1(c)) has only 2 pole pairs. Therefore for the same magnet thickness and hence the same airgap field produced by the magnets the maximum value of the flux in the yokes is smaller for a segment motor. Consequently the rotor and stator yokes may be thinner in a segment motor.

If the comparison was made for machines with the same number of pole pairs, the classical machine should have a much greater number of slots (for instance 30 for a machine with 5 pole pairs). The slot width would be strongly reduced and hence the filling factor as the thickness of the isolation material placed between the coil and the wall of the slot remains the same. Therefore for having the same copper surface per phase, the height of the slots should be strongly increased and also the diameter of the stator yoke as well as the teeth height.

### 3.3.9 Magnetic losses

Field variations induced by the motion of the rotor magnets and by the time variation of the stator currents induce hysteresis and eddy current losses in the stator and rotor yokes and in the rotor magnets.

On the stator side, in segment and non segment motors, the iron losses per unit of volume are bigger, as these motors require more pole pairs than classical motors. Indeed the losses increase with the electrical
Specific features and design issues of segment motors

frequency, which is proportional to the number of rotor pole pairs. But this increase is partly compensated by the fact that the stator yoke is thinner in segment and non-segment motors. Of course, a segment machine with \( n + 1 \) pole pairs has more losses than a machine with \( n - 1 \).

On the rotor side, if we neglect the magnet field variation due to the alternation of slots and teeth on the stator side, the losses due to the magnets are equal to zero. This is true for each type of motor.

Rotor losses come from the variation of the flux induced in the rotor by the currents flowing in the stator windings. For each type of motor, a part of the flux path remains in the air gap and do not interfere with the rotor. But another part goes through the magnets and through the iron rotor yoke and generates losses on the rotor side. The harmonic content of the MMF generated by the stator currents determines the importance of the rotor losses. Indeed the fundamental component of this MMF generates an airgap field which rotates at the same speed as the rotor and produces no flux variation while the harmonics generate airgap fields with a speed with respect to the rotor which increase with their rank. Hence the more important the harmonic content of armature reaction field, the more important the rotor losses. As shown in Figure 3.14, the segment motor has the greatest harmonic content, the classical the lowest.

The rotor losses must be carefully analyzed as they may cause a significative rise of temperature in the magnets, with an impact on their permanent magnetization if the temperature becomes close to the Curie point. Appropriate design processes have to be selected in order to avoid that. For example, magnets can be fragmented or partially fragmented [7]. This scarcely reduces the flux amplitude but highly reduces the losses in the magnets.

An analytical model of the losses in the magnets is presented in reference [8]. The model shows that, due in particular to the fact that a segment motor has more pole pairs and magnets small than classical ones, these losses are almost negligible.

3.3.10 Torque to mass ratio

In addition to the reliability, the torque to mass ratio of the actuator is the most important factor to maximize in aerospace applications. Hence the rightful question is: which motor type gives the best ratio?
Figure 3.14: MMF of the three type of motor
Specific features and design issues of segment motors

On the one hand, the comparison of segment motors with non-segment motors shows that the difference between these two types of motor are few. [9] shows that the main difference lies in the values of the mutual inductance, which are greater in the non-segment motor.

On the other hand, there are a lot of differences with the classical machine. The most important ones are:

- the number of rotor pole pairs which is much higher in a segment motor for a given number of stator slots;
- the length of the winding heads which are much shorter in a segment motor;
- the slot fill factor which is greater in segment motors;
- the pitch factor which is lower than 1 in segment motors.

Therefore in a segment motor in comparison with a classical one, the high number of rotor pole allows to significantly reduce the thickness of the stator and rotor yokes and thus the iron mass (as said in section 3.3.8). This reduction is still increased by the fact that the better slot fill factor allows to decrease the surface of the slots without decreasing the copper surface and hence to decrease the height of the slots and thus of the teeth. Besides, the reduced length of the winding heads reduces the mass of the windings for the same copper section per slot.

The resulting mass decrease is partly counterbalanced in terms of torque to mass ratio by the fact that the phase currents contribute to the torque production in a less efficient way due to the value of the pitch factor $k_p$ which is lower than 1. Therefore to obtain the same torque the currents in the segment motor must be increased by a factor $1/k_p$, yielding a variation of the Joule’s losses equal to $1/k_p^2$ times the mass reduction of the copper. If the copper mass reduction is not high enough it is possible that the currents may not be increased by the factor $1/k_p$, resulting in a decrease of the maximum torque the motor can develop.

Similarly the higher electrical frequency of the segment motor increases the losses in the yokes and the magnets so that even if the total iron mass is lower the losses may be greater imposing a further reduction of the currents in order to keep unchanged the total losses. Furthermore the increase of the iron losses with the frequency is not an important
### Motor Type Basics

<table>
<thead>
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<th>Motor type</th>
<th>LCM</th>
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<tr>
<td>segmented</td>
<td>60</td>
</tr>
<tr>
<td>non-segmented</td>
<td>60</td>
</tr>
<tr>
<td>classical</td>
<td>12</td>
</tr>
</tbody>
</table>

Table 3.4: The first non-zero harmonic of the cogging torque of the motors illustrated in Figure 3.1

This issue in direct drive applications as in this case the motor speed and hence the electrical frequency remain small.

Eventually for motors in the power range corresponding to our application, a gain in torque to mass ratio of about 10 to 20% may be expected. [2, 13–16]

#### 3.3.11 Cogging torque

The slot-tooth alternations create a variation of reluctance from the magnets point of view, and lead to an undesired oscillating torque. If not negligible, this torque can be critical for the control system, as it disturbs the magnetic torque and can hardly be compensated due to its high dynamic.

The cogging torque can be written as a sum of harmonics. The first non-zero harmonic is given by the Least Common Multiple (LCM) of the poles and slots numbers. Hence the higher the LCM the weaker the cogging torque [3]. Segment motors have a high number of poles and a number of slot which is close without being equal, which yields a high LCM and a low cogging torque.

As shown in Table 3.4, the segment motor shown in figure 3.1(a) and the non-segment one shown in Figure 3.1(b) have a LCM value of 60. The value is obviously the same as it does not depend on the winding configuration. The classic motor shown in Figure 3.1(c) has a LCM value of 12 and a higher cogging torque is therefore expected.

A segment motor with \( n + 1 \) pole pairs has always a greater LCM than the same one with \( n - 1 \) pole pairs, and expect a lower cogging torque. For example, the 6 phase 7 pole pairs motor shown in Figure 3.10(b) has a LCM equal to 84, which is higher than the LCM of 60 obtained by the 6 phase 5 pole pairs segment motor.

A detailed analysis of the cogging torque is performed in reference
Specific features and design issues of segment motors

[10] and confirms that this torque is much smaller in segment motors than in classical ones.

3.3.12 Radial forces

The electromagnetic interaction between the stator and the rotor can be separated in radial effects and tangential effects. The electromagnetic torque results from the tangential effects.

The radial effects can be expressed by the constraint field [11]:

\[
\sigma_r = \mu_0 \frac{1}{2} \left( H_r^2 - H_t^2 \right)
\]

with \( H_r \) and \( H_t \) the radial and tangential components in the air gap of the magnetization \( H \) due to the magnets and the stator currents. This constraint field along the rotor periphery is the radial force density and can be decomposed into modes. Figure 3.15 shows the four first modes and accentuates the resulting rotor deformations.

![Figure 3.15: The four first modes of rotor deformations](image)

Mode 1 is the only one critical because it is the only mode with a resulting force different from 0. This resulting force, called the unbalanced magnetic pull [12] can be dangerous if the speed of the motor coincides with a natural mode of vibration of the motor. This unbalanced magnetic pull also causes an accelerate aging of the bearings and vibrations which cause a noise, but those two drawbacks are not relevant for our application.

Radial force density modes depend on the number of slots and the number of poles. The only non-zero modes are the multiple of the Greatest Common Divisor (GCD) of the slot and pole numbers.
Segment motors have always an even number of poles and of slots. With a balanced set of currents, mode 1 is always equal to 0. In machines using the rule $p = n \pm 1$, the first mode is always mode 2. The repetition of the structure yields to a higher GCD and hence to less modes not equal to zero.

On the contrary non-segment motors can have an odd number of slot, and in that case can be subject to the unbalanced magnetic pull, as shown in the example of Figure 3.16(a). A structure with repetition has of course a first mode equal to zero. Figure 3.16(b) shows a non-segment machine with 72 slots and 60 poles, which is the version repeated 6 times of the non-segment motor shown in figure 3.1(b). The first non-zero mode is the twelfth.

Classical machines have always a number of slots which is a multiple of the number of pole pairs. The first mode is therefore the mode equal to the number of poles.

In any cases, after a failure such a disconnection of a phase of the machine, the MMF distribution in the airgap is no more balanced. In that fault mode, mode 1 can reappear for any type of motor and must be carefully considered.
Variants and improvements on the basic structure

3.4 Variants and improvements on the basic structure

3.4.1 Stator teeth widths and magnet width adaptation

Section 3.3.1 highlighted the reduction of the EMF amplitude in segment machines because the stator pitch do not match the rotor pitch. The rotor pitch cannot be modified, as the distribution of magnets has to be regular to induce a balanced set of EMFs in the stator windings. On the contrary, the stator pitch can be modified by adopting teeth with irregular widths and consequently an irregular distribution of the slots. If wound teeth are thicker, the non-wound ones must be narrowed so that the section of the stator attributed to one phase is kept unchanged. Tinner wound teeth may also be used with larger non-wound teeth.

The stator pitch of the teeth supporting the windings can therefore be equal to the rotor one. Wound teeth are widened if the number of rotor pole pairs is smaller than the number of phases, and narrowed in the other case. In the two cases the pitch factor of the wound teeth is brought to 1 as in classical motors. An example of stator pitch adaptation is shown in Figure 3.17 for a 6 phase machine with 5 pole pairs: wound teeth are thicker.

Figure 3.17: Stator pitch adaptation
If we may not modify the rotor pitch as said before, we may change the magnet width within the rotor pitch. Figure 3.18 shows the two adaptations: the teeth widths adaptation and the magnet width adap-
Variants and improvements on the basic structure

Parameters $\nu$ and $\tau$ are introduced for adapting the teeth width and the rotor magnet width respectively. On the rotor side, $\tau$ is limited between 0 and 1. For $\tau = 1$, the magnet width is equal to the rotor pitch.

On stator side:

- if $\nu = 1$, all teeth have equal width, as shown in the example of Figure 3.18(a), where $\tau$ is also equal to 1;
- if $\nu > 1$, wound teeth are wider, as shown in the example of Figure 3.18(b), where $\tau$ is smaller than 1;
- if $\nu < 1$, non-wound teeth are wider, as shown in the example of Figure 3.18(c).

The two adaptations modify the harmonic content of the EMFs induced in the stator phases. To get an idea of the modification of the harmonic content, we will suppose the magnet field ($B_e$) radial in the airgap:

$$B_e(\theta_m, \zeta) = \begin{cases} B & \forall \zeta \in \left[-\frac{\pi}{2p}, \frac{\pi}{2p}\right] + k\frac{2\pi}{p} + \theta_m, \\ 0 & \forall \zeta \in \left[\frac{\pi}{2p}, \frac{(2-\tau)\pi}{2p}\right] + k\frac{\pi}{p} + \theta_m, \\ -B & \forall \zeta \in \left[\frac{(2-\tau)\pi}{2p}, \frac{(2+\tau)\pi}{2p}\right] + k\frac{2\pi}{p} + \theta_m. \end{cases} \quad (3.10)$$

with $k \in \mathbb{N}$. $\theta_m$ is the angular position of a reference rotor pole regarding the reference stator wound tooth, as it is shown in Figure 3.18(a) and 3.18(b). $B$ is the field amplitude and $\zeta$ the polar coordinate of a point in the airgap. The field is shown in Figure 3.19.

Using Fourier series expansions, one gets:

$$B_e(\theta_m, \zeta) = \sum_{k=1}^{\infty} \frac{4}{\pi k} B \sin\left(k\frac{\tau \pi}{2}\right) \cos(kp(\zeta - \theta_m)) \quad (3.11)$$

with $k \in 2\mathbb{N} - 1$.

By introducing equation (3.11) in equation (3.4), the expression of
the flux induced by magnets in phase A becomes equal to\(^5\):

\[
\phi_{seg} = n_{sp} r_{si} L_m \int_{-\pi/2}^{\pi/2} B_e(\theta_m, \zeta) d\zeta
= n_{sp} r_{si} L_m \sum_{k=1}^{\infty} \frac{4}{\pi k} B \sin\left(\frac{k \tau \pi}{2}\right) \sin\left(\frac{kp \nu \pi}{2n}\right) \cos(k \theta_m)
\]  

(3.12)
with \(k \in 2N - 1\), \(r_{si}\) the stator inner radius and \(L_a\) the motor active length. The Fourier series expansion of the EMF in phase A is given by the expression:

\[
e_{seg} = -\frac{d\phi_{seg}}{dt} = -\frac{d\theta_m}{dt} \frac{d\phi_{seg}}{d\theta_m}
= \frac{d\theta_m}{dt} n_{sp} r_{si} L_m \sum_{k=1}^{\infty} \frac{8}{\pi k} B \sin\left(\frac{k \tau \pi}{2}\right) \sin\left(\frac{kp \nu \pi}{2n}\right) \sin(k \theta_m)
\]

(3.13)

Parameters \(\nu\) and \(\tau\) appear each one in one factor of the expression of the EMF. The amplitude of the fundamental is maximized if:

\[
\sin\left(\frac{p \nu \pi}{2n}\right) = 1 \quad \Rightarrow \quad \nu = \frac{n}{p}
\]

(3.14)

\[
\sin\left(\frac{\tau \pi}{2}\right) = 1 \quad \Rightarrow \quad \tau = 1
\]

(3.15)

As shown in Figure 3.20, equation (3.15) indicates that the magnet width must be maximized by taking a magnet width equal to the rotor

\(^5\)We only consider phase A as the fluxes in the other phases are identical with a phase shift of \((i - 1)\frac{2\pi}{n}\), with \((2 < i < n)\).
Variants and improvements on the basic structure

pitch. Equation (3.14) indicates that the stator pitch must be adapted to the rotor one.

\[
\theta_m = \frac{\pi}{2p}
\]

Figure 3.20: Adapted structure maximizing the EMF amplitude

The solution which maximizes the amplitude of the EMF also maximizes its harmonic content. Values of \( \tau \) and \( \nu \) can also be chosen in order to suppress some families of harmonic (i.e. an harmonic and its multiples) or to minimize the RMS value of a given number of harmonics.

3.4.1.1 Elimination of one family of harmonics

According to (3.13), harmonic \( k \) and harmonics multiple of \( k \) will be eliminated if:

\[
\sin \left( k \frac{\pi \tau}{2} \right) = 0 \tag{3.16}
\]

or if

\[
\sin \left( kp \frac{\nu \pi}{2n} \right) = 0 \tag{3.17}
\]

Acting on the value of \( \tau \) or on the value of \( \nu \frac{p}{n} \) will have the same effect and will modify in the same way the fundamental component. It is obvious that independently of suppressing the \( k^{th} \) family of harmonics, the maximization of the fundamental component of the EMFs remain an objective. So if we choose to act on \( \tau \) for performing the harmonic elimination, the value of the other parameter, \( \nu \frac{p}{n} \), must be equal to 1 for maximizing the amplitude of the fundamental component of the
EMFs. Similarly among the possible values of \( \tau \) solution of 3.16, we must chose the value which maximizes \( \sin \left( k \frac{\tau \pi}{2} \right) \) again for maximizing the fundamental component of the EMFs. This yields for \( \tau \) the following value

\[
\tau = 1 - \frac{1}{k}
\]  

(3.18)

Hence:

- \( \tau = 2/3 \) deletes family 3
- \( \tau = 4/5 \) deletes family 5
- \( \tau = 6/7 \) deletes family 7

For example the harmonic 3 family disappears with \( \tau = 0.67 \), as shown in Figure 3.21. That choice reduces the amplitude of the fundamental by a factor 0.87.

![Figure 3.21: Airgap magnet field annuling harmonic 3 and multiples](image)

It is obvious that setting:

\[
\tau = 1 \quad \& \quad \nu \frac{p}{n} = 1 - \frac{1}{k}
\]

(3.19)

will give the same result in terms of amplitude of fundamental component and harmonic elimination. However, \( \nu \frac{p}{n} \) can also be greater than 1 and choosing \( 1 + 1/k \) gives the same fundamental and the same harmonic content than \( 1 - 1/k \). One must choose the solution that limits
the modification of the teeth pitch, i.e. the solution that keeps $\nu$ close to 1. It yields:

$$\tau = 1 \quad \& \quad \frac{\nu_p}{n} = 1 + \frac{1}{k} \quad (3.20)$$

if $p > n$, and

$$\tau = 1 \quad \& \quad \frac{\nu_p}{n} = 1 - \frac{1}{k} \quad (3.21)$$

if $p < n$. For examples, for suppressing harmonic 3:

- a 6-phase 5 pole pairs motor requires $\nu$ equal to 0.8 (i.e. smaller wound teeth);
- a 6-phase 7 pole pairs requires $\nu$ equal to 1.14 (i.e. wider wound teeth).
- a 6-phase 4 pole pairs requires $\nu$ equal to 1 (i.e. a regular structure).

The last example highlights the fact that some regular structures suppress an harmonic family with $\nu = 1$. Another point to notice is that in the three examples, the pitch factor is equal to 0.87. Indeed parameter $\nu$ affects the pitch factor and therefore the amplitude of the fundamental.

From a technological point of view its better to reduce the magnet width than acting on wound teeth pitch. Indeed the model of a radial airgap field used for this analyses overvalue the contribution of the left and right ends of the magnet to the EMFs waveform. With a more accurate model, one would see that the air gap field in a region where a north pole is close to a south one is mainly tangential and confined in the airgap. Therefore it is preferable to reduce $\tau$ and to keep $\frac{\nu_p}{n} = 1$.

### 3.4.1.2 Elimination of two families of harmonics

Families $k$ and $l$ will be eliminated by selecting for instance $\tau$ and $\frac{\nu_p}{n}$ such as:

$$\sin \left( k \frac{\tau \pi}{2} \right) = 0 \quad (3.22)$$
and

\[ \sin \left( l \frac{\nu \pi}{2n} \right) = 0 \]  

(3.23)

Hence:

- \( \tau = 2/3 \) and \( \nu_n^p = 4/5 \) or \( 6/5 \) delete family 3 and 5;
- \( \tau = 4/5 \) and \( \nu_n^p = 2/3 \) or \( 4/3 \) delete family 3 and 5;
- \( \tau = 2/3 \) and \( \nu_n^p = 6/7 \) or \( 8/7 \) delete family 3 and 7;
- \( \tau = 6/7 \) and \( \nu_n^p = 2/3 \) or \( 4/3 \) delete family 3 and 7;
- \( \tau = 4/5 \) and \( \nu_n^p = 6/7 \) or \( 8/7 \) delete family 5 and 7;
- \( \tau = 6/7 \) and \( \nu_n^p = 4/5 \) or \( 6/5 \) delete family 5 and 7;

### 3.4.1.3 Best trade off method

Instead of choosing \( \tau \) and \( \nu_n^p \) in order to eliminate one family or two families of harmonics, they may be chosen as the best compromises between the maximization of the fundamental and the minimization of the harmonic content. In order to quantify those two criterion, we define the reduction factor of the fundamental as:

\[ Frf = \sin \left( \tau \frac{\pi}{2} \right) \sin \left( \nu_n^p \frac{\pi}{2} \right) \]  

(3.24)

and we evaluate the reduction of the harmonic content by the following factor upon the 99 first harmonics:

\[ Hrf = \frac{\sum_{k=3}^{99} \frac{1}{k} \left| \sin \left( k \tau \frac{\pi}{2} \right) \sin \left( k \nu_n^p \frac{\pi}{2} \right) \right|}{\sum_{k=3}^{99} \frac{1}{k}} \]  

(3.25)

Both \( Hrf \) and \( Frf \) are equal to 1 if \( \tau = \nu_n^p = 1 \). By looking for the best compromises between maximising \( Frf \) and minimising \( Hrf \) inside the domain of validity of \( \tau \) and \( \nu_n^p \), we find three supplementary solutions:

- \( \tau = 2/3 \) and \( \nu_n^p = 5/6 \) delete family 3 and choose the best compromise between keeping a high fundamental and reducing families 5 and 7;
Variants and improvements on the basic structure

<table>
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<th>#</th>
<th>$\tau$ or $\nu_{n}^{P}$</th>
<th>$Fr_f$</th>
<th>$Hrf$</th>
<th>$H3$</th>
<th>$H5$</th>
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<td>0.94</td>
<td>0.43</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>9</td>
<td>6/7</td>
<td>5/7</td>
<td>0.88</td>
<td>0.30</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 3.5: 9 best solutions

- $\tau = 4/5$ and $\nu_{n}^{P} = 9/10$ delete family 5 and choose the best compromise between keeping a high fundamental and reducing families 3 and 7;

- $\tau = 6/7$ and $\nu_{n}^{P} = 5/7$ delete family 7 and choose the best compromise between keeping a high fundamental and reducing families 3 and 5;

Of course, we may still interchange the values of $\tau$ and $\nu_{n}^{P}$, and choose the solution $1 + \frac{1}{k}$ instead of $1 - \frac{1}{k}$ for $\nu_{n}^{P}$.

3.4.1.4 Synthesis

The whole set of solutions is listed in Table 3.5. For simplicity, only the solutions with $\nu_{n}^{P} < 1$ are listed. The table also gives the value obtained for $Fr_f$ and $Hrf$. The last three columns describe actions taken for the three main families. '/' means that no action is taken, '0' means suppression and '-' means minimisation.

The first solution is the one maximizing the amplitude of the fundamental: the magnet width is equal to the rotor pole width and the stator pitch is equal to the rotor pitch. The three following solutions eliminate one family of harmonic, the three next ones eliminate two families and the three last ones are found by the best trade off method. As we can see, all the solutions get a $Fr_f$ higher than 0.8 while their $Hrf$ vary
Figure 3.22: Comparison of the 9 best solutions

from 0.3 to nearly 0.7. The spectra of the solutions are given in Figure 3.23.

Figure 3.22 illustrates the solutions in a plot as a function of the two criterion, $Frf$ on the $x$-axis and $Hrf$ on the $y$-axis. The unrealistic sinusoidal solution is also represented (with $Frf = 1$ and $Hrf = 0$), as it is the solution we ideally want to reach. Solution 0 is the one maximising the fundamental and the harmonic content (with $Frf = 1$ and $Hrf = 1$). As we can see, the other solutions lies between them in two front. One front is made of the three solutions cancelling one family of harmonics. The second one is made of the solutions cancelling two families of harmonics and of the solutions seeking the best trade off. Those latter solutions are more efficient, as $Hrf$ is quite lower while $Frf$ is kept high (higher than 0.8). A high harmonic reduction can be reached without harshly deteriorating the fundamental.

To sum up, the two parameters $\nu$ and $\tau$ can be chosen aiming for two objectives. On the one hand they can be chosen in order to maximize
Variants and improvements on the basic structure

Figure 3.23: Spectra of solutions 1 to 9 in comparison with spectrum of solution 0
the amplitude of the fundamental of the EMFs and alleviate the loss of efficiency due to the pitch factor, but this also maximizes the harmonic content. On the other hand, the parameters can also be chosen to reduce the harmonic content and even to suppress an harmonic family. But this choice will also affect the amplitude of the fundamental. Because the number of phases cannot be equal to the number of rotor pole pairs, some regular structures suppress or minimize harmonics families without any adaptation (i.e. with $\nu = \tau = 1$), but have also a reduction of the fundamental component due to the pitch factor.

### 3.4.2 To lighten wider teeth

If wound teeth are narrowed due to an adaptation, then non-wound-teeth are oversized. A way of improvement of the efficiency is therefore to remove the non useful iron mass of non-wounded teeth for retrieving the same field intensity in both wound and non-wound teeth. Figure 3.24 outlines the mass removal, the optimal profil stays to be optimised.

![Figure 3.24: Removing unnecessary iron mass](image)

The recovered space can be filled with a material with better thermal quality aspects (Figure 3.25). For example a higher specific heat limitates the temperature rise during the mission.

![Figure 3.25: Adding heat sink](image)

Another solution consists in retriving the same width of the wound and non-wound tooth by widening the slot surfaces. This is shown in Figure 3.26. The slots-tooth alternance returns to a regular distribution.
However the slot openings stay irregularly distributed along the stator inner periphery.

![Diagram](image1)

(a) Lightened structure  
(b) Structure with widened slots

Figure 3.26: Widening the teeth to get back a regular slots-teeth alternance

The advantage of this solution is that it allows to increase the copper section in the same proportion than the slot surface. This keep unchanged the EMFs induced and the phase inductances, but it decreases the phase resistance, and hence the Joule’s losses.

Authors of [19] have analysed a segment motor with an irregular distribution of the slot openings, given in Figure 3.27(a). The structure of the machine studied in [19] is shown in figure 3.27(b) before the slot opening adaptation. It is a 4-phase machine with 3 pole pairs. By modifying the slot opening distribution as shown in figure 3.27(a), the stator pitch of a wound tooth comes closer to the rotor pole pitch. That influences the harmonic content of the MMF in the airgap, and the magnetic losses on the rotor side. In their case, the magnetic losses have been reduced of 29% on the rotor side.

![Diagram](image2)

(a) Slot opening adaptation  
(b) 4-phase machine with 3 pole pairs

Figure 3.27: Irregular distribution of the slot openings
3.4.3 Slot subdivision

The basic structure of the segment motor has two slots per phase. But one phase can consist in more than one coil and fills more than two slots. In order to keep the physical separation of the phases, these coils and slots must be confined in the \(1/n\)th part of the machine dedicated to the phase. The motors with that type of arrangement of the windings are named modular machines because each phase is composed of a module of several consecutive slots.

3.4.3.1 Subdivision with a segmented structure

One phase is composed of \(k\) consecutive windings. Figure 3.28 shows the example of a phase whom the turns are spread into three pair of slots. To keep a high pitch factor, the number of pole pairs is also multiplied by three.

In the more basic case all the teeth have the same width. The flux induced in one of the \(k\) windings has a small phase shift with respect to the others. This is equivalent to the use of spread windings in classical machines. This reduces the amplitude of the fundamental of the flux in the phase, but also reduces the harmonic content.

The head coils are shorter, reducing the volume of copper and the Joule’s losses. However, the volume of isolating material separating the windings from the iron increases. The slot fill factor diminishes.
One has also to take into account that to keep the same mechanical speed, the electrical speed must be multiplied by $k$, which require a control with a higher dynamic.

With the same total number of turns in the subdivided structure with $k$ coils than in the basic structure with one coil, the self inductance is weaker with the subdivided structure. Indeed, the inductance of the basic structure is:

$$L = \frac{\mu_0 \mu_r n_{sp}^2 S}{h}$$

with $S$ the cross section of a wound tooth, $h$ the height of the apparent airgap, $n_{sp}$ the number of turns and $\mu_r$ the relative permeability of the iron.

When the phase is subdivided in $k$ coils, the inductance of the coils in series is equal to:

$$L' = k \frac{\mu_0 \mu_r \left( \frac{n_{sp}}{k} \right)^2 S}{h} = \frac{L}{k^2}$$

and is smaller than the inductance without subdivision. Besides, the flux induced by each coil has a mutual effect on the other coils of the phase, decreasing even more the flux of the whole phase.

### 3.4.3.2 Subdivision with a non-segmented structure

The second subdivision subdivides the phase in a non-segment structure of several coils. As shown in Figure 3.29, the two slots of one phase are splitted in several half slots. In order to keep the physical separation of the phases, we have to add iron 'dead' zones between half slots of different phases. These zones imposes to slightly reduce the width of the teeth. This subdivision is therefore mostly useful for motors with a rotor pitch smaller than the stator one. The reduction of the width imposed by adding dead zones has the effect to make closer the stator and rotor pitch, and hence increases the flux induced in the windings.

In the example shown in Figure 3.29, the coil of the basic structure is divided in 3 coils wound around 3 consecutive teeth, and the stator pole pitch is reduced to fit the rotor one.

This structure has the advantage to reduce even more the coils head than the previous subdivision. The total inductance is also reduced,
even if the mutual effect between the coils of one phase increases it. The mutual inductances between different phases are also greatly reduced, in a more important way than the self inductance.

### 3.5 Equivalent circuit

In section 3.3.2 we introduced two models for computing the mutual inductances between the phases of a segment machine.

According to the second model, which is the simplest, the mutual inductances are all equal to zero. From the electrical point of view, the $n$ stator phases of a segment motor can then be seen in this case as $n$ decoupled circuits, each made of a resistance $R$ and an inductance $L$ in series with a back EMF $e$ induced by the magnets, as shown in Fig. 3.30. The electrical equation of phase $k$ is given by:

$$u_k = Ri_k + L \frac{di_k}{dt} + e_k$$

(3.28)

with $k \in [1, \ldots, n]$. With magnets properly shaped we can make the assumption that the fluxes $\psi_k$ induced in the phases by the magnets vary sinusoidally with the rotor position and hence that the EMFs they induce in the phases are also sinusoidal:

$$e_k = \dot{\theta}_m \frac{d\psi_k}{d\theta_m} = \dot{\theta}_m \frac{d}{d\theta_m} \left[ \psi_0 \cdot \cos \left( p\theta_m - \frac{k-1}{n} \frac{2\pi}{n} \right) \right]$$

(3.29)
$\psi_0$ is the amplitude of the flux induced by the magnets in one phase, $p$ the number of rotor pole pairs and $\theta_m$ the rotor mechanical position chosen in such a way that, for $\theta_m = 0$, the flux induced in phase 1 is maximum.

According to the first model, the mutual inductances between the phases have all the same value equal to:

$$M = \frac{-1}{2n-1} L$$

(3.30)

The matrix of inductance, linking the flux induced in the windings with the currents, is:

$$\begin{bmatrix} L & M & \cdots & M \\ M & L & \cdots & M \\ \vdots & \vdots & \ddots & \vdots \\ M & M & \cdots & L \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_n \end{bmatrix}$$

$$= (L - M) \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_n \end{bmatrix} + \begin{bmatrix} M & M & \cdots & M \\ M & M & \cdots & M \\ \vdots & \vdots & \ddots & \vdots \\ M & M & \cdots & M \end{bmatrix} \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_n \end{bmatrix}$$

(3.31)

$$= (L - M) \begin{bmatrix} i_1 \\ i_2 \\ \vdots \\ i_n \end{bmatrix} + M \sum_{k=1}^{n} i_k$$

Under this last form, one may see that an interaction of the flux with the currents of the other phases comes down to a term depending on the sum of the currents.
If the sum of the phase currents is equal to zero (i.e. if the phases are star connected with a neutral point, or if the control imposes that constraint), one gets back the independence of the phases:

\[
\begin{bmatrix}
\psi_{c1} \\
\psi_{c2} \\
\vdots \\
\psi_{cn}
\end{bmatrix}
= (L - M) \begin{bmatrix}
i_1 \\
i_2 \\
\vdots \\
i_n
\end{bmatrix}
\]

(3.32)

Using equation (3.30), we eventually get:

\[
\begin{bmatrix}
\psi_{c1} \\
\psi_{c2} \\
\vdots \\
\psi_{cn}
\end{bmatrix}
= \frac{2n}{2n-1} L \begin{bmatrix}
i_1 \\
i_2 \\
\vdots \\
i_n
\end{bmatrix}
\]

(3.33)

Hence when the sum of the currents is equal to zero, one may rewrite equation (3.28) taking into account the mutual inductances as follow:

\[
u_k = Ri_k + \frac{2n}{2n-1} L \frac{d}{dt} i_k + e_k
\]

(3.34)

We have also pointed out that the reality lies somewhere between the two cases: the mutual inductances are not equal to zero but their value is much smaller that the values given by the first model. Besides if all the mutual inductances does not have exactly the same values, they does not differ significantly from each others.\(^6\) In that case, we take the mutual inductance into account as follow:

\[
u_k = Ri_k + (L - M) \frac{d}{dt} i_k + e_k
\]

(3.35)

Therefore equations (3.28), (3.34) and (3.35) show that it is always possible to consider that the motor equivalent circuit is corresponding to \(n\) decoupled phases having each one a resistance, a self inductance and an EMF. But, they also show that when the mutual inductances are not fully negligible, this independency is only achieved when the sum of the currents is equal to zero. This constraint is for instance automatically verified when the phases are star connected with an isolated neutral point as already said.

\(^6\)This will be validated in chapter 9.
3.6 Conclusions

The segment motor has been presented in this chapter. From that analysis, we were able to highlight the main benefits and drawbacks of the segment motor in comparison with the other type of permanent magnet synchronous motors. The conclusion is that this type of motor is really fitted for aerospace applications.

From the reliability point of view, any type of motor can be affected by the loss of one phase. But segment motor have a higher level of reliability than any other motor types due to the physical separation between the phases. There is no risk of fault between two phases or of propagating a fault from a phase to another in the motor. Besides, the detection and localization of a fault is easier, due to the electric and magnetic separation of the phases.

The non overlapping configuration of the windings makes easy the manufacturing of motors with more than 3 phases, what still reduces the length of the winding heads and the mismatch between the stator and rotor pole pitches. Furthermore the higher the number of phases, the lower the impact of the loss of feeding of one phase.

The use of more than 3 phases has also the effect of increasing the number of pole pairs, which reduces the cogging torque, but increases the electrical frequency and impacts negatively the magnetic losses. The magnetic losses density is greater in segment motor than in classical motors. However the stator and rotor yokes are smaller and the magnetic losses may possibly be smaller.

After that, we have presented some improvements and variants of the basic structure of segment motors. Performances can drastically be increased with a more complex design (such the use of Halbach arrays, prepressed of preformed windings), or by modifying some design parameters such as the tooth width, the magnet width, the angular position of the slot openings, ...

Eventually, the electrical equivalent circuit of the motor has been presented and discussed. This equivalent circuit will be used in the next section devoted to control issues.
3.7 Appendix : More specific features about machine manufacturing

These two appendices deal with some design techniques allowing to increase the motor performance but imposing to modify the motor manufacturing process.

3.7.1 Use of preformed or prepressed windings

Prepressed or preformed windings can enhance the fill factor. Figure 3.31(a) shows that a prepressed coils tends to take the form of an alveolar structure. The thermal resistance of the windings is substantially reduced and this increases considerably the performances. Authors of [4] cite a slot fill factor of 0.78% and a reduction of 76% of the thermal resistance.

Authors of [5] also reach a slot fill factor of 78%, not with prepressed windings but with preformed ones with rectangular conductors (Figure 3.31(b)). However both prepressed and preformed windings require to fractionate the stator core. Windings have to be inserted as one block around the tooth.

In the case of the prepressed windings, the authors build the parts of the stator with powdered iron (Figure 3.32), for a non-segment mo-
Appendix: More specific features about machine manufacturing

tor. The stator made of powdered iron is less efficient than a laminated stator. The level of magnetic saturation is lower. The teeth and yoke widths have to be larger. But prepressed windings, with a higher fill factor and a better heat evacuation, counterbalance the loss of efficiency due to the use of powdered iron. For the same thermal limits, the motor produces nearly 2 times more torque than an equivalent machine without prepressed windings and with laminated iron parts. From the

![Image of motor design with powdered iron stator](image)

(a) Scattered view (b) Phase assembly (c) Electromagnetic part of the stator

Figure 3.32: A motor design with powdered iron stator (picture from [4])

reliability point of view, on the one hand, the compression brings closer the coils and increases the probability of a short circuit occurrence. On the other hand, the compression homogenizes the insulation filler, making disappear the imperfection of the insulation like cracks or vacuum bubbles. The coils form a rigid block not sensitive to alteration and wear from operation vibration.

In the case of the preformed windings, the authors uses grain-oriented silicon iron sheets for the teeth. This is only possible with fragmented stator as shown in Figure 3.33. This type of material has less saturation and lower losses caused by the flux in the grain orientation [6]. However, as the slot is rectangular, the width of the teeth cannot be constant. Both methods can be extended to the design of segment motors. The fractioning shown in 3.32(c) has to be adapted as the windings fills in the entire slot. On the contrary the fractioning shown in 3.33 can be the same for segment and non segment motors, as the slot is rectangular.
3.7.2 Use of Halbach array arrangement of the magnets

3.7.2.1 Description of the Halbach array used in synchronous motors

*Halbach array* is an arrangement of the magnets that reinforces the field on one side and weakens it on the other side.

Figures 3.34(a) and 3.34(b) give a schematic view of respectively a classical arrangement and an Halbach array. Figure 3.34(b) shows that a halbach array can be seen as the combination of two patterns of magnet: one with an alternation of up-down magnetization, and one with an alternation left-right. The superposition of the fields of the two patterns gives the field of the Halbach array.

The Halbach array gives the following benefits:

- The amplitude of the fundamental of the flux is increased by a factor that depends on the magnet width. Hence the torque is improved by the same factor. [21] gives, for a classical 3-phase machine, an increase of the fundamental of 11% with a ratio width/height
Appendix: More specific features about machine manufacturing

(a) Classical arrangement

(b) Halbach array

Figure 3.34: Linear arrangements of magnets

of the magnets equal to 1/2.

- The field distribution of a Halbach array has also the property to carry a weaker harmonic content, which produces hence less magnetic losses in the stator yoke, and smooths the undesired fluctuations of the electromagnetic torque.

- The flux density is weaker inside the rotor yoke, which can therefore be thinner. Its mass and inertia are decreased. The dynamic is increased and peaks of currents in transient states are weaker.

The Halbach array needs always more than two magnets by pole pairs. The structure shown in Figure 3.34(b) carries four magnets per pole pair. More complex structures carry even more magnets. Figure 3.35 shows structures with four to ten magnets per pole pair. The magnet magnetization is shifted by $2\pi/a$ radians between each magnet, with $a$ the number of magnets per pole pair.

With eight magnets per pole pair, [22] gives an improvement of 30% of the torque given by a classical machine with 3 pole pairs, and a decrease of the cogging torque by a factor 3.
The more the number of magnets per pole pair, the more the arrangement tends to the ideal Halbach arrangement, illustrated in figure 3.36. It corresponds to a magnet ring whom the magnetization orientation varies as a function of the angular position. This ring is made from magnet powder, oriented during the manufacturing. It results from that ring an EMF purely sinusoidal.

Figure 3.37 gives an idea of the distribution of the no load airgap field and the waveform of the EMF, for machines with two to ten magnets per pole pair, as well as for a machine with an ideal Halbach array.


3.7.2.2 Study of a simple case

To get a more precise idea of the interests of the Halbach array, a study using an FEM computation has been carried out on a simplified model of a permanent magnet synchronous motor. As we are not interested by the slot effect on the airgap field, we neglect the slots and approximate the stator by a uniform iron ring. With the simplified stator, we use the symmetries of the model and only consider an angular section of the machine going from the center of one pole to the center of the next one. For a machine with 6 pole pairs, the section is corresponding to one twelfth of the machine as shown in Figure 3.38(a). Magnets are \( \text{NdFeB} \) with a remanent field of 1.16 \( T \) and the magnetization curve of the iron...
Figure 3.37: On the left: distribution of the airgap magnetic field. On the right: waveforms of the phase EMF (in bold) and of the line EMF (a: 4 pole pairs, b: 6 pole pairs, c: 8 pole pairs, d: 10 pole pairs, e: ideal Halbach magnetisation, f: 2 pole pairs). Picture from [21]
Appendix: More specific features about machine manufacturing

<table>
<thead>
<tr>
<th>number of magnets per pole pair</th>
<th>fundamental of the field</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>0.84 T (100%)</td>
</tr>
<tr>
<td>4</td>
<td>0.86 T (102%)</td>
</tr>
<tr>
<td>6</td>
<td>0.91 T (108%)</td>
</tr>
<tr>
<td>8</td>
<td>0.93 T (110%)</td>
</tr>
</tbody>
</table>

Table 3.6: Fundamental component values of the airgap field

of the stator and rotor yokes is shown in Figure 3.38(b).

![Diagram of Halbach array arrangement](image)

(a) Structure and dimensions  (b) B-H curve of the stator and rotor yokes

Figure 3.38: Study case of Halbach array arrangements

The field distribution is computed using the software \textit{Femm2D} for various structures of the magnet ring, going from the classic structure with one magnet per pole to Halbach array with 8 magnets per pole pair. For each configuration, Figure 3.39 shows the magnet distribution, the result of the FEM analysis and the airgap field in the airgap at the stator periphery.

As said before, the first interest of the Halbach array lies in the increase of the value of the fundamental component of the field as shown in table 3.6.

The second interest of the Halbach array is the weaker harmonic...
Segment motor basics

(a) Two magnets per pole pair (no Halbach array)

(b) Four magnets per pole pair

(c) Six magnets per pole pair

(d) Eight magnets per pole pair

Figure 3.39: Airgap field for various magnet configurations
content. We can see in Figure 3.39 that the field distribution is closer to a sinusoidal distribution with a higher number of magnets per pole pair. Figure 3.40 gives the first harmonics of the airgap field for the four cases under study. With 4 magnets only harmonic 5 remains not negligible. With 6 magnets per pole pair, it is harmonic 7. With a number $a$ of magnets per pole pair, the harmonic that are not negligible at the stator inner periphery are of order:

$$ka + 1, \quad k \in \mathbb{N}$$

(3.36)

When $a$ tends to the infinity, equation (3.36) shows that the structure tends to have no harmonic, and therefore tends to create a sinusoidal distribution of the field in the airgap. The width of the magnets tends to zero and tends to the ideal Halbach array.

Harmonic 3 is always negligible when an Halbach array is used, regardless the number of magnets per pole pair, if the magnets are all of the same width.

At the interface between magnets and the rotor yoke, the field distri-
bution has another harmonic content. As shown in figure 3.41, except for the fundamental, which is reduced by 75% in the three cases of Halbach arrays, harmonics that are not negligible are the one of order:

\[ ka - 1, \quad k \in \mathbb{N} \quad (3.37) \]

The third interest of Halbach array is the weaker field intensity inside the rotor yoke. In figure 3.39 the FEM results show clearly that the field intensity in the rotor yoke is much more important without an Halbach array. The critical location is between two rotor poles (at an angular position of 15° in the study case), as shown in figure 3.42(a). Figure 3.42(b) gives the intensity of the field radially from the inner periphery to the outer periphery of the rotor yoke.

Without Halbach array, the field is mainly higher than 1.5\( T \) and close to the saturation field of the rotor yoke (Figure 3.38(b)). With an Halbach array, the field is a lot smaller. Table 3.7 gives the value of the
Appendix: More specific features about machine manufacturing

(a) Path of the flux

(b) Intensity of the field in the rotor yoke as a function of the radius

Figure 3.42: Field in the rotor yoke

<table>
<thead>
<tr>
<th>number of magnets per pole pair</th>
<th>peak flux in the rotor yoke</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>7.92e-3 Wb (100%)</td>
</tr>
<tr>
<td>4</td>
<td>2.21e-3 Wb (28%)</td>
</tr>
<tr>
<td>6</td>
<td>2.23e-3 Wb (28%)</td>
</tr>
<tr>
<td>8</td>
<td>1.81e-3 Wb (23%)</td>
</tr>
</tbody>
</table>

Table 3.7: Magnet flux between two poles in the iron yoke

flux between the two poles of the rotor (for a length of the motor equal to 1 meter).

The number of magnets per pole pair has not a great influence on the results. With the use of any Halbach array, the flux going from one north pole to one south pole is reduced by a factor 4. In a design process, the rotor yoke could therefore be reduced by the same factor.
3.8 Appendix: Properties of the link between the number of rotor pole pairs and the number of phases of segment motors

The demonstration is split into three parts:

- First, it is demonstrated that if \( n \) and \( p \) are not coprime, the equation is not solvable.

- Second, it is shown that, under the first condition, \( mp = qn + 1 \) has always one and only one solution with \( m < n \) and \( q < p \).

- Third, it is shown that if \( mp = qn + 1 \) has one solution with \( m < n \) and \( q < p \), the dual equation \( m'p = q'n - 1 \) has also one and only one solution with \( m' < n \) and \( q' < p \), and that either \( m' \) and \( q' \), or \( m \) and \( q \), are respectively smaller than \( n/2 \) and \( p/2 \).

**Theorem 3.8.1** Assuming the following Diophantine equation:

\[
mp = qn \pm 1 \tag{3.38}
\]

with \( m, p, q, n \) natural numbers. The equation has solutions for \( m \) and \( q \) if and only if \( n \) and \( p \) are coprime.

**Proof** Let assume that \( n \) and \( p \) are any natural numbers. Then we can write:

\[
p = kp' \\
n = kn'
\]

with \( n' \) and \( p' \) coprime, and \( k \) the greatest common multiple of \( n \) and \( p \). If \( k = 1 \) then \( n \) and \( p \) are coprime. If not they are not.

We can hence rewrite equation (3.38) as:

\[
mkp' = qkn' \pm 1 \tag{3.40}
\]

\[
\Rightarrow k(mp' - qn') = \pm 1 \tag{3.41}
\]

This equation has solutions only if \( k = 1 \). Therefore \( p \) and \( n \) are respectively equal to \( p' \) and \( n' \), and are consequently coprime.

Note that the demonstration can be easily adapted to show that:
Appendix : Properties of the link between phases and rotor pole pairs

- $n$ is also coprime with $m$,
- $p$ is also coprime with $q$,
- and $q$ is also coprime with $m$.

**Theorem 3.8.2** There is one and only one solution for $m < n$ and $q < p$, such that

$$mp = qn + 1 \quad (3.42)$$

with $m, q, n$ and $p$ natural, and $n$ and $p$ coprime.

**Proof**

- **First case :** $n < p$

Assuming the following series:

$$z_m = (mp) \mod n \quad (3.43)$$

with $m = 0, 1, ..., n - 1$.

We have $z_m < n$. All $z_m$ are different as, otherwise, we would have:

$$m_1p = z + q_1n \quad (3.44)$$
$$m_2p = z + q_2n \quad (3.45)$$
$$\Rightarrow (m_2 - m_1)p = (q_2 - q_1)n \quad (3.46)$$

We suppose $m_2 > m_1$. As $p$ and $n$ are coprime, the minimal values for $(m_2 - m_1)$ and $(q_2 - q_1)$ that resolve equation (3.46), are respectively:

$$m_2 - m_1 = n \quad (3.47)$$
$$q_2 - q_1 = p \quad (3.48)$$

Therefore:

$$m_2 = n + m_1 \quad (3.49)$$
$$q_2 = p + q_1$$

This would imply that $m_2 > n$ and $q_2 > p$, what is not possible. Therefore the $z_m$ are all different.
As they are all different and smaller than $n$, one and only one of them is necessarily equal to 1, such that

$$mp = 1 + qn$$  \hspace{1cm} (3.50)

As $m < n$, we have:

$$mp - 1 < pn$$  \hspace{1cm} (3.51)

$$
\Rightarrow \quad qn < pn \\
\Rightarrow \quad q < p$$  \hspace{1cm} (3.52)

- **Second case**: $p < n$

Assuming the following series:

$$z_q = (qn) \mod p$$  \hspace{1cm} (3.54)

with $q = 0, 1, ..., p - 1$.

We have $z_q < p$. All $z_m$ are different as, otherwise, we would have:

$$q_1 n = z + m_1 p$$  \hspace{1cm} (3.55)

$$q_2 n = z + m_2 p$$  \hspace{1cm} (3.56)

$$
\Rightarrow \quad (q_2 - q_1) n = (m_1 - m_2) p$$  \hspace{1cm} (3.57)

We suppose $q_2 > q_1$. As $p$ and $n$ are coprime, the minimal values for $(m_2 - m_1)$ and $(q_2 - q_1)$ that resolve equation (3.57), are respectively:

$$(m_2 - m_1) = n$$  \hspace{1cm} (3.58)

$$(q_2 - q_1) = p$$  \hspace{1cm} (3.59)

Therefore:

$$m_2 = n + m_1$$  \hspace{1cm} (3.60)

$$q_2 = p + q_1$$

This would imply that $m_2 > n$ and $q_2 > p$, what is not possible. Therefore the $z_m$ are all different.

As they are all different and smaller than $n$, one and only one of them is necessarily equal to $p - 1$, such that

$$qn = p - 1 + mp$$  \hspace{1cm} (3.61)

$$(m + 1)p = qn + 1$$  \hspace{1cm} (3.62)

$$mp = qn + 1$$  \hspace{1cm} (3.63)
Appendix: Properties of the link between phases and rotor pole pairs

As \( q < p \), we have:

\[
mp < pn + 1 \quad (3.64)
\]

\[
\Rightarrow qn + 1 < pn + 1 \quad (3.65)
\]

\[
\Rightarrow m < n \quad (3.66)
\]

**Theorem 3.8.3** If the following equation:

\[
mp = qn + 1 \quad (3.67)
\]

has a solution for \( m < n \) and \( q < p \), with \( m, q, n \) and \( p \) natural, then the dual equation:

\[
m'p = q'n - 1 \quad (3.68)
\]

has a solution for \( m' < n \) and \( q' < p \), with \( m', q' \) natural. Besides either \( q < p/2 \) and \( m < n/2 \), or \( q' < p/2 \) and \( m' < n/2 \).

**Proof** Let start with the following equation:

\[
mp = qn + 1 \quad (3.69)
\]

If we subtract each part by \( np \), we get:

\[
(m - n)p = (q - p)n + 1
\]

\[
\Rightarrow (n - m)p = (p - q)n - 1 \quad (3.70)
\]

\[
\Rightarrow m'p = q'n - 1
\]

with \( m' = n - m \) and \( q' = p - q \).

This last equation is the dual equation of equation (3.69) (with -1 instead of +1). As \( n \) and \( m \) are coprime, \( m \) cannot be equal to \( n/2 \). Therefore, either \( m \) or \( m' \) is smaller than \( n/2 \). Similarly, either \( q \) or \( q' \) is smaller than \( p/2 \). But we still have to show that if \( m' < n/2 \), then it is \( q' \) (and not \( q \)) which is smaller than \( p \).

- **First case**: \( m < n/2 \) and \( q < p/2 \)

\[
m < \frac{n}{2}
\]

\[
\Rightarrow mp < \frac{n}{2}p
\]

\[
\Rightarrow qn + 1 < \frac{n}{2}p
\]

\[
\Rightarrow qn < \frac{n}{2}p
\]

\[
\Rightarrow q < \frac{p}{2}
\]
• **Second case:** $m' < n/2$ and $q' < p/2$

\[ q' < \frac{p}{2} \]
\[ \Rightarrow q'n < \frac{p}{2}n \]
\[ \Rightarrow q'n - 1 < \frac{p}{2}n \]
\[ \Rightarrow m'p < \frac{p}{2}n \]
\[ \Rightarrow m' < \frac{n}{2} \] (3.72)
CHAPTER 4

Power converter architectures

4.1 Basic structures

In embedded systems, the electrical power comes from a DC bus or a battery and the power electronics relies on voltage sourced inverter working in PWM. The two basic structures of power electronics for feeding a machine with $n$ phases are thus the $n$-leg inverter (Figure 4.1) which implies star connected phases, or a separate feeding of the phases by $n$ 'H' bridges (Figure 4.2).

Figure 4.1: Example of a 6-leg inverter feeding a 6 phase motor
The VA rating of the two solutions are the same, even if for the solution based on \(n\) 'H' bridges the number of switches is twice that of a \(n\)-leg inverter. Indeed for the same DC input voltage \(U\), the maximum value of the phase voltage that the \(n\)-leg inverter can deliver (\(\pm U/2\)) is half that of the 'H' bridge (\(\pm U\)). Therefore when using a \(n\)-leg inverter, the motor must be designed for a nominal phase voltage twice smaller than when using \(n\) 'H' bridges. Consequently the motor phase current is multiplied by two. The current rating of the switches of the \(n\)-leg inverter must be twice that of the switches of the \(n\) 'H' bridges. This means that the total amount of silicon needed is the same.

For illustrating this, Figure 4.3 shows a picture of a power module corresponding to an inverter leg in which each transistor is made of the parallel connection of two chips and each “diode” by the parallel connection of four chips. [23] So this leg can be converted into a 'H' bridge having a nominal current two times smaller simply by modifying the connections between the chips inside the module. The power that can be delivered by the inverter leg and the 'H' bridge is therefore the same.

However, it should be noted that the \(n\)-leg inverter imposes to the motor phase currents to have a zero valued sum while the \(n\) 'H' bridges do not. Nevertheless with a \(n\)-leg inverter the constraint on the sum of the currents can be removed by adding a supplementary leg connected to the neutral point (Figure 4.4).
4.2 Consequence of the failure of one switch inside the power electronics

A switch failure may result in a permanent short circuit state or a permanent open circuit state of the switch. For the diodes a fault always results from a failure at the semiconductor level, while for the transistors it may also come from a failure at the gate drive unit level.

When a switch goes into open circuit, it is possible to limit the consequences of the fault to the loss of feeding of the phase, which has one terminal connected to the faulted switch, simply by applying permanently an OFF signal to the switches of the corresponding leg. When a switch goes into short circuit, turning OFF permanently the other
switch of the corresponding leg allows to avoid to short circuit the DC source, but not to avoid that the phase, which has a terminal connected to the faulted switch, will be periodically short circuited through the faulted switch and the diodes of the other switches linked to the same DC terminal. The resulting short circuit currents will perturb the working of the machine in an unacceptable way, as it is shown for instance in Figure 4.5 for a 3-phase machine with a transistor short circuited in the leg of one phase.\footnote{This example supposes a machine running at constant speed. Of course, the short circuit currents will induce a pulsating torque that will affect the speed of the machine, but this is out of the scope of this simple example.} For more details, the computation is developed in appendix 4.6.

![Schematic diagram of a short circuited transistor of a 3-phase inverter](image1)

(a) Schematic diagram of a short circuited transistor of a 3-phase inverter

![Short-circuit currents flowing into the phases of the machine at constant speed](image2)

(b) Short-circuit currents flowing into the phases of the machine at constant speed

Figure 4.5: Study of a short-circuited switch

### 4.3 Making the power electronics switches tolerant against a fault at the semiconductor level

Figure 4.6(a) shows a switch architecture fully tolerant against a failure at a semiconductor level (including the gate drive unit for the transistors).
It is easy to verify that in the case of a failure in short circuit or in open circuit of any semiconductor, the switch remains fully operational. In the preceding section, we have shown that it is imperative to avoid that a switch goes into short circuit but that we can tolerate that a switch goes into open circuit provided that we accept to lose the feeding of the corresponding phase and make the machine able to run on a reduced number of phases. In that case we only need to make the switches tolerant against a short circuit failure of a semiconductor. This yields the switch architecture shown in Figure 4.6(b) which needs only two transistors and two diodes instead of four transistors and four diodes.

4.4 Other power electronics architectures safe against a short circuit failure of a semiconductor

In the case of \( n \) 'H' bridges feeding the machine, a remedial against the periodic short circuit of a phase resulting from a failure in short circuit of one semiconductor, consists in replacing the diodes by thyristors as shown in Figure 4.7(a).

In normal operation gate signals are applied to the thyristors, making them work as simple diodes. When a semiconductor fails in short circuit the gate signals on the thyristors are suppressed as well as drive signals on the transistors. After a short transient, all the semiconductors go
into OFF state but the faulted one. The phase is isolated as shown in figure 4.7(b) since the three non faulted switches are now in open circuit state.

Another way to avoid any risk of short circuit of the DC source or at the motor level is to disconnect the leg in which a fault occurs by using fuses. But in order to be sure that the fuses will blow up, this solution generally imposes the use of crowbar thyristors as shown in Figure 4.8.
4.5 Conclusions

This brief overview of the possible power electronic architectures shows that the solution allowing to keep the switches fully operational in case of failure of one semiconductor is at least twice more bulky than those that prevent only short circuit failures. The latter, in the case of an open circuit failure, imply to lose the feeding of the corresponding phase.

Therefore it is worthy to consider a control architecture and a motor design allowing an operation on a reduced number of phases associated with a power electronics which is safe against a switch short circuit failure, but not against a switch open circuit failure.

With the considered architectures, if the failure occurs in a semiconductor when it was feeding a phase, the current will continue to flow in the remaining healthy components until it crosses zero. For example, the turns off of the thyristor in Figure 4.7 will happen at the current zero crossing. This is necessary in order to avoid a transient peak voltage that could damage other components of the power electronics.

4.6 Appendix : Determination of the phase currents in case of a switch short circuit failure

A short circuit failure of a switch is critical because of the apparition of short circuit currents circulating into all the phases. Even if the transistors of the remaining healthy switches are turned OFF, the short-circuit current is not eliminated because it can flow through the diodes of the switches connected to the same DC terminal as the faulted one.

In the case of a three phase system with the switch failure shown in Figure 4.5, the equivalent circuit when all the remaining transistors are turned OFF, is represented in Figure 4.9. Each phase of the motor is represented by a phase resistance, a phase inductance and a voltage source representing the EMFs induced by the rotor magnets.

By defining $v_b$ and $v_c$ respectively the voltages at the terminal of the diodes of phase $b$ and $c$, and with the direction of the currents defined as in Figure 4.9, the two following equations describe the behaviour of
the circuit:

\[

e_b - e_a = sL_i_b + R_i_b + v_b(i_b) - sL_i_a - R_i_a \tag{4.1}
\]

\[

e_c - e_a = sL_i_c + R_i_c + v_c(i_c) - sL_i_a - R_i_a \tag{4.2}
\]

In these equations, \( s \) is the Laplace operator.

Using the two following combinations: \( 2*(4.1)-(4.2) \) and \( 2*(4.2)-(4.1) \), we get:

\[
\begin{align*}
2e_b - e_a - e_c &= sL(2i_b - i_a - i_c) + R(2i_b - i_a - i_c) + 2v_b(i_b) - v_c(i_c) \\
2e_c - e_a - e_b &= sL(2i_c - i_a - i_b) + R(2i_c - i_a - i_b) + 2v_c(i_c) - v_b(i_b) \tag{4.3}
\end{align*}
\]

and eventually using \( i_a = -i_b - i_c \) and isolating the state variables, we get:

\[
\begin{align*}
2e_b - e_a - e_c &= sL(2i_b - i_a - i_c) + R(2i_b - i_a - i_c) + 2v_b(i_b) - v_c(i_c) \\
2e_c - e_a - e_b &= sL(2i_c - i_a - i_b) + R(2i_c - i_a - i_b) + 2v_c(i_c) - v_b(i_b) \tag{4.3}
\end{align*}
\]

\[
\begin{align*}
sL_i_b &= \frac{2e_b - e_a - e_c}{3} - R_i_b - \frac{2}{3}v_b(i_b) - \frac{1}{3}v_c(i_c) \\
sL_i_c &= \frac{2e_c - e_a - e_b}{3} - R_i_c + \frac{2}{3}v_c(i_c) - \frac{1}{3}v_b(i_b) \tag{4.4}
\end{align*}
\]

The Simulink diagram equivalent to the system (4.4) is shown in Figure 4.10. Of course, with sinusoidal EMFs, we can use the fact that the sum of the EMFs is equal to zero and simplify the two expressions:

\[
\begin{align*}
sL_i_b &= e_b - R_i_b - \frac{2}{3}v_b(i_b) - \frac{1}{3}v_c(i_c) \\
sL_i_c &= e_c - R_i_c + \frac{2}{3}v_c(i_c) - \frac{1}{3}v_b(i_b) \tag{4.5}
\end{align*}
\]
One must still define the voltages at the terminal of the two diodes, $v_c(i_c)$ and $v_b(i_c)$, as a function of the current flowing into them. In our case the following modelisation has been adopted:

- if the current flows in the forward direction of the diode, the diode is modelized by a forward resistance $R_{ON}$ (Figure 4.11(a));
- if the current flows in the reverse direction of the diode, the diode is modelized by a reverse resistance $R_{OFF}$. For the convergence of the computation at the transition from $R_{ON}$ to $R_{OFF}$, we add a snubber capacitance $C$, in parallel with the $R_{OFF}$ (Figure 4.11(b)).

The Simulink model of the diodes is shown in Figure 4.11(c).

In the simulation shown in Figure 4.5, we assume that the machine rotates at a constant speed. The phase EMFs are then given by:

$$
e_a = k_t \sin(\omega t)$$
$$
e_b = k_t \sin\left(\omega t - \frac{2\pi}{3}\right)$$
$$
e_c = k_t \sin\left(\omega t - \frac{4\pi}{3}\right)$$

(4.6)
Power converter architectures

(a) Model of the passing diode

(b) Model of the blocking diode

(c) Simulink subsystem for modelling the diode

Figure 4.11: Modeling the diode

with \( \omega \) the electrical speed and \( k_t \) the speed constant of the machine.

The parameters used for the simulation are shown in Table 4.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>phase resistance</td>
<td>( R = 100 \text{ m}\Omega )</td>
</tr>
<tr>
<td>phase inductance</td>
<td>( L = 550 \text{ } \mu \text{H} )</td>
</tr>
<tr>
<td>EMF coefficient</td>
<td>( k_t = 0.2 \text{ V/rad/s} )</td>
</tr>
<tr>
<td>forward resistance</td>
<td>( R_{ON} = 1 \text{ m}\Omega )</td>
</tr>
<tr>
<td>phase resistance</td>
<td>( R_{OFF} = 1 \text{ k}\Omega )</td>
</tr>
<tr>
<td>electrical speed</td>
<td>( w = 60 \text{ rad/s} )</td>
</tr>
</tbody>
</table>

Table 4.1: Parameters used for the simulation shown in Figure 4.5

The hypothesis of using a constant speed may be questioned as the short circuit current generates a disturbance torque that perturbs the machine running. The model shown in Figure 4.10 could be easily integrated in an extended model taking into account this disturbance, but this is beyond the scope of this chapter. We may also point out that the principle described here can be easily extended to the case of machines with more than 3 phases.
References


[16] W. Bing & al., “Optimal design of a high power density PM motor with discrete halbach array and concentrated windings,” in:
Appendix : Phase currents with a switch short circuit failure


- II -

Control issues
CHAPTER 5

Control laws

5.1 Introduction

With an appropriate motor technology i.e. the use of polyphase segment motors and a proper redundancy at the control and power electronics levels [1], the most probable fault is losing the feeding of one phase resulting from a failure at the inverter or current sensors level. The reliability of the actuation system can then be significantly increased as it is possible to alleviate such a fault. Indeed the performance of the actuation system can be kept unchanged by a proper reconfiguration of the control of the currents flowing in the remaining phases. This reconfiguration, leading to a fault tolerant operation, mainly relies on a redefinition of the link between the motor phase currents and the torque after fault occurrence.

First of all, the general expression of the optimal currents in normal operation mode is given on the basis of a Lagrangian formulation. It is then extended to the fault tolerant mode. In both cases we determine two expressions, depending on whether the sum of the currents must be zero-valued or not. Indeed when the motor phases are star-connected with an isolated neutral point, the sum of the currents flowing into the phases is equal to zero.
In particular for machines with sinusoidal EMFs, the Lagrangian formulation of the optimal currents yields sinusoidal currents for normal operation mode, but not for any fault tolerant mode. Hence, for that type of machines, in order to make the current control easier, we reformulate the problem in order to obtain sinusoidal currents both in normal and fault-tolerant operation mode [2]. Even if this implies that the solutions found are suboptimal, they lead to a easier reconfiguration of the control strategy. Therefore we have four possibilities for the current expressions in fault-tolerant mode, depending on:

- whether the sum of the currents must be zero-valued or not;
- whether the waveforms of the currents are optimal or sinusoidal.

Eventually the four solutions resulting from the reconfiguration of the currents in fault-tolerant operation mode are compared in terms of increase of Joule’s losses and of motor phase peak current. Hence we may evaluate the oversizing of the motor and the power electronics that results from those increases for any application case.

### 5.2 Obtaining the optimal reference currents for developing a given torque

#### 5.2.1 Optimal reference currents in normal operation mode

##### 5.2.1.1 General expression of optimal currents

Surface mounted permanent magnet synchronous motors (in particular segment motors [3]) have very small reluctant and cogging torques. These torque components may therefore be neglected. Hence the torque expression can be reduced to its electrodynamic component yielding for a $n$-phase motor:

$$T = \sum_{k=1}^{n} i_k f_k$$

with $i_k$ the current in phase $k$ and $f_k$ the derivative, in function of rotor mechanical position $\theta_m$, of the flux $\psi_k$ induced in phase $k$ by the
Obtaining the optimal reference currents for a given torque

magnets:

\[ f_k = \frac{d\psi_k}{d\theta_m} \]  \hspace{1cm} (5.2)

Coefficients \( f_k \) are linked to the EMFs induced by the magnets in phase \( k \) by:

\[ e_k = p\dot{\theta}_m f_k \]  \hspace{1cm} (5.3)

with \( p \) the number of pole pairs on the rotor side and \( p\theta_m \) the electrical position.

References [4] or [6] show that the currents allowing to obtain a given torque \( T_{ref} \) with minimal Joule’s losses are given\(^1\) by:

\[ i_k = T_{ref} \frac{f_k}{\sum_{\kappa=1}^{n} f_{\kappa} f_{\kappa}} \]  \hspace{1cm} (5.4)

Note that index \( \kappa \) is used in the denominator to avoid any confusion with index \( k \) indicating the number of the phase.

For a given torque reference, equation (5.4) gives the current waveforms minimizing the Joule’s losses. With a precise measurement of functions \( f_k \), for any position of the rotor, one may compute the optimal currents taking into account the full harmonic content of functions \( f_k \).

5.2.1.2 Optimal currents with a zero-valued sum

The sum of the currents given by expression (5.4) is directly linked to the sum of functions \( f_k \) which is not necessarily equal to zero as the phase EMFs may show an homopolar component. Therefore expression (5.4) is not applicable if the motor windings are star connected with an isolated neutral point. In that case the optimal expression of the currents has to be computed again, adding the supplementary constraint of keeping their sum equal to zero. This yields\(^2\):

\[ i_k = T_{ref} \frac{f_k - \frac{1}{n} \sum_{\kappa=1}^{n} f_{\kappa}}{\sum_{\kappa=1}^{n} f_{\kappa}^2 - \frac{1}{n} \sum_{\kappa=1}^{n} f_{\kappa} \sum_{\kappa=1}^{n} f_{\kappa}} \]  \hspace{1cm} (5.5)

\(^1\)See appendix 5.5.1.1 for the demonstration.

\(^2\)See appendix 5.5.1.2 for the demonstration.
Equation (5.5) differs from equation (5.4) by the two terms added on the numerator and on the denominator. Those terms involve the sum of the $f_k$'s. Therefore for $f_k$'s waveforms without an homopolar components the two formulas (5.4) and (5.5) are identical. For instance for a 3-phase motor with sinusoidal $f_k$’s, both expressions lead to the obvious solution of sinusoidal currents isomorphic to their respective $f_k$’s. Adding an harmonic 3 in the $f_k$’s (i.e. an homopolar component) yields different current waveforms according to the fact that the sum of the current must be equal to zero or not, as shown in Fig.5.1. Indeed when the sum of the currents differs from zero, the homopolar component of the $f_k$’s contributes to the torque production.
5.2.2 Optimal reference currents in fault-tolerant operation mode

5.2.2.1 General expression of the optimal currents

Expression (5.4) gives the optimal solutions of the currents what may be the values of functions $f_k$. So it remains true if we set to zero the $f_k$'s of the lost phases in order to discard their contribution to the torque:

$$i_k = T_{ref} \frac{f_k}{\sum_{\kappa=1}^{n} f_{\kappa}^2} \quad \text{with } f_k = 0 \text{ for lost phases.} \quad (5.6)$$

This yields:

Current references in the faulted phases equal to 0. This is in accordance with the fact that these phases are in open circuit and do not participate anymore to the torque production.

Current references in the remaining healthy phases consequently modified to maintain the torque to its reference value. Indeed, as the sum in the denominator of (5.6) depends on the sum of the square of functions $f_k$, this denominator and hence each current expression are modified when some of the $f_k$ are set to zero.

Without any constraint on the value of their sum, the expressions of optimal currents in fault-tolerant mode can thus be immediately derived from the solution in normal mode. The only adaptation needed is to set the $f_k$'s of the faulted phases to zero in expression (5.4).

5.2.2.2 Optimal currents with a zero-valued sum

In this case the trick of setting to 0 the $f_k$’s of the faulted phases does not work as it does not give currents equal to zero in the faulted phases. We have to compute the optimal currents by considering only the non-faulted phases in the computation process described in appendix 5.5.1.2. With $n_h$ the number of remaining active phases, this yields the following expression:

$$i_k = T_{ref} \frac{f_k - \frac{1}{n_h} \sum_{\kappa=1}^{n_h} f_{\kappa}}{\sum_{\kappa=1}^{n} f_{\kappa}^2 - \frac{1}{n_h} \sum_{\kappa=1}^{n_h} f_{\kappa} \sum_{\kappa=1}^{n} f_{\kappa}} \quad \text{with } f_k = 0 \text{ for lost phases.} \quad (5.7)$$
5.2.3 Application to machines with sinusoidal EMFs

5.2.3.1 Optimal currents in normal operation mode

In most of actuation systems involving an accurate position control, the motor is designed to have sinusoidal or nearly sinusoidal EMFs. For such machines, by discarding the small harmonic content of the EMFs if any, flux $\psi_k$ induced in phase $k$ can be written as:

$$\psi_k = \psi_0 \cos\left(p\theta_m - \frac{k-1}{n}2\pi\right)$$  \hspace{1cm} (5.8)

with $\psi_0$ the amplitude of the fundamental component of the flux. Its derivative in function of the rotor mechanical position, $\theta_m$, is therefore equal to:

$$f_k = -p\psi_0 \sin\left(p\theta_m - \frac{k-1}{n}2\pi\right)$$  \hspace{1cm} (5.9)

Using expression (5.9) in equation (5.4) gives the optimal currents for sinusoidal machines:

$$i_k = T_{ref} \frac{2}{np^2\psi_0^2} f_k$$

$$= -T_{ref} \frac{2}{np\psi_0} \sin\left(p\theta_m - \frac{k-1}{n}2\pi\right)$$  \hspace{1cm} (5.10)

$$= -I \sin\left(p\theta_m - \frac{k-1}{n}2\pi\right)$$

with $I$ defined as the current amplitude:

$$I = \frac{2T_{ref}}{np\psi_0}$$  \hspace{1cm} (5.11)

Optimal currents are also sinusoidal and isomorphic to their respective EMFs. As the currents are a balanced set of sinusoidal functions of the rotor position, their sum is equal to zero. Hence the optimal solution with the constraint of a zero-valued sum of the currents is the same as the general solution. Indeed, introducing (5.9) in equation (5.5) yields the same expression (5.10) since the sum of the $f_k$’s, which are a balanced set of sinusoidal functions, is equal to zero. This is in accordance with the fact that $f_k$’s with no homopolar components yields the same currents expression with or without the constraint of a sum equal to zero.
5.2.3.2 Optimal currents in fault-tolerant operation mode

Introducing expression (5.9) in equations (5.6) and (5.7) gives the optimal currents in fault-tolerant mode for machines with sinusoidal EMFs. The waveforms of the general expressions of the optimal currents for machines with 3 to 8 phases when one phase is lost are shown in Figures 5.2-5.7. Each figure shows:

- on top the EMFs in the remaining active phases;
- on bottom left the optimal currents in these phases when their sum may differ from zero;
- on bottom right the optimal currents in these phases when their sum is constrained to be zero valued.

Note that the two solutions are no more equal, even with sinusoidal $f_k$'s. Besides, the solution given in Figure 5.2(b) is not physically feasible because requiring an infinite current amplitude.

For machines with sinusoidal EMFs, the optimal currents in fault-tolerant mode lose the sinusoidal shape they have in normal operation mode. This makes the control of currents flowing in the motor more difficult. However, it must be noted that the greater the number of phases, the more the shapes of the currents tend to be close to sinusoidal shapes. Therefore it may be worthwhile looking for solutions which keep the sinusoidal shapes of the currents, as they would be easier to control and, with a sufficiently high number of phases, close to the optimal solutions.

5.3 Keeping the currents sinusoidal in fault-tolerant mode

This section determines for machines with sinusoidal fluxes the expressions of suboptimal currents which still aim to develop the needed torque while minimizing Joule’s losses, but with the constraints of remaining sinusoidal in fault-tolerant mode. The analysis is restricted to a single fault operation. We need to reformulate the criteria of:

- developing the same torque as in normal operation mode;
Control laws

Figure 5.2: Fault tolerant mode of a 3-phase motor with phase 1 lost in open circuit: EMFs (on top) and optimal currents (on bottom) in the 2 remaining phases

Figure 5.3: Fault tolerant mode of a 4-phase motor with phase 1 lost in open circuit: EMFs (on top) and optimal currents (on bottom) in the 3 remaining phases
Keeping the currents sinusoidal in fault-tolerant mode

Figure 5.4: Fault tolerant mode of a 5-phase motor with phase 1 lost in open circuit: EMFs (on top) and optimal currents (on bottom) in the 4 remaining phases.

Figure 5.5: Fault tolerant mode of a 6-phase motor with phase 1 lost in open circuit: EMFs (on top) and optimal currents (on bottom) in the 5 remaining phases.
Control laws

Figure 5.6: Fault tolerant mode of a 7-phase motor with phase 1 lost in open circuit: EMFs (on top) and optimal currents (on bottom) in the 6 remaining phases

Figure 5.7: Fault tolerant mode of a 8-phase motor with phase 1 lost in open circuit: EMFs (on top) and optimal currents (on bottom) in the 7 remaining phases
• minimizing Joule’s losses;
• possibly a zero valued sum of the currents;

with the idea of imposing sinusoidal waveforms to the currents.

5.3.1 Reformulation of the torque expression

The loss of one phase in open-circuit means that its EMF can not any-
more contribute to the torque production. In other word, the term of
the sum in equation (5.1) including its \( f_k \) has to be removed. However
this loss can be compensated if the \( f_k \) of the lost phase can be recompo-
sed from a linear combination of the \( f_k \)'s of the remaining active phases.
Assuming that phase 1 is lost\(^3\), we can find \( c_k \) factors such that:

\[
f_1 = \sum_{i=2}^{n} c_k f_k
\]  \hspace{1cm} (5.12)

With a balanced set of sinusoidal \( f_k \), each of the remaining \( f_k \) can
be divided in a component \( f_{k\parallel 1} \) in phase with \( f_1 \) and a component \( f_{k\perp 1} \)
in quadrature:

\[
f_1 = \sum_{i=2}^{n} c_k f_{k\parallel 1} + \sum_{i=2}^{n} c_k f_{k\perp 1} .
\]  \hspace{1cm} (5.13)

Defining \( f_{\perp 1} \) the angular variation of flux a phase in quadrature with
phase 1 would encircle, one may write:

\[
f_{k\parallel 1} = f_1 \cos\left(\frac{k-1}{n} 2\pi\right)\]
\[
f_{k\perp 1} = f_{\perp 1} \sin\left(\frac{k-1}{n} 2\pi\right)
\]  \hspace{1cm} (5.14)

as it is shown in Figure 5.8 for a 6 phase machine.

Introducing equations (5.14) in equation (5.13), one eventually gets:

\[
f_1 = \sum_{i=2}^{n} c_k \cos\left(\frac{k-1}{n} 2\pi\right) f_1 + \sum_{i=2}^{n} c_k \sin\left(\frac{k-1}{n} 2\pi\right) f_{\perp 1} . \]  \hspace{1cm} (5.15)

\(^3\)We will see in section 5.3.6 how to generalize the results to the loss of any phase.
The first term of the right part of equation (5.15) must be equal to \( f_1 \) while the second term must be equal to zero. Therefore the coefficients \( c_k \) must fulfill two constraints:

\[
\sum_{i=2}^{n} c_k \cos \left( \frac{k-1}{n} \frac{2\pi}{n} \right) = 1 \quad \& \quad \sum_{i=2}^{n} c_k \sin \left( \frac{k-1}{n} \frac{2\pi}{n} \right) = 0 \tag{5.16}
\]

Starting from equation (5.1) which gives the electromagnetic torque in healthy operation mode, we may write:

\[
T_{ref} = \sum_{i=1}^{n} i_k f_k \\
= i_1 f_1 + \sum_{i=2}^{n} i_k f_k \\
= i_1 \sum_{i=2}^{n} c_k f_k + \sum_{i=2}^{n} i_k f_k \tag{5.17}
\]

The last expression of the torque does not anymore depend on phase 1. The loss of phase 1 can be compensated by adding current components
isomorphic to $i_1$ to the currents which should flow in normal operation mode in the still active phases. This yields:

$$
\begin{align*}
    i_1' & = 0 \\
    i_2' & = i_2 + c_2 i_1 \\
    & \vdots \\
    i_n' & = i_n + c_n i_1
\end{align*}
$$

(5.18)

The added components restore the part of the torque given by phase 1 in normal mode. The main interest of the method is that, for a given torque, currents $i_2'$ to $i_n'$ are sinusoidal as a function of the rotor position, as they are a combination of currents which are sinusoidal in normal operation mode.

### 5.3.2 Reformulation of the minimal Joule’s losses criterion

With only constraints (5.16) of torque conservation, coefficients $c_k$ are not univocally defined. We have an infinity of solutions in the general case of an $n$-phase motor. Therefore we add the criterion of finding the one which minimizes the Joule’s losses.

With $I$ the current amplitude in normal mode (see equation (5.10), the amplitudes of the currents in fault-tolerant mode are given by:

$$
I_k' = I \sqrt{1 + 2c_k \cos \left( \frac{k-1}{n} 2\pi \right) + c_k^2}
$$

(5.19)

Thus the expression of Joule’s losses is proportional to:

$$
\sum_{i=2}^{n} \left( \frac{I_i'}{I} \right)^2 = \sum_{i=2}^{n} \left( 1 + 2c_k \cos \left( \frac{k-1}{n} 2\pi \right) + c_k^2 \right)
$$

(5.20)

The solution we are looking for must minimize the value of (5.20).

### 5.3.3 Reformulation of the zero valued sum of currents criterion

If the sum of the currents must be equal to zero, this adds a supplementary constraint on the values of coefficients $c_k$. Adding term by term
equations of system (5.18), we can make appear that constraint on the left part of the following equation:

\[
\sum_{i=1}^{n} i_k' = \sum_{i=2}^{n} i_k + \sum_{i=2}^{n} c_k i_1
\]

(5.21)

\[
\Rightarrow \sum_{i=2}^{n} c_k = 1
\]

(5.22)

5.3.4 Solution without a zero valued sum of the currents when phase 1 is lost

Using Lagrange’s principle, we aim to minimize equation (5.20) under the constraints given by equations (5.16):

\[
\mathcal{L}(c_k, \lambda_1, \lambda_2) = \sum_{k=2}^{n} \left( 1 + 2 c_k \cos \left( \frac{k-1}{n} 2\pi \right) + c_k^2 \right)
\]

\[
+ \lambda_1 \left( \sum_{k=2}^{n} c_k \cos \left( \frac{k-1}{n} 2\pi \right) - 1 \right) + \lambda_2 \left( \sum_{k=2}^{n} c_k \sin \left( \frac{k-1}{n} 2\pi \right) \right)
\]

(5.23)

Calculations are detailed in appendix 5.5.2 and yield the following values for coefficients \(c_k\):

\[
c_k = \frac{2}{n-2} \cos \left( \frac{k-1}{n} 2\pi \right)
\]

(5.24)

Factors \(c_k\) depend on number of phases \(n\) and indices \(k\) of the phases. The table 5.1 gives the values for motors with a number of phases from 3 to 8.

Introducing these values of \(c_k\) in equation (5.18) gives the currents expressions in fault mode. The phasor representation of the currents as well as their evolution as a function of the rotor position are shown in Figure 5.9 to 5.14 for machines with 3 to 8 phases. As we can see, the importance of the reconfiguration of the currents decreases with the number of phases. The reconfiguration of the 8-phase motor is marginal. By accepting a small pulsating torque, the reconfiguration could even be neglected.

The reconfiguration of currents obtained for the 3-phase motor does not actually imply a minimization of the Joule’s losses because with only
Keeping the currents sinusoidal in fault-tolerant mode

(a) (on top) $f_k$ in the 2 remaining phases; (on bottom) sinusoidal currents

(b) currents diagram

Figure 5.9: Sinusoidal fault-tolerant mode of a 3-phase motor with phase 1 lost in open circuit and without zero valued sum of currents

(a) (on top) $f_k$ in the 3 remaining phases; (on bottom) sinusoidal currents

(b) currents diagram

Figure 5.10: Sinusoidal fault-tolerant mode of a 4-phase motor with phase 1 lost in open circuit and without zero valued sum of currents
Figure 5.11: Sinusoidal fault-tolerant mode of a 5-phase motor with phase 1 lost in open circuit and \textbf{without} zero valued sum of currents

Figure 5.12: Sinusoidal fault-tolerant mode of a 6-phase motor with phase 1 lost in open circuit and \textbf{without} zero valued sum of currents
Keeping the currents sinusoidal in fault-tolerant mode

Figure 5.13: Sinusoidal fault-tolerant mode of a 7-phase motor with phase 1 lost in open circuit and without zero valued sum of currents

Figure 5.14: Sinusoidal fault-tolerant mode of a 8-phase motor with phase 1 lost in open circuit and without zero valued sum of currents
two parameters ($c_2$ and $c_3$) and the two constraints equations (5.16) the solution is unique and can be found in [7]. But the generalized method presented here works also in that case.

We may ask what the sum of currents $i'_\Sigma$ is:

\[
i'_\Sigma = \sum_{k=2}^{n} i_k + \sum_{k=2}^{n} c_k i_1
\]

\[
= -i_1 + \sum_{i=2}^{n} c_k i_1
\]

\[
\Rightarrow \frac{i'_\Sigma}{i_1} = -1 + \frac{2}{n-2} \sum_{k=2}^{n} \cos \left( \frac{k-1}{n} \frac{2\pi}{2} \right)
\]

\[
\Rightarrow \frac{i'_\Sigma}{i_1} = \frac{-n}{n-2}
\]

With a great number of phases, the sum of currents tends to the opposite of the lost phase current. This is because the reconfiguration of the remaining currents becomes negligible.

5.3.5 Solution with a zero valued sum of the currents when phase 1 is lost

We redefine Lagrange’s equation by adding constraint (5.22) of a zero valued sum of currents. This yields:

\[
\mathcal{L}'(c_k, \lambda_0, \lambda_1, \lambda_2) = \mathcal{L}(c_k, \lambda_1, \lambda_2) + \lambda_0 \left( \sum_{i=2}^{n} c_k - 1 \right)
\]
we get\footnote{See appendix 5.5.2.1 for calculation details}:

\[ c_k = \frac{1}{n-3} \left( 1 + 2 \cos \left( \frac{k-1}{n} 2\pi \right) \right) \quad (5.27) \]

The new values of the \( c_k \) factors are given in Table 5.2. There is no solution for the 3-phase machine. Indeed a 3-phase motor cannot run on two phases with a sum of currents equal to zero and a constant torque.

The phasor representations of the currents as well as their evolution as a function of the rotor position are given in Figures 5.15 to 5.19 for machines with 4 to 8 phases. A more important reconfiguration of the currents results with the zero valued sum constraint. Particularly we can see that for the 4-phase motor (see figure 5.15) the currents of phases 2 and 4 (in quadrature with the phase lost) are modified while they were not with the previous sinusoidal reconfiguration (see Figure 5.10). The components added to phases 2 and 4 do not contribute to the torque production, they just allow the sum of currents to be equal to zero.

### 5.3.6 General formulation of the solutions as a function of the index of the lost phase

Until now all computations have been made assuming that the lost phase is phase 1. In fact the lost phase may be any of the \( n \) phases. If the lost phase is phase \( k \), the modification of the currents can still be obtained from equations (5.18) provided that we replace the index 1 by \( k \), the index 2 by \( k + 1 \) (\( k + 1 - n \) if \( k + 1 \) greater than \( n \)) and so on.
By defining for each phase a factor $\alpha_k$ equal to 0 or 1 according to the fact that the phase is healthy or not, we can define a matrix $C_k$ which ensures the above mentioned permutations:

$$C_k = \begin{pmatrix} -\alpha_1 & \alpha_2 c_n & \ldots & \alpha_n c_2 \\ \alpha_1 c_2 & -\alpha_2 & \ldots & \alpha_n c_3 \\ \vdots & \vdots & \ddots & \vdots \\ \alpha_1 c_n & \alpha_2 c_{n-1} & \ldots & -\alpha_n \end{pmatrix}$$

and write in a compact form:

$$i'_{ref} = i_{ref} + C_k i_{ref} = (I + C_k) i_{ref}$$

(5.29)

with $I$ the $n \times n$ unity matrix.

In the normal operation mode, matrix $C_k$ is equal to zero. When one phase is lost, the corresponding column of $C_k$ redefines the currents consequently.
Keeping the currents sinusoidal in fault-tolerant mode

![Diagram](image)

(a) (on top) $f_k$ in the 4 remaining phases; (on bottom) sinusoidal currents

(b) currents diagram

Figure 5.16: Sinusoidal fault-tolerant mode of a 5-phase motor with phase 1 lost in open circuit and with zero valued sum of currents

![Diagram](image)

(a) (on top) $f_k$ in the 5 remaining phases; (on bottom) sinusoidal currents

(b) currents diagram

Figure 5.17: Sinusoidal fault-tolerant mode of a 6-phase motor with phase 1 lost in open circuit and with zero valued sum of currents
Figure 5.18: Sinusoidal fault-tolerant mode of a 7-phase motor with phase 1 lost in open circuit and with zero valued sum of currents

Figure 5.19: Sinusoidal fault-tolerant mode of a 8-phase motor with phase 1 lost in open circuit and with zero valued sum of currents
5.3.7 Remarks

5.3.7.1 Impact of neglecting the harmonic content of the EMFs in the computation of the currents

If the fluxes induced by the magnets are not purely sinusoidal, they may be split into a fundamental component and harmonics. The same applies to their derivatives with respect to $\theta_m$. In this case the optimal currents given by (5.4) are no more a balanced set of sinusoidal functions and their sum is generally no more equal to zero.

Nevertheless it is still possible to obtain sinusoidal reference currents by introducing in (5.4) only the fundamental components of the $d\psi_k/d\theta_m$. But then if the interaction of the reference currents with the fundamental components of $d\psi_k/d\theta_m$ produces the reference value of the torque, their interactions with the harmonics present in the $d\psi_k/d\theta_m$ will generate a pulsating torque component which depends on the ranks of the harmonics present in the $d\psi_k/d\theta_m$ and on their amplitudes.

5.3.7.2 Are additional components of currents in quadrature with the current of the lost phase useful?

When looking for sinusoidal currents in fault-tolerant mode, the Lagrangian expression computed to find the currents reconfiguration minimizing Joule’s losses assumed that the components added to the remaining currents are isomorphic to the current of the lost phase. One could say that this restricts the space of solution. Why not add components in quadrature with the lost current in each remaining current:

$$
\begin{align*}
    i'_1 &= 0 \\
    i'_2 &= i_2 + c_2i_1 + d_2i_{11} \\
    \vdots \\
    i'_n &= i_2 + c_ni_1 + d_ni_{11}
\end{align*}
$$

(5.30)

as shown in Figure 5.20 for the 5-phase case. Coefficients $d_k$, as well as coefficients $c_k$, have to be determined.

Those components do not participate to the compensation of the torque production because they are in quadrature with the lost current.
This yields the two following constraints:

\[
\sum_{i=2}^{n} d_k \cos \left( \frac{k-1}{n} 2\pi \right) = 0 \quad \& \quad \sum_{i=2}^{n} d_k \sin \left( \frac{k-1}{n} 2\pi \right) = 0
\]  

But they allow to scan the space of all solutions and could reduce the amplitudes of the remaining currents. The Lagrangian expression taking them into account is written as:

\[
\mathcal{L}(c_k, \lambda_1, \lambda_2, \mu_1, \mu_2) = \\
\sum_{k=2}^{n} \left( 1 + 2c_k \cos \left( \frac{k-1}{n} 2\pi \right) + 2d_k \sin \left( \frac{k-1}{n} 2\pi \right) + c_k^2 + d_k^2 \right) \\
+ \lambda_1 \left( \sum_{k=2}^{n} c_k \cos \left( \frac{k-1}{n} 2\pi \right) - 1 \right) + \lambda_2 \left( \sum_{k=2}^{n} c_k \sin \left( \frac{k-1}{n} 2\pi \right) \right) \\
+ \mu_1 \left( \sum_{k=2}^{n} d_k \sin \left( \frac{k-1}{n} 2\pi \right) \right) - \mu_2 \left( \sum_{k=2}^{n} d_k \cos \left( \frac{k-1}{n} 2\pi \right) \right)
\]  

without a zero valued sum of currents, and:

\[
\mathcal{L}'(c_k, \lambda_0, \lambda_1, \lambda_2, \mu_0, \mu_1, \mu_2) = \mathcal{L}(c_k, \lambda_1, \lambda_2, \mu_1, \mu_2) \\
+ \lambda_0 \left( \sum_{k=2}^{n} c_k - 1 \right) + \mu_0 \sum_{k=2}^{n} d_k
\]
with that constraint.

Computations are heavy and will not be detailed but they yield:

$$d_k = 0$$  \hspace{1cm} (5.34)

which means that components in quadrature with the lost current do not reduce the Joule's losses. Adding only isomorphic components gives the optimal sinusoidal solutions. The control strategies that will be presented in the next chapter\textsuperscript{5} have been developed by taking into account this specificity.

\section*{5.4 Comparison of the various solutions}

This section focuses on the impact of keeping the same torque in fault-tolerant mode for the four solutions developed previously: the optimal and sinusoidal solutions, with or without a zero valued sum of the currents.

The comparison is made under the assumption that the EMFs are purely sinusoidal.

\subsection*{5.4.1 Comparison on the basis of the increase of Joule’s losses}

We want to express the increase of the Joule’s losses between the normal operation mode and the fault-tolerant operation mode. For that purpose, we first compute the Joule’s losses expression in normal operation mode. Then we compute the Joule’s losses in fault-tolerant mode using the currents obtained for the various fault-tolerant solutions, and eventually we determine the increase factors by comparing the fault-tolerant solutions with the normal one.

\subsubsection*{5.4.1.1 Joule’s losses in normal operation mode}

With sinusoidal EMFs, the normal operation mode currents are defined by the following equation (see section 5.2.3.1):

$$i_k = -I \sin \left( p \theta_m - \frac{k - 1}{n} 2\pi \right)$$  \hspace{1cm} (5.35)

\textsuperscript{5}Control strategies are treated in chapter 6
with $I$ the current amplitude as a function of the torque reference:

$$I = \frac{2 \ T_{\text{ref}}}{n \ p \psi_0} \quad (5.36)$$

Therefore the Joule’s losses in normal operation mode, $P_J$, can be written as a function of the torque reference:

$$P_J = R \sum_{k=1}^{n} i_k^2$$

$$= \frac{n}{2} R I^2 \quad (5.37)$$

$$= R \frac{T_{\text{ref}}^2}{n \ p^2 \psi_0^2}$$

### 5.4.1.2 Joule’s losses with optimal fault-tolerant currents without a zero valued sum

To facilitate the computation, we assume that phase 1 is the lost phase. Hence $f_1$ does not participate to the torque production (see section 5.2.2.1):

$$i'_k = T_{\text{ref}} \frac{f_k}{\sum_{k=2}^{n} f_k^2} \quad (5.38)$$

Then the Joule’s losses expression in fault-tolerant mode, named $P'_f$, becomes:

$$P'_f = R \sum_{k=2}^{n} i'_k^2$$

$$= R T_{\text{ref}}^2 \frac{\sum_{k=2}^{n} f_k^2}{\left(\sum_{k=2}^{n} f_k^2\right)^2}$$

$$= R T_{\text{ref}}^2 \frac{1}{\sum_{k=2}^{n} f_k^2}$$

$$= R T_{\text{ref}}^2 \frac{1}{\sum_{k=1}^{n} f_k^2 - f_1^2}$$

$$= R T_{\text{ref}}^2 \frac{n}{2 \ p^2 \psi_0^2 - f_1^2} \quad (5.39)$$
Comparison of the various solutions

with $f_1$ the angular variation of the flux induced in phase 1 in normal operation mode:

$$f_1 = -p\psi_0 \sin(p \theta_m) \quad (5.40)$$

Therefore:

$$P'_J = R^2 \frac{T_{ref}^2}{n p^2 \psi_0^2} \frac{1}{1 - \cos(2p \theta_m)} \frac{n}{n}$$

$$= R^2 \frac{T_{ref}^2}{n p^2 \psi_0^2} \frac{n}{n - 1 + \cos(2p \theta_m)} \quad (5.41)$$

Introducing equation (5.37) in equation (5.41), we can express the Joule’s losses expression in fault-tolerant mode as a function of the Joule’s losses in normal operation mode:

$$P'_J = P_J \frac{n}{n - 1 + \cos(2p \theta_m)} \quad (5.42)$$

The expression of the Joule’s losses varies as a function of the rotor angular position $\theta_m$, from 1 time to $n/(n - 2)$ times the Joule’s losses in normal operation mode. The mean value on an electrical cycle is equal to:

$$<P'_J> = P_J \int_0^{2\pi/p} \frac{n}{n - 1 - \cos(2p \theta_m)} d\theta_m \quad (5.43)$$

$$= P_J \sqrt{\frac{n}{n - 2}} \quad (5.44)$$

5.4.1.3 Joule’s losses with optimal fault-tolerant currents with a zero valued sum

The mathematical development is the same as in the preceding section. But the optimal currents expression is given by (see section 5.2.2.2):

$$i_k = \frac{f_k - \frac{1}{n-1} \sum_{k=2}^{n} f_k}{\sum_{k=2}^{n} f_k^2 - \frac{1}{n-1} \sum_{k=2}^{n} f_k \sum_{k=2}^{n} f_k} \quad (5.45)$$

$$= \frac{f_k + \frac{1}{n-1} f_1}{\frac{n}{2} p^2 \psi_0^2 - \frac{n}{n-1} f_1^2}$$
The Joule’s losses are hence given by:

\[ P'_{J} = R \sum_{k=2}^{n} i_k'^2 \]

\[ = RT_{ref}^2 \frac{\sum_{k=2}^{n} f_k^2 + 2 \sum_{k=2}^{n} f_k \frac{1}{n-1} f_1 + \sum_{k=2}^{n} \frac{1}{(n-1)^2} f_1^2}{\left( \frac{\pi}{2} p^2 \psi_0^2 - \frac{n}{n-1} f_1^2 \right)^2} \]

\[ = RT_{ref}^2 \frac{n}{2} p^2 \psi_0^2 - f_1^2 - \frac{2}{n-1} f_1^2 + \frac{1}{n-1} f_1^2 \]

\[ \left( \frac{n}{2} p^2 \psi_0^2 - \frac{n}{n-1} f_1^2 \right)^2 \]

\[ = \frac{1}{2} \sum_{k=2}^{n} \frac{n-1}{n-1} f_1^2 \]

\[ = \frac{1}{2} \sum_{k=2}^{n} \frac{n-1}{n-1} f_1^2 \]

\[ = \frac{1}{2} \sum_{k=2}^{n} \frac{n-1}{n-1} f_1^2 \]

\[ = \frac{P_J}{n-2 - \cos(2p\theta_m)} \]

and their mean value on an electrical cycle:

\[ < P'_{J} > = P_J \int_0^{2\pi/p} \frac{n-1}{n-2 - \cos(2p\theta_m)} d\theta_m \]

\[ = P_J \sqrt{\frac{n-1}{n-3}} \]

5.4.1.4 Joule’s losses with sinusoidal fault-tolerant currents without a zero valued sum

With phase 1 lost, the fault-tolerant sinusoidal currents are given by:

\[ i'_k = i_k + c_k i_1 \]

(5.48)

with

\[ c_k = \frac{2}{n-2} \cos \left( \frac{k-1}{n} 2\pi \right) \]

(5.49)

\[ i_k \text{ and } i'_k \text{ are respectively the normal operation mode current and the fault-tolerant operation mode current of phase } k. \]
Comparison of the various solutions

The Joule’s losses expression can therefore be formulate as:

\[ P'_J = R \sum_{k=2}^{n} i'_k^2 \]

\[ = R \sum_{k=2}^{n} (i_k + c_k i_1)^2 \]

\[ = R \sum_{k=2}^{n} i_k^2 + 2R \sum_{k=2}^{n} i_k c_k i_1 + R \sum_{k=2}^{n} c_k^2 i_1^2 \]

The first one depends on the components of currents which are the same in normal and fault-tolerant operation mode:

\[ <1> = R \sum_{k=1}^{n} i_k^2 - R i_1^2 \]

\[ = P_J - R i_1^2 \]

and is equal to the Joule’s losses in normal operation mode minus those in phase 1. The second and third terms depend on coefficients \(c_k\). Using equation (5.35), we get:
\[<2> = 2R_i \sum_{k=2}^{n} i_k c_k\]
\[= \frac{2R_i}{n-2} \sum_{k=2}^{n} -I \sin \left( p\theta_m - \frac{k-1}{n} 2\pi \right) 2 \cos \left( \frac{k-1}{n} 2\pi \right)\]
\[= \frac{2R_i}{n-2} \left( \sum_{k=2}^{n} -I \sin(p\theta_m) + \sum_{k=2}^{n} -I \sin \left( p\theta_m - 2\frac{k-1}{n} 2\pi \right) \right)\]
\[= \frac{2R_i}{n-2} \left( \frac{(n-1)i_1 - i_1}{2(n-2)} \right)\]
\[= 2R_i^2 (5.52)\]

\[<3> = R_i^2 \sum_{k=2}^{n} c_k^2\]
\[= \frac{R_i^2}{(n-2)^2} \sum_{k=2}^{n} 4 \cos \left( \frac{k-1}{n} 2\pi \right)^2\]
\[= R_i^2 \frac{2}{n-2}\]

Eventually, the sum of the three terms gives the Joule’s losses in fault-tolerant operation mode with sinusoidal currents:

\[P_J' = <1> + <2> + <3>\]
\[= P_J - R_i^2 + 2R_i^2 + R_i^2 \frac{2}{n-2}\]
\[= P_J + \frac{n}{n-2} R_i^2\]

Using equations (5.35) and (5.37):

\[P_J' = P_J + \frac{n}{n-2} R_i^2 \frac{1 - \cos(2p\theta_m)}{2}\]
\[= P_J + P_J \frac{1 - \cos(2p\theta_m)}{n-2}\]
\[= P_J \frac{n-1 - \cos(2p\theta_m)}{n-2}\]
Comparison of the various solutions

The Joule’s losses increase factor with sinusoidal current s varies from 1 to \(\frac{n}{n - 2}\), like the equivalent ones with optimal currents given by equation (5.44), but their dependance as a function of the rotor position is not the same. The mean value on an electrical cycle is a little higher:

\[
< P'_J > = P_J \int_0^{2\pi/p} \frac{n-1 - \cos(2p\theta_m)}{n-2} d\theta_m
\]

\[
= P_J \frac{n-1}{n-2}
\]

(5.55)

5.4.1.5 Joule’s losses with sinusoidal fault-tolerant currents with a zero valued sum

With a zero valued sum, the expression of the Joule’s losses is still given by equation (5.50), but with coefficients \(c_k\) given by:

\[
c_k = \frac{3}{n-3} \left(1 + \cos\left(\frac{k-1}{n}2\pi\right)\right)
\]

(5.56)

The first term \(< 1 >\) is the same as for currents without a zero valued sum:

\[
< 1 > = R \sum_{i=1}^{n} i_k^2 - Ri_1^2
\]

\[
= P_J - Ri_1^2
\]

(5.57)

and the second term \(< 2 >\) gives the same result as previously although coefficients \(c_k\) are different:

\[
< 2 > = 2Ri_1 \sum_{k=2}^{n} i_k c_k
\]

\[
= \frac{2Ri_1}{n-3} \sum_{k=2}^{n} -I\sin\left(p\theta_m - \frac{k-1}{n}2\pi\right) \left(1 + 2\cos\left(\frac{k-1}{n}2\pi\right)\right)
\]

\[
= \frac{2Ri_1}{3-n} \left(\sum_{k=2}^{n} I\sin\left(p\theta_m - \frac{k-1}{n}2\pi\right) + \sum_{k=2}^{n} I\sin(p\theta_m) + \sum_{k=2}^{n} I\sin\left(p\theta_m - \frac{2k-1}{n}2\pi\right)\right)
\]

\[
= \frac{2Ri_1}{3-n} (i_1 - (n-1)i_1 + i_1)
\]

\[
= 2Ri_1^2
\]

(5.58)
The last term is different and gives higher losses:

\[ <3> = R_i^2 \sum_{k=2}^{n} c_k^2 \]

\[ = \frac{R_i^2}{(n-3)^2} \sum_{k=2}^{n} \left( 1 + 2 \cos \left( \frac{k-1}{n} 2\pi \right) \right)^2 \]

\[ = \frac{R_i^2}{(n-3)^2} \left( \sum_{k=2}^{n} 1 + 4 \sum_{k=2}^{n} \cos \left( \frac{k-1}{n} 2\pi \right) + 4 \sum_{k=2}^{n} \cos \left( \frac{k-1}{n} 2\pi \right)^2 \right) \]

\[ = R_i^2 \frac{3}{n-3} \]

Eventually, the sum of the three terms gives:

\[ P_J' = <1> + <2> + <3> \]

\[ = P_J - R_i^2 + 2R_i^2 + R_i^2 \frac{3}{n-3} \]

\[ = P_J + \frac{n}{n-3} R_i^2 \]

\[ = P_J \frac{n-2 - \cos(2p\theta_m)}{n-3} \]

With the constraint of keeping a zero valued sum of the currents, the Joule’s losses variations are higher: from 1 to \((n-1)/(n-3)\) times the normal operation Joule’s losses. The mean value is:

\[ <P_J'> = P_J \int_0^{2\pi/p} \frac{n-1 - \cos(2p\theta_m)}{n-3} d\theta_m \]

\[ = P_J \frac{n-2}{n-3} \]

### 5.4.1.6 Synthesis

The four expressions of Joule’s mean losses in fault mode are always equal to losses in normal mode times a factor depending on the number of phases \(n\). Table 5.3 gives those factors computed previously:

Figure 5.21 plots and gives the values of the factors for a number of phases from 3 to 8. Of course, the increase tends to 1 with the number
Comparison of the various solutions

<table>
<thead>
<tr>
<th>Fault tolerant solutions</th>
<th>without zero valued sum</th>
<th>with zero valued sum</th>
</tr>
</thead>
<tbody>
<tr>
<td>optimal</td>
<td>$\sqrt{\frac{n}{n-2}}$</td>
<td>$\sqrt{\frac{n-1}{n-3}}$</td>
</tr>
<tr>
<td>sinusoidal</td>
<td>$\frac{n-1}{n-2}$</td>
<td>$\frac{n-2}{n-3}$</td>
</tr>
</tbody>
</table>

Table 5.3: Increase factors of the Joule’s losses of the various solutions

of phases, as the contribution of the lost phase tends to zero. The main point is that there is no significative difference between the optimal solutions and their corresponding sinusoidal ones. The increase factor is not significantly higher when using the sinusoidal solutions instead of the optimal ones.

Another ascertainment is that the increase factor of a solution for $n$ phases with the constraint of keeping a zero valued sum is equal to the increase factor of the solution for $n-1$ phases without the constraint, both with sinusoidal and optimal solutions. In other words, for a fixed admissible threshold of Joule’s losses increase, a solution needing a zero valued sum of currents will require one more phase than a solution without.

Figure 5.21: Joule’s losses for the various configuration of currents
5.4.2 Comparison on the basis of the increase of the maximum current amplitude

After the Joule’s losses, the maximum current amplitude increase is an important factor to be considered. Indeed it is directly linked to the power electronic. As the motor sizing depends on the Joule’s losses, the power electronics sizing depends on the maximum current amplitude.

The phase with the higher amplitude of current depends on:

- the index of the lost phase;
- the number of phases;
- the solution adopted in fault-tolerant mode.

For example, for a 5-phase machine with phase 1 lost, the maximum current amplitude is found in phases 2 and 5 if the solution adopted is the sinusoidal one with a zero valued sum. Without zero valued sum, it is phases 3 and 4. In a 6-phase machine, it is the phase in opposition with the lost phase that carries the higher current without a zero valued sum, and the two neighboring phases with zero valued sum.

Therefore, as there is no general rule to find the phase with the highest current, the factor of increase of the current amplitude is found
Comparison of the various solutions

numerically for each number of phases and each solution. Figure 5.22 plots the maximum current amplitude increase and gives the value of the increase factor for 3- to 8-phase motors.

For a given number of phases, the various solutions have similar factors of increase. They are even exactly equal in the 4-phase case. The factor is smaller and smaller with the number of phases, but not in a smoothly way as for Joule’s losses, as the maximum current amplitude depends on the three conditions cited above.

Lastly, even if the differences are small, it is interesting to notice that the sinusoidal solutions are in most of the cases better than the optimal, and the sinusoidal solution with a zero valued sum, which was the worst solution in terms of Joule’s losses, is the best in terms of maximum current amplitude. Indeed imposing a zero valued sum somewhat balance the compensation between the remaining phases.

5.4.3 Comparison with solutions found in the literature

Some researches have already been done on the subject of finding currents reconfiguration to alleviate the loss of a phase. The particular case of running a 3-phase motor on the two remaining phases is already well-known [7, 8].

Authors of [9] found the sinusoidal solution with zero valued sum for a 4-phase motor (Figure 5.15) and [10] for the 5-phase one (Figure 5.16). For those cases the number of degrees of freedom in reconfiguring the remaining phase currents is low enough to find the solution without calling for a minimization criterion.

Authors of [9] also developed a method to find sinusoidal currents configurations minimizing Joule’s losses after the loss of one phase. They expect that Joule’s losses are minimized when the currents have all the same amplitude and a zero valued sum. These assumptions provides enough constraints for bypassing the use of a minimization method such as a Lagrangian’s principle. Their solutions are close to the ones found here, but the Joule’s losses are a few percents greater, as shown in Figure 5.23(a).

However the current maximum amplitude is lower. In fact we can suppose that they have found the best solution from the minimization of the current amplitude point of view.

To understand that, let start from the solution minimizing Joule’s
losses in a 5-phase motor with zero valued sum (Figure 5.16). Currents $i_2$ and $i_5$ are greater than $i_3$ and $i_4$. If we want to reduce the two greater currents, we’ll have to increase the two smaller for keeping the reconfiguration balance. The best solution is obtained when the four currents are of the same amplitude. This reasoning can be recursively applied for motors with greater number of phases. The solution minimizing the currents maximum amplitude must be the one with currents with the same amplitude but this has still to be demonstrated.

Eventually the method given in [9] does not give a generalized formulation. The computations have to be done from the beginning for each number of phases.
5.5 Appendix : Obtaining reference currents : detailed calculations

5.5.1 Optimal currents

5.5.1.1 General expression of optimal currents

The optimal currents are defined as those minimizing the Joule’s losses. These losses are proportional to:

$$\sum_{k=1}^{n} \frac{i_k i_k}{2}$$ (5.62)

and the torque expression is given by:

$$T = \sum_{k=1}^{n} i_k f_k$$ (5.63)

with $f_k$ the $\theta_m$-variation of the induced flux in phase $k$ by the magnets:

$$f_k = \frac{d\psi_k}{d\theta_m}$$ (5.64)

The optimal currents are found using a Lagrangian expression aiming to minimize the Joule’s losses under the constraint of developing a reference torque $T_{ref}$. Using equations (5.63) and (5.62), the Lagrangian expression is therefore:

$$\mathcal{L}(i_k, \lambda) = \sum_{k=1}^{n} \frac{i_k i_k}{2} - \lambda \left( \sum_{k=1}^{n} i_k f_k - T_{ref} \right)$$ (5.65)

The $k$ partial derivatives of equation (5.65) as a function of currents $i_k$ give the following constraints:

$$0 = \frac{\partial \mathcal{L}}{\partial i_k} = i_k - \lambda f_k$$ (5.66)

Multiplying each constraint by the corresponding $f_k$ and then summing them allows to use the constraint of a constant torque to find the
Lagrangian multiplier $\lambda$:

\[
0 = i_k f_k - \lambda f_k^2
\]

\[
0 = \sum_{k=1}^{n} i_k f_k - \lambda \sum_{k=1}^{n} f_k^2
\]

\[
0 = T_{ref} - \lambda \sum_{k=1}^{n} f_k^2
\]

\[
\Rightarrow \lambda = T_{ref} \frac{1}{\sum_{k=1}^{n} f_k^2}
\]

Introducing (5.67) in (5.66) gives the optimal currents in normal mode:

\[
i_k = T_{ref} \frac{f_k}{\sum_{k=1}^{n} f_k^2}
\]

### 5.5.1.2 Optimal currents with a zero-valued sum

The optimal currents are defined as those minimizing the Joule’s losses. These losses are proportional to:

\[
\sum_{k=1}^{n} \frac{i_k i_k}{2}
\]

and the torque expression is given by:

\[
T = \sum_{k=1}^{n} i_k f_k
\]

with $f_k$ the $\theta_m$-variation of the induced flux in phase $k$ by the magnets:

\[
f_k = \frac{d\psi_k}{d\theta_m}
\]

Adding the supplementary constraint of keeping their sum equal to zero:

\[
\sum_{k=1}^{n} i_k = 0
\]
Appendix : Obtaining reference currents : detailed calculations

the Lagrangian expression becomes:

\[
L(i_k, \lambda_0, \lambda_1) = \sum_{k=1}^{n} \frac{i_k i_k}{2} - \lambda_0 \left( \sum_{k=1}^{n} i_k f_k - T_{ref} \right) - \lambda_1 \sum_{k=1}^{n} i_k \tag{5.73}
\]

The k derivatives of (5.73) with respect to \(i_k\) give the following constraints:

\[
0 = \frac{\partial L}{\partial i_k} = i_k - \lambda_0 f_k - \lambda_1 \tag{5.74}
\]

Firstly adding these constraints gives the following equation:

\[
0 = \sum_{k=1}^{n} i_k - \lambda_0 \sum_{k=1}^{n} f_k - n \lambda_1
\]

Hence introducing the constraint (5.72) yields:

\[
\lambda_1 = -\frac{1}{n} \lambda_0 \sum_{k=1}^{n} f_k \tag{5.75}
\]

Then multiplying each constraint by the corresponding \(f_k\) before summing them gives:

\[
0 = \sum_{k=1}^{n} i_k f_k - \lambda_0 \sum_{k=1}^{n} f_k^2 - \sum_{k=1}^{n} \lambda_1 f_k
\]

Using (5.70) this equation allows to take into account the constraint of developing a constant torque:

\[
T_{ref} = \lambda_0 \sum_{k=1}^{n} f_k^2 + \lambda_1 \sum_{k=1}^{n} f_k \tag{5.76}
\]

Equations (5.75) and (5.76) determine the Lagrangian multipliers:

\[
\lambda_0 = T_{ref} \left( \frac{1}{\sum_{k=1}^{n} f_k^2 - \frac{1}{n} \sum_{k=1}^{n} f_k \sum_{k=1}^{n} f_k} \right) \tag{5.77}
\]

\[
\lambda_1 = T_{ref} \left( \frac{-1}{\sum_{k=1}^{n} f_k^2 - \frac{1}{n} \sum_{k=1}^{n} f_k \sum_{k=1}^{n} f_k} \right)
\]

and eventually the optimal currents with a sum of the currents equal to zero:

\[
i_k = T_{ref} \left( \frac{f_k - \frac{1}{n} \sum_{\kappa=1}^{n} f_\kappa}{\sum_{\kappa=1}^{n} f_\kappa^2 - \frac{1}{n} \sum_{\kappa=1}^{n} f_\kappa \sum_{\kappa=1}^{n} f_\kappa} \right) \tag{5.78}
\]
5.5.2 Fault tolerant sinusoidal currents

5.5.2.1 With a zero valued sum of the currents

Using Lagrange principle, one may find to minimize the total Joule’s losses by minimizing the criterion given by equation (5.20) under the constraints of obtaining the same torque as in normal mode (equation (5.16)) and with a zero sum of currents (equation (5.22)). That is to say:

$$L(c_k, \lambda_0, \lambda_1, \lambda_2) = \sum_{k=2}^{n} \left( 1 + 2c_k \cos\left(\frac{k-1}{n}2\pi\right) + c_k^2 \right) + \lambda_0 \left( \sum_{k=2}^{n} c_k - 1 \right)$$

$$+ \lambda_1 \left( \sum_{k=2}^{n} c_k \cos\left(\frac{k-1}{n}2\pi\right) - 1 \right) + \lambda_2 \left( \sum_{k=2}^{n} c_k \sin\left(\frac{k-1}{n}2\pi\right) \right)$$

(5.79)

Partial derivatives according to $c_k$, aimed to be null, are therefore given by:

$$0 = \frac{\partial L}{\partial c_k} = 2 \cos\left(\frac{k-1}{n}2\pi\right) + 2c_k + \lambda_0 + \lambda_1 \cos\left(\frac{k-1}{n}2\pi\right) + \lambda_2 \sin\left(\frac{k-1}{n}2\pi\right)$$

(5.80)

The partial derivative gives a system of $n-1$ equations, for $k$ from 2 to $n$:

$$0 = 2c_k + \lambda_0 + (\lambda_1 + 2) \cos\left(\frac{k-1}{n}2\pi\right) + \lambda_2 \sin\left(\frac{k-1}{n}2\pi\right)$$

(5.81)

The sum of those $n-1$ equations allows to use the constraint of the zero sum of currents:

$$0 = 2 \sum_{k=2}^{n} c_k + \lambda_0 \sum_{k=2}^{n} \cos\left(\frac{k-1}{n}2\pi\right) + (\lambda_1 + 2) \sum_{k=2}^{n} \cos\left(\frac{k-1}{n}2\pi\right) + \lambda_2 \sum_{k=2}^{n} \sin\left(\frac{k-1}{n}2\pi\right)$$

$$\Rightarrow \lambda_1 = \lambda_0 (n-1)$$

(5.82)

Multiplying the constraints by $\cos\left(\frac{k-1}{n}2\pi\right)$ and then summing them, one may apply the first constraint of obtaining the same torque as in
normal mode:

\[ 0 = \sum_{i=2}^{n} 2 \cos \left( \frac{k-1}{n} 2\pi \right) c_k + \lambda_0 \sum_{i=2}^{n} \cos \left( \frac{k-1}{n} 2\pi \right) \]

\[ + (\lambda_1 + 2) \sum_{i=2}^{n} \cos \left( \frac{k-1}{n} 2\pi \right)^2 + \frac{1}{2} \lambda_2 \sum_{i=2}^{n} \sin \left( \frac{k-1}{n} 4\pi \right) \]  

(5.83)

\[ \Rightarrow \lambda_0 = (n-2) \frac{\lambda_1}{2} + n \]

Multiplying the constraints by \( \sin \left( \frac{k-1}{n} 2\pi \right) \) and then summing them, one may apply the second constraint of obtaining the same torque as in normal mode:

\[ 0 = \sum_{i=2}^{n} 2 \sin \left( \frac{k-1}{n} 2\pi \right) c_k + \lambda_0 \sum_{i=2}^{n} \sin \left( \frac{k-1}{n} 2\pi \right) \]

\[ + (\lambda_1 + 2) \sum_{i=2}^{n} \sin \left( \frac{k-1}{n} 4\pi \right) + \frac{1}{2} \lambda_2 \sum_{i=2}^{n} \sin \left( \frac{k-1}{n} 2\pi \right)^2 \]  

(5.84)

\[ \Rightarrow \lambda_2 = 0 \]

Eventually, using (5.81), (5.82), (5.83) and (5.84), one may determine all the Lagrangian multipliers:

\[ \lambda_0 = \frac{2}{3-n} \]  

(5.85)

\[ \lambda_1 = \frac{2n-1}{3-n} \]  

(5.86)

\[ \lambda_2 = 0 \]  

(5.87)

\[ c_k = \frac{1}{n-3} \left( 1 + 2 \cos \left( \frac{k-1}{n} 2\pi \right) \right) \]  

(5.88)

### 5.5.2.2 Without a zero valued sum of the currents

The resolution process is the same as without the zero valued sum. The equations are the same except that we impose \( \lambda_0 = 0 \) in Lagrangian equation (5.79) to remove the constraint of the zero sum. The Lagrangian
parameter $\lambda_1$ is then straightforwardly given by equation (5.83):

$$\lambda_1 = \frac{2n}{2 - n} \quad (5.89)$$

And from the system of equations (5.81), we get coefficients $c_k$:

$$c_k = \frac{2}{n - 2} \cos \left( \frac{k - 1}{n} 2\pi \right) \quad (5.90)$$
Control strategies

6.1 Introduction

In order to apply to the motor the current references determined in the previous chapter, we need to define the adequate control scheme for generating the voltages applied to the machine windings by the voltage sourced inverter feeding it. The simplest closed-loop controller for tracking the reference currents is the sigma-delta controller. As shown in Figure 6.1, the current references are computed in the stator frame, depending on the torque reference $T_{\text{ref}}$ and the rotor mechanical position $\theta_m$ according to the laws presented in the preceding chapter. Each reference current is compared to the measured one, and the error is send, as shown in Figure 6.1, to an hysteresis comparator which directly generates the PWM signal for controlling the transistors of the leg or of the 'H' bridge feeding the corresponding phase.

The advantages of this type of control is that any of the current laws defined in the previous chapter can be used for the motor currents, the optimal ones as well as the sinusoidal ones. The reconfiguration to adapt the control to the loss of a phase (i.e. for going from the normal operation mode to the fault tolerant operation mode) just implies to modify the current laws determining the current references. Most of the
authors dealing with fault tolerant operation mode of PM synchronous
machine use a Sigma-Delta controller to validate their fault tolerant
current laws, see [13] for instance. Indeed the Sigma-Delta control is
the best suited one for applying the optimal current references, without
needing a high computation process.

As said in the previous chapter, for machines with sinusoidal EMFs,
in fault tolerant mode, the solutions imposing sinusoidal currents do
not increase substantially the Joule’s losses in comparison with those
obtained with the optimal currents. Besides, sinusoidal currents are
easier to control than the optimal ones since they offer the possibility
of developing the control scheme in the rotor frame, by using a pro-
per variable transformation $T$ on the measured currents and its inverse
on the reference voltages computed by the controller. In that frame,
control variables are independent of the rotor position and we can build
a vector control similar to that used for three phase machines (Figure
6.2), with 'PI' controllers. The voltages computed by the vector control
are converted into PWM signals at a constant frequency to control the
power electronics.

In comparison, the Sigma-Delta control requires a variable switching
frequency with a greater maximum value, and hence more losses in the
power electronics. Besides, with an independent control of the phases
and the use of $n$ 'H' bridges, the harmonic content of the voltages can still
be reduced with the PWM signals by introducing a three level control.
Eventually, the control in the rotor frame offers some advantages for the
fault detection, as it will be explained in this chapter.

Of course there are some more complex Sigma-Delta control strate-
gies, using a vector approach to limit the commutation frequency and
the associated commutation losses [14–16], but their complexity of im-
Implementation is similar to that of the vector control.

For developing our fault-tolerant strategies in this chapter, we chose the vector control in the rotor frame.

In literature, we can find how to build generalized transformations to translate the measured currents in variables controlled in the rotor frame, in normal and fault tolerant operation mode [17, 18, 21]. We also published one generalized transformation [24], that we applied to the control of 5 and 6 phase machines [11, 12].

We have shown that this approach has a major drawback. In order to keep the same reference currents in the rotor frame in normal and in fault tolerant operation mode, we need to define two transformations.

The first transformation is defined to translate, in normal operation mode, the reference currents in the stator frame into reference currents in the rotor frame, which are independent of the rotor position. With this transform, in the rotor reference frame, the equations of the machine used for building the vector control are independent of the rotor position.\footnote{It will be shown in the next section that those equations are equivalent to the equations of an equivalent DC machine with two armature windings and of the equations of some RL circuits. The whole set of equation is independent of the rotor position.}

The second transformation is defined to translate, in fault tolerant operation mode, the reference currents in the stator frame into the same reference currents in the rotor frame as in the normal operation mode.

Figure 6.2: Control scheme in the rotor frame
Control strategies

Unfortunately, it can be shown that due to the unbalance born with the lost phase, the equations of the machine in the rotor frame, which are used for building the vector control, still depend on the rotor position\(^2\), what reduces the benefits of controlling the currents in the rotor frame.

In the first part of this chapter, a control in the rotor frame which is not subject to this drawback is presented. It will be shown that, by an adequate modification of the measured currents and of the reference voltages, the generalized transformation computed for the normal operation mode can also be used in fault tolerant operation mode, without modifying the reference values and the vector control (Figure 6.3).

The second part is dedicated to a control strategy resolving the problem in another way. Instead of defining a generalized transformation for the whole system of variables, we define one different transformation for each phase. We will show that it is then possible to control each current in an appropriate rotor reference frame and to reconfigure the control in fault-tolerant operation mode only with an action on the reference values of the currents in the rotor frame of each phase. This control strategy has been published for the case of the 3-phase machine in [7]. We extend it to the case of a \(n\)-phase machines.

\(^2\)These equations are not equivalent to those of a DC machine, and all of them include terms that vary at twice the electrical position.
6.2 Vector control in an extended Park reference frame

This section is divided in three parts. The first part explains the way to build a control in the rotor frame in normal operation mode for a \( n \)-phase machine. Then the second part explains how to extend the same vector control scheme to be able to run the machine in fault-tolerant mode, after the loss of one phase. Eventually the last part is a short note about a state of the art about the fault detection and the determination of the faulted phase, necessary to adapt the control scheme from the normal operation mode to the correct fault-tolerant one.

Before we start, it is important to note that for developing the control, we assume that the phases are electrically independent from the others. It means that the mutual inductances:

- either are negligible and the phases are magnetically decoupled;
- or are not fully negligible but are not significantly different from each other. That means that, provided that the sum of the phase currents is equal to zero, we can define a cyclical inductance, as stated in section 3.5, to recover the independence of the phases.

In the second case we must consequently use a control law of the currents which imposes that the sum of the currents is equal to zero.

The electrical equations used for developing the control are thus:

\[
\begin{align*}
\dot{u}_k &= R_i_k + L \frac{d}{dt}i_k + e_k \\
T_{em} &= \sum_i i_k \frac{d\psi_k}{d\theta_m} = \frac{1}{p\theta_m} \sum i_k e_k
\end{align*}
\]  

(6.1)

with:

- \( u_k \) the voltage of phase \( k \),
- \( i_k \) the current of phase \( k \),
- \( e_k \) the EMF of phase \( k \),
- \( \psi_k \) the flux induced by the magnets in phase \( k \),
- \( R \) the phase resistance,
Control strategies

- $L$ the phase self (or cyclical) inductance,
- $T_{em}$ the electromechanical torque,
- $p$ the number of pole pairs,
- $\theta_m$ the mechanical rotor position,

6.2.1 Control in normal operation mode

As stated in section 5.2.3.1, for machines with sinusoidal fluxes, in the normal operation mode, the optimal reference currents for a given torque are isomorphic to the fluxes variations induced in the phases by the magnets. They are corresponding, like the EMFs, to a balanced set of $n$ sinusoidal functions of the rotor position. Their amplitudes are proportional to the reference torque:

$$i_k = T_{ref} \frac{2}{np^2\psi_0^2} f_k$$

$$= -T_{ref} \frac{2}{np\psi_0} \sin \left( p\theta_m - \frac{k-1}{n} 2\pi \right)$$

$$= -I \sin \left( p\theta_m - \frac{k-1}{n} 2\pi \right)$$

with $I$:

$$I = \frac{2}{n} \frac{T_{ref}}{p\psi_0}$$

Their sum, as for the EMFs, is equal to zero and they can be represented by a set of phasors rotating at a speed $p\dot{\theta}_m$ (see section 5.3). As we consider a magnetic decoupling of the phases, we may therefore consider as a generalization of the Concordia transform any transformation which translates any set of voltages or currents which corresponds to a set of $n$ sinusoidal functions into a balanced set of two sinusoidal components in quadrature (named the $\alpha$ and $\beta$ components) and $n-2$ "homopolar" components all equal to zero, as shown in Figure 6.4. In a mathematical form, one may write:

$$x_{\alpha\beta ij} = Cx_{1,n}$$
Vector control in an extended Park reference frame

Figure 6.4: Generalized Concordia transformation

with:

\[ x_{1n} = \begin{pmatrix} x_1 & x_2 & x_3 & \cdots & x_n \end{pmatrix}^T \quad (6.5) \]

the vector of the values of the variables in the stator frame and

\[ x_{\alpha\beta0j} = \begin{pmatrix} x_\alpha & x_\beta & x_{01} & \cdots & x_{0n-2} \end{pmatrix}^T \quad (6.6) \]

the values of the variables in the corresponding \( \alpha\beta \) frame:

There is not a unique generalized Concordia transform. The two first lines of the transformation generate the \( \alpha\beta \) components and are univocally defined, as they are corresponding to the sum of the projection of the phasors on two perpendicular axes stationary with respect to the stator. However the remaining \( n - 2 \) combinations of the above mentioned set of phasors are not univocally defined. Each one must fulfill the constraints of being equal to zero when applied to a balanced set of variables, and of being independent from the other combinations. One
may select for instance the transform proposed in [17, 18]:

\[
C = \sqrt{\frac{2}{n}} \begin{pmatrix}
1 & \cos\left(\frac{2\pi}{n}\right) & \cdots & \cos\left(\frac{n-1}{n}2\pi\right) \\
0 & \sin\left(\frac{2\pi}{n}\right) & \cdots & \sin\left(\frac{n-1}{n}2\pi\right) \\
\vdots & \vdots & \ddots & \vdots \\
1 & \cos\left(\frac{n-1}{2}2\pi\right) & \cdots & \cos\left(\frac{n-1}{2}n-1\right)2\pi \\
0 & \sin\left(\frac{n-1}{2}2\pi\right) & \cdots & \sin\left(\frac{n-1}{2}n-1\right)2\pi \\
1 & \frac{1}{\sqrt{2}} & \cdots & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \cdots & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \cdots & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \cdots & \frac{1}{\sqrt{2}} \\
\end{pmatrix}
\]  

(6.7)

for a machine with an odd number of phases \(n\), and

\[
C = \sqrt{\frac{2}{n}} \begin{pmatrix}
1 & \cos\left(\frac{2\pi}{n}\right) & \cdots & \cos\left(\frac{n-1}{n}2\pi\right) \\
0 & \sin\left(\frac{2\pi}{n}\right) & \cdots & \sin\left(\frac{n-1}{n}2\pi\right) \\
\vdots & \vdots & \ddots & \vdots \\
1 & \cos\left(\frac{n-1}{2}2\pi\right) & \cdots & \cos\left(\frac{n-1}{2}n-1\right)2\pi \\
0 & \sin\left(\frac{n-1}{2}2\pi\right) & \cdots & \sin\left(\frac{n-1}{2}n-1\right)2\pi \\
1 & \frac{1}{\sqrt{2}} & \cdots & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \cdots & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \cdots & \frac{1}{\sqrt{2}} \\
\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \cdots & \frac{1}{\sqrt{2}} \\
\end{pmatrix}
\]  

(6.8)

for machine with an even number of phases.

Applying any generalized Concordia transform to the motor electrical equations yields the same set of equations in the \(\alpha\beta\) frame:

\[
\begin{align*}
u_\alpha &= Ri_\alpha + L\frac{di_\alpha}{dt} + e_\alpha \\
u_\beta &= Ri_\beta + L\frac{di_\beta}{dt} + e_\beta \\
u_{0j} &= Ri_{0j} + L\frac{di_{0j}}{dt}, \quad 1 \leq j \leq n - 2 \\
T_{em} &= \frac{1}{p\dot{\theta}_m} (i_\alpha e_\alpha + i_\beta e_\beta)
\end{align*}
\]  

(6.9)

These equations can be seen as those of an equivalent two phase synchronous machine and of \((n - 2)\) \(R\)-\(L\) circuits which do not contribute to the torque production. When applied to the optimal reference currents given by equation (6.2), any generalized Concordia transform
Vector control in an extended Park reference frame

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gives for the $\alpha\beta$ components a two-phase balanced set of sinusoidal currents in phase with $e_\alpha$ and $e_\beta$, and $n-2$ homopolar components equal to zero. This is obvious since only the components of $i_\alpha$ and $i_\beta$ in phase with $e_\alpha$ and $e_\beta$ are contributing to the torque production.

Some authors have proposed another type of generalized Concordia transform [19, 20]. This type of transformation lies on a division of the polyphase machine into a set of fictitious 1- or 2-phase machines, in order to be able to diagonalize the matrix of inductances. But this is beyond the scope of this thesis as this matrix is inherently diagonal for segment motors.

In order to obtain the machine equations in the frame linked to the rotor, an extended Park transformation is applied to the $\alpha\beta0j$ components of voltages and currents, in order to transform them in their $dq0j$ components:

$$
x_{dq0j} = Px_{\alpha\beta0j} = PCx_{1n}
$$

The extended Park transformation $P$ is given by:

$$
P = \begin{pmatrix}
\cos (p\theta_m) & \sin (p\theta_m) & 0 & \ldots & 0 \\
-\sin (p\theta_m) & \cos (p\theta_m) & 0 & \ldots & 0 \\
0 & 0 & 1 & \ldots & 0 \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & \ldots & 1
\end{pmatrix}
$$

The generalized Park transform modifies only the $\alpha\beta$ components, as the homopolar ones are already independent of the rotor position. In the $dq$ frame\footnote{The rotor frame defined by the Park transform.}, all the components of the optimal reference currents are equal to zero, excepted the $q$ axis component which is proportional to the torque. It is thus very easy to build from $T_{ref}$ the reference values of $i_{dq0j,ref}$:

$$
i_{dq0j,ref} = \begin{pmatrix}
0 & \sqrt{\frac{T_{ref}}{n \cdot p\psi_0}} & 0 & \ldots & 0
\end{pmatrix}^T
$$

If we take a look back at Figure 6.2, which represents the vector control in normal operation mode, we can see that:

- Equation (6.12) is the control law in the rotor frame.
• The transformation $T$ is equal to the consecutive transformations of Concordia and Park $PC$. $T^{-1}$ is equal to $C^{-1}P^{-1}$.

Therefore, we just need to define the machine equations in the $dq$ frame in order to build in normal operation mode the vector control in the rotor frame:

$$
\begin{align*}
    u_d &= Ri_d + L \frac{di_d}{dt} - p\dot{\theta}_m L i_q \\
    u_q &= Ri_q + L \frac{di_q}{dt} + p\dot{\theta}_m L i_d + \sqrt{\frac{n}{2}} \psi_0 p\dot{\theta}_m \\
    u_{0j} &= Ri_{0j} + L \frac{di_{0j}}{dt}, \quad 1 \leq j \leq n - 2 \\
    T_{em} &= \sqrt{\frac{n}{2}} p\psi_0 i_q 
\end{align*}
$$

(6.13)

In the $q$ axis equation, the term depending on $\psi_0$ is a representative of the EMFs induced in the phases by the rotor magnets. If we assume that these EMFs will be compensated in the voltages applied to the phases by proper feed-forward actions we may discard this term in (6.13) for synthesizing the controller. Under this assumption, all the circuits reduce to first order ones with the same resistance $R$ and inductance $L$ excepted that in the $dq$ ones, coupling terms between the two circuits are added. This explains the structure of the controller used for controlling the Park components of the motor currents depicted in Figure 6.5. A multivariable PI controller with feed forward actions compensating the coupling terms is used for the $dq$ components and $n - 2$ simple PI controllers for the homopolar components. The same proportional and integral gains may be used for all the PI controllers.

In details, the architecture of the motor torque controller depicted in Figure 6.5 is the following one:

• The Concordia and Park transform ($PC$ block) translating the measured phase currents in their $dq$ components.

• The “MPI” block containing $n$ PI discrete controllers.

• The inverse Park and Concordia transformations ($C^{-1}P^{-1}$ block) translating the $u_{dq,\text{ref}}$ voltages generated by the controller in the phase voltages $u_{1n,\text{ref}}$.

*Like the classic vector control of three phase machines.
Vector control in an extended Park reference frame

Figure 6.5: Extended vector control with zoom inside the multivariable PI controller
• The feed forward terms $C^{-1}P^{-1}\dot{e}_{dq}$ corresponding to the EMFs induced by the magnets ($\dot{e}_{dq}$ is a vector with all its components equal to zero excepted the q one which is equal to $\sqrt{n/2}\dot{\psi}_0\theta_m$).\textsuperscript{5}

Voltages $u_{1,n}$ are the voltages that must be applied to the motor phases.

With the use of a $n$-leg inverter with a star connection of the windings, the sum of the currents, which corresponds to the last line of the generalized Concordia transform, is automatically set to zero. The last PI controller must be removed.

### 6.2.2 Control in fault-tolerant mode

In the present section we show how to extend the vector control from the normal operation mode to a fault-tolerant operation mode, able to feed the machine with the normal operation mode currents as well as with the fault-tolerant ones, after the loss of any phase in open circuit. The control will be upgraded from the one represented in Figure 6.2 to the one represented in Figure 6.3.

In order to do that, we must define in what consist the modifications of the inputs and of the outputs of the controller.

#### 6.2.2.1 Modification of the inputs

In normal operation mode, the input vector of the MPI is the error signal between the reference currents and the measured currents in the $dq$ frame. The signal error may be written as follow:

$$\epsilon_i = i_{dq,ref} - PCI$$

(6.14)

By writing the reference currents in the stator frame, one gets:

$$\epsilon_i = PC(i_{1n,ref} - i)$$

(6.15)

To be able to run with a lost phase, the reference currents are re-defined by equation (5.29) defined in the previous chapter\textsuperscript{6}, and given below:

$$i'_{1n,ref} = i_{1n,ref} + C_ki_{1n,ref}$$

(6.16)

\textsuperscript{5}When the motor EMFs are not purely sinusoidal, the feed forward terms must include the harmonic content of the EMFs in order to compensate their effect on the evolution of the currents.

\textsuperscript{6}in section 5.3.6
Vector control in an extended Park reference frame

with $C_k$ a matrix that modifies the currents in fault-tolerant operation mode according to the lost phase.

Introducing equation (6.16) in equation (6.15), the error signal $i'_{\text{ref}} - i$ between the reference currents in fault-tolerant operation mode and the measured phase currents can be written as follow:

$$
\epsilon_i = PC(i'_{1n,\text{ref}} - i) \\
= PC(i_{1n,\text{ref}} + C_ki_{1n,\text{ref}} - i) \\
= PCi_{1n,\text{ref}} - PC(i - C_ki_{1n,\text{ref}}) \\
= i_{dq,\text{ref}} - PC(i - C_kC^{-1}P^{-1}i_{dq,\text{ref}})
$$

Under that form, the error signals are defined as a function of the reference currents in normal operation mode. To do that, we just need to subtract $C_kC^{-1}P^{-1}i_{dq,\text{ref}}$ to the measured currents before their Concordia-Park transformation.

The modification is highlighted by the difference between Figures 6.6 and 6.7. Figure 6.6 represents the part of the controller scheme in normal operation mode upstream of the MPI block. The control scheme represented in Figure 6.7 subtracts $C_kC^{-1}P^{-1}i_{dq,\text{ref}}$ to the measured currents, so that the reference currents in the rotor frame remain unchanged from the normal operation mode.
6.2.2.2 Modification of the outputs

The modification of the input of the controller allows us to use the MPI block with the same reference currents in normal and in fault-tolerant operation mode. It means that, in fault-tolerant operation mode, we introduce an error signal in the PI controller defined for the normal operation mode. The controller hence generates the $dq,ref$ voltages needed for aligning the currents $(i - C_k i_{ref})$ on $i_{ref}$. In the machine, we want to align $i$ on $(i_{ref} + C_k i_{ref})$.

By the Rule of Three, if the $u_{1n,ref}$ generated by the PI controller align the currents on $i_{ref}$, then, $u_{1n,ref} + C_k u_{1n,ref}$ align them on $(i_{ref} + C_k i_{ref})$. Indeed this is obvious since as long as the phase EMFs are discarded the $n$ circuits in which we want to control the currents are identical.

In practice, this simply implies to add $C_k u_{1n,ref}$ to the $u_{1n,ref}$ generated by the PI controller, as shown in Figure 6.8.

One can notice that in normal operation mode, all the entries of $C_k$ are equal to zero, what nullifies the modifications of the input and outputs of the controller. The control scheme of Figure 6.8 goes back to the controller of Figure 6.6.

6.2.3 Note on the fault detection strategies

The vector control has been enhanced to be able to run with a lost phase. The last step to make the control fault-tolerant is the detection of the fault and the determination of the faulted phase in order to properly reconfigure the control.
A lot of methods have been published in the literature, a very good survey may be find in [26]. Most of the fault diagnosis methods use the information given by the measured currents and require no additional sensors or devices. Some of them are listed here below:

- the Park’s Vector Method [27];
- the Normalised DC Current Method [28];
- the Modified Normalised DC Current Method [29];
- the Slope Method [30];
- the Current Signature Analysis [31];
- the EKF-based observer [32];
- the Normalised Absolute Current Method [33].

Some authors also developed methods based on the voltage analysis. [34,35] Those methods have lower detection time, but require additional sensors that complicate the power electronics, and make the system less reliable, as these sensors are also subject to a failure occurrence.

With those methods, one can achieve a fault detection below the electrical period, and easily locate the phase undergoing the fault. But a detailed analysis of the fault detection strategies and their implementation are not performed in the frame of this thesis and could be a post doctoral research.

To interface the implementation of one of these detection strategies with the fault-tolerant control scheme presented in this section, we just need to define the output of the detection system as \( k \) factors \( \alpha_k \) all equal to 0 in normal operation mode. When a phase is considered as faulted, its coefficient is set to 1.

Defined in that way, and according to equation (5.28), those factors are the ones needed to adapt the matrix \( C_k \) to the adequate fault-tolerant mode. The whole control strategy including the detection system is depicted in Figure 6.9.
6.3 Vector control using a Park reference frame per phase

In opposition with the control strategy introduced in the previous section, the control presented in this section aims to control the phase independently. In order to do that, this control associates to each phase a controller with its own rotor frame, allowing to control each current with its own $q$-axis and $d$-axis references as shown in Figure 6.10.

It is important to notice that, as the phase currents are controlled independently, the machine cannot be fed by a $n$-leg inverter with a star connexion of the phases, as it imposes a zero valued sum of the currents and generates a dependance between the currents. A power architecture with $n$ ‘$\Pi$’ bridges\(^7\) as shown in Figure 6.10 ensures the independence of the currents.\(^8\)

The main advantage of this control scheme is that it is exactly the same in normal and in fault-tolerant operation mode. By setting adequately the $q$-axis and $d$-axis reference currents in the controller of each

---

\(^7\)For example, we may choose the architecture with thyristor presented in section 4.4.

\(^8\)This can also be obtained by adding to a $n$-bridge inverter a $(n+1)$th leg connected to the neutral point of the star connected phases.
Vector control using a Park reference frame per phase

Controller 1

Controller 2

Controller n

Figure 6.10: Global control scheme

phase, we can control the phase currents to align them with the current reference defined for the normal operation mode as well as for the fault-tolerant operation mode.

6.3.1 Current references in normal operation mode

As shown in Figure 6.11, we align the $q$-axis of each of the $n$ reference frames used with the phasor associated to the corresponding reference current in normal operation mode. The $d$-axis of each phase is aligned on the phasor associated to the flux $\psi_k$ encircled by the phase. Therefore, the projections of the reference currents on the $d$ and $q$ axes of the rotor
reference frames associated to each phase, are equal to:

\[ i_{qk,ref} = |i_{k,ref}| \]
\[ i_{dk,ref} = 0 \]  \hspace{1cm} (6.18)

or, according to equation (6.2):

\[ i_{qk,ref} = \frac{2}{np\psi_0} T_{ref} \]
\[ i_{dk,ref} = 0 \]  \hspace{1cm} (6.19)

6.3.2 Reference currents in fault-tolerant mode

In fault-tolerant mode, assuming that phase 1 is lost, we have shown that to alleviate the loss of the phase we must add components equal to \( c_ki_{1,ref} \) to the reference currents of the remaining healthy phases. For machines with negligible mutual inductances between the phases we may use the coefficients \( c_k \) given by Table 5.1. When the mutual inductances can not be neglected we have seen in section 3.5 that it is possible to diagonalize the inductance matrix by introducing a cyclical inductance provided that the sum of the phase currents is equal to zero. In that case it is mandatory to use for the \( c_k \) the values given in table 5.2.

Therefore, we find the reference currents in fault-tolerant mode by adding to the reference currents in normal operation mode the projection of the phasor associated to \( c_ki_{1,ref} \) onto the \( d \) and \( q \) axes of the frame associated to each phase. This yields:

\[ i_{qk,ref} = \frac{2}{np\psi_0} T_{ref} \left( 1 + c_k \cos \left( \frac{k-1}{n} \pi \right) \right) \]
\[ i_{dk,ref} = \frac{2}{np\psi_0} T_{ref} c_k \sin \left( \frac{k-1}{n} \pi \right) \]  \hspace{1cm} (6.20)

as shown in Figure 6.12 with the phasor representation of the reference currents of the 3, 4, 5 and 6 phase machines with coefficients \( c_k \) given by Table 5.1.

Of course, when the lost phase is phase \( j \) instead of phase 1 we need to make the proper permutation of the phase indexes, as indicated in section 5.3.6.
Vector control using a Park reference frame per phase

6.3.3 Control scheme

For ensuring the control of $i_k$ in the reference frame associated to phase $k$ according to the reference values given by equation (6.19) or (6.20), we need to determine its Park components. But this is not possible on the basis of the sole measurement of this current.

In [7], the authors presents a way to overcome the problem in the case of a three phase machine. They consider that each phase is the phase $\alpha$ of a fictitious two phase machine and they estimate inside the controller the current in the fictitious phase $\beta$ on the basis of a model
Figure 6.12: Phasor representation of the reference currents of 4 to 6 phase machines both in normal and in fault-tolerant operation mode when using the $c_k$ given in Table 5.1
Vector control using a Park reference frame per phase

The model used for the fictitious phase consists of the series connection of a resistance $R$, of an inductance $L$ and of a back EMF equal to 

$$\dot{\psi}_k(\theta_{mk}+\pi/2)$$

where $R$ is the value of the machine resistances, $L$ the value of the machine phase inductances, and $\psi_k(\theta_{mk}+\pi/2)$ the value of flux which should be induced by the magnets in a phase shifted of $\pi/2$ with respect to phase $k$. $\theta_{mk}$ is defined for each phase as:

$$\theta_{mk} = \theta_m - \frac{k-1}{n}2\pi$$

(6.21)

In order to get one rotor frame per phase, we define a different Park transform for each phase:

$$P_k = \begin{pmatrix} \cos p\theta_{mk} & \sin p\theta_{mk} \\ -\sin p\theta_{mk} & \cos p\theta_{mk} \end{pmatrix}$$

(6.22)

By applying it on the measured current $i_{\alpha k}$ and on the estimated current $i_{\beta k}$, we get two measured currents $i_{dk}$ and $i_{qk}$, as shown in Figure 6.13. The vector control used for aligning currents $i_{dk}$ and $i_{qk}$ on the reference currents $i_{dk,ref}$ and $i_{qk,ref}$ corresponds to the following equation in the $dq$ frame of a two phase machine:

$$u_d = Ri_d + L\frac{di_d}{dt} - p\dot{\theta}_mL_i_q$$

$$u_q = Ri_q + L\frac{di_q}{dt} + p\dot{\theta}_mL_i_d + \sqrt{\frac{n}{2}}\psi_0p\dot{\theta}_m$$

(6.23)

and is shown in Figure 6.14. This control is the same for all the phases, as it is independent of the rotor position. The voltages generated by the vector control are translated in the stator frame by the inverse Park transform:

$$P_k^{-1} = \begin{pmatrix} \cos p\theta_{mk} & -\sin p\theta_{mk} \\ \sin p\theta_{mk} & \cos p\theta_{mk} \end{pmatrix}$$

(6.24)

and the voltages $u_{\alpha k}$ and $u_{\beta k}$ are applied respectively to the real phase and the fictitious one.

A limitation of that control strategy is that the fictitious current is estimated with some error due to the fact that the model is a simplified one: the parameters $R$, $L$ and $\psi_k$ used for the model may differ from the
Control strategies

Figure 6.13: Control using a fictitious phase $\beta$

Figure 6.14: Vector control used for each phase
Vector control using a Park reference frame per phase

real ones and effects such as the variation of the DC bus voltage or the voltage drops on the semiconductors of the inverter are not considered. As the equation in the Park frame are coupled, that error shall perturb the control of the current of the real phase and consequently the contribution of the phase to the torque generation. As each phase contributes independently to the torque production, the total torque is just the sum of the torques generated by all the phases, including the perturbations due to the error on the estimation of the fictitious currents.

A more efficient way to perform the control consists in assuming that the \( i_{\beta k} \) are equal to their reference values, directly computed on the basis of the reference currents \( i_{dk,ref} \) and \( i_{qk,ref} \), as shown in Figure 6.15. The \( i_{\beta k} \) are computed from an inverse Park transform applied to the reference \( dq \) currents.

\[
    i_{\beta k} = \sin (p\theta_{mk}) i_{dk,ref} + \cos (p\theta_{mk}) i_{qk,ref} \tag{6.25}
\]

That way, it is as if current \( i_{\beta k} \) of the fictitious phases were perfectly controlled. They do not perturb the control of the currents of the real phases, and add no error to the torque generation.
6.3.4 Fault detection strategies

In normal operation mode, the system of equations (6.19) shows that the q axis reference current of each phase is identical. It means that the measured currents $i_{qk}$ should tend to that same reference value, with a behavior quite similar in each phase. Therefore, it is easy to detect when a failure occurs in any phase as the measured current $i_{qk}$ of that phase will diverge from the other ones.

The simplest detection system is a majority vote on these measured values of the currents $i_{qk}$. It ensures the detection of the faulted phase and takes the decision to change the reference currents in the remaining phases with the proper reconfiguration. The whole control scheme is represented in Figure 6.16. This has already been implemented and tested with a three phase machine by authors of [36].

6.4 Appendix: Stability of the control with independent controllers

If we consider that all the mutual inductances have the same value, the electrical state equation of a n phase machine is given by:

$$
s \begin{bmatrix}
L & M & \ldots & M \\
M & L & \ldots & M \\
\vdots & \vdots & \ddots & \vdots \\
M & M & \ldots & L
\end{bmatrix} 
\begin{bmatrix}
i_1 \\
i_2 \\
\vdots \\
i_n
\end{bmatrix} = 
\begin{bmatrix}
u_1 \\
u_2 \\
\vdots \\
u_n
\end{bmatrix} - 
\begin{bmatrix}
e_1 \\
e_2 \\
\vdots \\
e_n
\end{bmatrix} - 
\begin{bmatrix}
i_1 \\
i_2 \\
\vdots \\
i_n
\end{bmatrix} - 
R \begin{bmatrix}
i_1 \\
i_2 \\
\vdots \\
i_n
\end{bmatrix}$$

(6.26)

If we assume that all the phases are controlled independently by PI controllers in the stator frame, based on the errors between the reference currents and the measured currents, and if we suppose that the EMFs are perfectly compensated by feed forward terms, we can write for the voltages:

$$u_k = \frac{sK_P + K_I}{s} (i_{k,ref} - i_k) + e_k$$

(6.27)

where $K_P$ and $K_I$ are respectively the proportional gain and the integral gain of the controller.

We suppose $i_{k,ref}$ equal to 0 as it has no influence on the stability of
the system. We can hence rewrite equation (6.26) as follow:

\[
\begin{bmatrix}
  sL + R + \frac{sK_p+K_i}{s} & \ldots & sM & sM & sM \\
  \vdots & \ddots & \vdots & \ddots & \vdots \\
  sM & \ldots & sL + R + \frac{sK_p+K_i}{s} & sM & sM \\
  sM & \ldots & sM & sL + R + \frac{sK_p+K_i}{s} & sM \\
\end{bmatrix}
\begin{bmatrix}
  i_1 \\
  i_2 \\
  \vdots \\
  i_n
\end{bmatrix} = 0
\]

(6.28)
The \( n \) eigenvalues of such a system are equal to:

\[
\begin{align*}
    s(L - M) + R + \frac{sK_P + K_I}{s} \\
    : \\
    s(L - M) + R + \frac{sK_P + K_I}{s} \\
    s(L + (n - 1)M) + R + \frac{sK_P + K_I}{s}
\end{align*}
\]

(6.29)

Therefore, to ensure the stability, the real part of the roots of the two following polynomial:

\[
\begin{align*}
    (L - M)s^2 + (R + K_P)s + K_I \\
    (L + (n - 1)M)s^2 + (R + K_P)s + K_I
\end{align*}
\]

(6.30)

must be negative.

As \( L, -M, R, K_P, K_I \) are all positive, the first polynomial has always negative roots. For the last one, which represents the homopolar component, this is also true for any positive value of \( K_P, K_I \) and \( R \) if:

\[
L + (n - 1)M > 0
\]

(6.31)

or:

\[
M > -\frac{L}{n - 1}
\]

(6.32)

This is always true in segment motors as, as explained in section 3.5, the minimum value for the mutual inductance is given by:

\[
M = -\frac{L}{2n - 1}
\]

(6.33)

which is greater than the limit value of stability. Therefore the system is intrinsically stable.

This remains true for a control in the rotor frame, as the change of frame simply implies a linear change of variables.
Implementation and validation of the torque control strategies

In this chapter, the two torque control strategies presented in chapter 6 will be validated both by simulations and by experimental results. The latter are obtained by an implementation of the strategies on a Dspace workstation that controls a 6-phase motor. The simulation models are built with Matlab Simulink and modelize the experimental bench. The parameters of the simulation models are the same as the parameters of the experimental bench.

7.1 Experimental setup

7.1.1 Description of the experimental setup

The experimental setup, depicted in Figure 7.1, consists in:

- a 6-phase segment PMSM motor with star connected windings, mechanically loaded by a PM three phase PM synchronous motor feeding a three phase resistive load;
- a 14 bits position encoder connected on the shaft of the motor;
Implementation and validation of the torque control strategies

Figure 7.1: Schematic of the whole setup

- a 6-leg PWM voltage source inverter, working at the PWM frequency of 10 kHz;
- Current sensors (LEM type) on each phase of the motor;
- a DC power source for feeding the inverter with a constant voltage;
- a Dspace workstation DS1005, coupled with Matlab Simulink 2006, supporting the control algorithms corresponding to the control strategies shown in Figure 6.8 and 6.16 with a sampling period of 1 ms. This workstation is interfaced with a DS2201 Multi-IO board for the acquisitions from the sensors and the generation of the PWM signals controlling the inverter legs.

Photographs of the test bench are shown in Figure 7.2. On the two photographs, we can see a balance, which will be used to determine the torque developed by the motor at standstill. The balance measures the force that the shaft exerts on a lever arm.

7.1.2 Parameters of the motor

The motor chosen for testing the control strategies is a 4.5kW 6-phase motor. It has 10 pole pairs and a nominal current equal to 13A. The electrical parameters of the machine are:

- Phase resistance : $R = 0.95 \, \Omega$
- Self inductance : $L = 2.45 \, mH$
Experimental setup

Figure 7.2: Experimental setup

(a) Photograph of the motor and the load

(b) Photograph of the control setup
Implementation and validation of the torque control strategies

- Amplitude of the fundamental component of the induced flux: 
  \[ \psi_0 = 0.0335 \text{ Wb} \]

The mutual inductances are negligible. Indeed, we have chosen a motor in which each winding is split in three coils in series. As presented in section 3.4.3.2, this subdivision of the windings greatly reduces the mutual effects. In the simulations, we will introduce a mutual inductance between the phases equal to 1% of the self inductance.

The harmonic content of the EMFs is not negligible. Table 7.1 gives the amplitude of the first harmonics (up to the 9th) in % of the fundamental.

<table>
<thead>
<tr>
<th>harmonic</th>
<th>% of the fundamental amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>6.4</td>
</tr>
<tr>
<td>5</td>
<td>−4.3</td>
</tr>
<tr>
<td>7</td>
<td>−3.2</td>
</tr>
<tr>
<td>9</td>
<td>−0.9</td>
</tr>
</tbody>
</table>

Table 7.1: Harmonic content of the back EMFs of the test bench motor

Figure 7.3 shows, as a function of the rotor electrical position, the waveform of \( d\psi_1/d\theta \), which is equal to the back EMFs of phase 1 divided by the electrical speed, and of its fundamental component.

7.2 Validation by simulation of the torque control in the extended Park reference frame

This section aims to validate the control in the extended Park reference frame developed in section 6.2.

7.2.1 Simulation model

To be able to compare the results of the simulations and of the experimental setup, the Simulink model is built in order to predict the behavior of the 6-phase motor used in the test bench. The model of simulation is structured as shown in Figure 7.4.

The torque control block emulates the implementation in the DSpace workstation of the vector control in the extended Park reference frame.
developed in section 6.2, and shown in Figure 6.8. It computes the voltage references to apply to the machine in order to obtain a torque equal to the torque reference.

The *power electronics* block emulates the 6-phase inverter. It must
be noted that, in the experimental setup, the current measurements are almost ripple-free as they are properly synchronized on the PWM signals. Therefore it is not mandatory to compute the PWM signals from the reference voltages in the simulation model, as it costs an important computation time. The sole action of the power electronics block is hence to limit the voltages applied to the machine to ± the half of the bus voltage.

The motor block emulates the behavior of the 6-phase motor. More precisely it computes the phase currents as a function of the voltages applied to the motor under the constraint of a star connection of the windings. The equivalent electrical scheme is shown in Figure 7.5. A small mutual inductance (1% of the self inductance) is taken into account into the computation. The block also computes the torque $T$ generated by the motor. As we neglect the cogging torque resulting from the slots-teeth alternation, it comes:

$$T_{\text{real}} = \frac{1}{p\dot{\theta}} \sum_{k=1}^{n} i_k e_k$$  \hfill (7.1)

where $p$ is the number of pole pairs and $\dot{\theta}$ the mechanical speed.

![Figure 7.5: Computation of the phase currents](image)

The load block emulates the behavior of the mechanical part. It computes from the torque generated by the motor the mechanical speed.
Simulation of the control in the extended Park frame

and position of the shaft. As the load of the experimental setup is a passive one (a synchronous motor coupled with a wye-connected resistive load), the mechanical part can be modeled by an inertia and a viscous torque, as the load torque (i.e. the power dissipated in the load resistances) is proportional to the square of the amplitude of the EMFs of the load motor which are proportional to the speed. The mechanical speed $\omega_{\text{real}}$ and position $\theta_{\text{real}}$ of the rotor are simply obtained by the following equations:

$$
\omega_{\text{real}} = \frac{1}{J s + f} T_{\text{real}}
$$

$$
\theta_{\text{real}} = \frac{1}{s} \omega_{\text{real}}
$$

(7.2)

with $J$ the inertia of the rotating parts, and $f$ the viscous torque coefficient of the load.

The acquisition block emulates the 14 bits position encoder, the currents sensors and the acquisition of the measurements by the DSpace workstation. The position measurement is quantified with the precision of the encoder, and the speed measurements is obtained by derivating and filtering the position measurement\(^1\). Besides, a blank noise with a variance equal to 1A (i.e. 7.7% of the motor nominal current) is added onto the current measurements.

As the torque control block and the measurements blocks represent the code implemented in the Dspace workstation, they are computed with a sample time equal to 500µs while the other blocks, which modelize the continuous state of the physical systems, are computed every 10µs.

The simulation results presented in the next section will show the behavior of the system in three operation modes:

- in normal operation mode;
- in fault operation mode: one phase is lost in open circuit but no reconfiguration of the control is performed;
- in the fault-tolerant operation mode: the adequate reconfiguration of the controller is performed in order to alleviate the loss of the faulted phase.

\(^1\)This is why we make a distinction between the real speed and the real position of the rotor with their measured values.
We willingly chose to not implement any detection system that would have automatically reconfigured the control into the fault-tolerant operation mode, as we also wanted to analyze the behavior of the system in the fault operation mode. Figure 7.6 gives the time evolution of the reference torque that the motor must develop and the duration of the various operation modes.

An overview of the structure used for the model of simulation is shown in Figure 7.7.

7.2.2 Simulation results

7.2.2.1 Case of the loss of one phase in open circuit

To simulate the loss of one phase in open circuit, we impose to the voltage applied to phase 4 to be equal to the voltage of the neutral point. This is equivalent to the loss of this phase in open circuit as the current flowing into it goes rapidly to 0.

Figure 7.8 shows the simulation results assuming that the EMFs are purely sinusoidal. In these simulations, the harmonic content of the EMFs given in table 7.1 is not taken into account. The figures show four plots:

- The reference torque that the motor must develop and the torque really applied by the motor to the shaft.
Simulation of the control in the extended Park frame

Figure 7.7: Structure of the Simulink model with an Extended Park control

- The speed response of the shaft.
- The currents flowing into the 6 phases of the motor.
- The derivatives of the flux induced by the magnets in the 6 phases.
  In other words, the EMFs of the 6 phases divided by the rotor speed.

Figure 7.9 shows the simulation results when the harmonics of the EMFs are taken into account, as shown in the fourth plot. Feed forward terms in the reference phase voltages generated by the controller include those harmonics. We can see that the current sinusoidal shapes are perturbed, as the discretization of the feed forward terms yields some errors in the compensation of the harmonics. Nevertheless, the torque and speed responses are very similar to the torque and speed responses obtained when assuming sinusoidal EMFs.

In normal operation mode, we can see in both Figures that the torque generated by the motor follows the torque reference, despite the deviations due to the discretization of the control\(^2\) and the noises added on the measurements. The speed estimated on the basis of the position

\(^2\)An analysis of the effects of the discretization of the control in the Park frame has been performed in \([37, 38]\).
Figure 7.8: One phase lost in open circuit
Simulation of the control in the extended Park frame

Figure 7.9: Results with the real EMFs of the machine
Figure 7.10: Results with a controller with an higher dynamics
measurement has a delay in the transient states, but the vector control keeps the currents to a balanced set of sinusoidal functions, isomorphic to their respective EMFs.

In fault operation mode, we can see that the torque adopts an oscillating behavior which is not negligible and affects the speed response. The oscillation is at twice the electrical frequency as the contribution of the lost phase to the torque generation is maximum two times in an electrical period. The controller tries to compensate this loss by increasing the currents in the remaining healthy phases but owing to its limited bandwidth it fails to do so. Nevertheless due to the integral actions in the controller the average value of the torque is restored at its reference value.

To go from the fault operation mode to the fault-tolerant one, we simply modify adequately the measured currents and the reference voltages, as shown in Figure 6.8, through the adaptation of matrix $C_k$. In the normal and the fault operation modes, we do not modify the inputs and outputs of the controller. In equation (5.28), all the $\alpha_k$ are equal to zero, and the computation of $C_k$ is straightforward:

$$C_k = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$ (7.3)

In fault-tolerant operation mode, $\alpha_4$ is set to 1 to compensate the loss of phase 4. Matrix $C_k$ becomes:

$$C_k = \begin{pmatrix} 0 & 0 & 0 & c_4 & 0 & 0 \\ 0 & 0 & c_5 & 0 & 0 & 0 \\ 0 & 0 & c_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & c_2 & 0 & 0 & 0 \\ 0 & 0 & c_3 & 0 & 0 & 0 \end{pmatrix}$$ (7.4)

Using coefficients $c_k$ defined by equation (5.27) to keep the sum of the remaining currents equal to zero, we get:

$$C_k = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1/3 & 0 & 0 & 0 \\ 0 & 0 & 2/3 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 2/3 & 0 & 0 & 0 \\ 0 & 0 & -1/3 & 0 & 0 & 0 \end{pmatrix}$$ (7.5)
As we can see, in fault-tolerant mode the torque and speed responses of the main variables are close to the ones in normal operation mode. The important oscillations found in fault operation mode disappear. The waveforms of the currents are close to the value that they must take in fault-tolerant operation mode. These values have been determined in section 5.3 and shown in Figure 5.17.

Figure 7.10 shows the results in the same conditions as for the results shown in Figure 7.9, but with a controller with a higher dynamics. The proportional gain and the integral gain of the controllers, which where equal to:

\[
\begin{align*}
    k_p &= 0.25 \\
    k_i &= 100
\end{align*}
\]  

are now five times greater:

\[
\begin{align*}
    k_p &= 1.25 \\
    k_i &= 500
\end{align*}
\]  

As we can see, the response is faster in normal operation mode as well as in fault-tolerant operation mode. In fault mode, the oscillating behavior is reduced, as the controller is now able to compensate partially the contribution of phase 4 to the torque generation as a function of the electrical position.

However, the control in fault mode generates far more Joule’s losses than the control in fault-tolerant mode. The mean values of the Joule’s losses in each mode for a torque of 5Nm are given in table 7.2. In fault operation mode, the mean value of the Joule’s losses is twice the one in normal operation mode. In fault-tolerant mode, they are only increased by a factor 1.34, as predicted in section 5.4.1.

<table>
<thead>
<tr>
<th>operation mode</th>
<th>Joule’s losses</th>
</tr>
</thead>
<tbody>
<tr>
<td>normal</td>
<td>72 W (100%)</td>
</tr>
<tr>
<td>fault</td>
<td>143 W (200%)</td>
</tr>
<tr>
<td>fault-tolerant</td>
<td>96 W (134%)</td>
</tr>
</tbody>
</table>

Table 7.2: Mean Joule’s losses in each mode
7.2.2.2 Case of the loss of one current sensor

To simulate the loss of a current sensor, we do not modify the model of the motor, we simply set to zero one current in the measurement block. Figure 7.11 shows the results for the loss of measurement of the current in phase 4, with the full harmonic content of the EMFs.

In comparison with the previous case, we can see here that the behavior in fault operation mode is far less satisfactory. As the control tries to compensate a fictitious loss, it will increase the current flowing into the phase which has lost its current sensor until the voltage reaches the limit fixed in the controller.

When going into the fault-tolerant operation mode, the same matrix $C_k$ defined by equation (7.5) reconfigures the remaining healthy phases and restores the torque and speed response as in the normal operation mode. The faulted phase is disconnected from the system, and the false information delivered by its current sensor is no more used in the control.

7.3 Experimental validation of the torque control in the extended Park reference frame

7.3.1 Tests at standstill

A first set of tests have been made with the rotor at standstill in normal operation mode (Figure 7.12) and in fault-tolerant mode (Figure 7.13), for various rotor positions. During these tests, the phase back EMFs are equal to zero. The good agreement between the reference values of the currents and the measured ones demonstrates that the proposed controller will correctly track the reference currents in normal and fault operation mode, provided that the phase back EMFs are properly compensated by the corresponding feed forward terms when the motor is rotating.

The torque developed by the motor is determined by the measurement with the balance. This torque exhibits small deviations around its reference value, as a function of the rotor position. These deviations, of about 8%, come mainly from the parasitic torque linked to the slotting effect and to the harmonic content of fluxes $\psi_k$ and of their derivatives $d\psi_k/d\theta$, shown in Figure 7.3.
Implementation and validation of the torque control strategies

Figure 7.11: One current sensor lost
Figures 7.12 and 7.13 also show that the estimation of the torque on the basis of the measured $q$ axis current effectively gives a satisfactory image of the torque.

### 7.3.2 Tests at 250 rpm

A second set of measurements have been made with the motor running at 250 rpm, in order to verify that the control maintains its effectiveness when the phase back EMFs are correctly compensated by the corresponding feed forward action. The load torque is adjusted in order to reach the speed of 250 rpm when the electromagnetic torque developed by the motor has a value of 5.2 Nm corresponding to an $i_{q,ref}$ of 9 A. With this value of the torque, the motor phase reference currents in normal operation mode are a balanced set of sinusoidal functions with an amplitude of 5.2 A.

The experimental results presented in Figure 7.14(a) and the simulation results presented in Figure 7.14(b) are corresponding to the normal operation mode. We can see that the vector control in the extended Park reference frame still ensures correctly the tracking of the references currents and that the ripple on $i_q$ (and therefore of the torque) does not exceed 9.4%.

Figure 7.15 and Figure 7.16 show the results obtained when the phase 4 is lost.

In Figure 7.15, the controller is not reconfigured to compensate the loss of phase 4 and is still the one used in the normal operation mode. As said in section 7.2.2 the controller tries to compensate the impact of the lost phase on the measured value of $i_q$ by modifying the currents in the remaining phases, but in a way which is limited by its bandwidth. Nevertheless, due to the integral action, the impact of the lost phase on the average value of the measured $i_q$ is fully cancelled, but not its oscillations around this value. The measured value of $i_q$ is alternatively smaller and bigger than its reference one. This comes from the fact that the contribution of each phase to the torque production (and hence to the value of $i_q$) varies with the rotor position at a frequency which is outside of the controller bandwidth.

In Figure 7.16, the controller is reconfigured to alleviate the loss of phase 4. This almost fully eliminates the impact of the lost phase on the measured value of $i_q$ (and hence on the torque). In particular, it can be
Figure 7.12: Torque and currents in normal operation mode ('-' references values, '●' experimental points)
Figure 7.13: Torque and currents in fault-tolerant operation mode (‘−’ reference values in fault-tolerant mode, ‘⋯’ reference values in normal mode, ‘●’ experimental points)
Implementation and validation of the torque control strategies

Figure 7.14: Normal operation mode
Experimental validation of the control in the extended Park frame

(a) Measured currents

(b) Simulated currents

Figure 7.15: Fault operation mode
Implementation and validation of the torque control strategies

Figure 7.16: Fault tolerant operation mode
seen that the measured and simulated phase currents are close to their reference values (shown in Figure 5.17). This proves the effectiveness of the proposed controller reconfiguration.

The small deviations of $i_q$ around its reference value comes mainly from the fact that the harmonics present in the machine back EMFs are not perfectly compensated due to the discretization introduced by the sampling rate of the controller.

7.4 Validation by simulation of the torque control with one Park reference frame per phase

This section aims to validate by simulation the control strategy with one Park reference frame per phase developed in section 6.3.

7.4.1 Adaptation of the model

The structure of the model used for the validation is shown in Figure 7.17 and is quite similar to the one used for the control in an extended Park reference frame.

![Figure 7.17: Structure of the Simulink model for the control with one Park reference frame per phase](image)
The torque control block is now the implementation of the control strategy with one Park reference per phase developed in section 6.3. As this strategy controls independently the phases, the feeding of the machine by a 6-leg inverter with a star connection of the windings is no more suited, as this puts a constraint on the currents that will create an interaction between the 6 controllers. We suppose therefore that each phase is fed independently\(^3\). The current of phase \(k\) is simply obtained by the following equation:

\[
i_k = \frac{1}{sL + R}(u_k - e_k)
\] (7.8)

In the fault-tolerant operation mode, we keep using the reference currents with a zero valued sum, so that the results can be easily compared with the results of the control strategy using an extended Park transform, but we could have used the reference currents without the zero valued sum (see section 5.3).

### 7.4.2 Simulation results

The simulation conditions are again those shown in Figure 7.6. The proportional gain and the integral are the one with the higher dynamics:

\[
k_p = 1.25
\]
\[
k_i = 500
\] (7.9)

#### 7.4.2.1 Case of the loss of a phase in open circuit

Figure 7.18 validates the control strategy, as the torque follows its reference value in the normal operation mode and in the fault-tolerant operation mode. In the fault operation mode, the contribution of the lost phase to the torque is in no way compensated as the controls of the phase currents are independent from each other. Hence, the motor torque oscillates around an average value which is lower than its rated value.

For the general case of a \(n\) phase machine, the reference currents in each Park frame in the normal operation mode and in the fault-tolerant

\(^3\)For example by an 'H' bridge. In that case the limiter must maintain the voltage \(u_k\) between \(\pm\) the DC bus voltage.
Simulation of the torque control with one Park frame per phase

Figure 7.18: Results for the control with a Park reference frame per phase for the case of phase 4 lost in open circuit
Figure 7.19: Measured and reference $i_q$ currents in the Park frames
Simulation of the torque control with one Park frame per phase

operation mode are given by equations (6.19) and (6.20) respectively. In the case of the 6 phase machine under study, we get for the normal operation mode:

\[
i_{qk,\text{ref}} = \frac{1}{3p\psi_0} T_{\text{ref}}
\]
\[
i_{dk,\text{ref}} = 0
\]  

(7.10)

and for the fault-tolerant operation mode:

\[
i_{q1,\text{ref}} = \frac{4}{9p\psi_0} T_{\text{ref}}
\]
\[
i_{d1,\text{ref}} = \frac{-4}{9p\psi_0} T_{\text{ref}}
\]
\[
i_{q2,\text{ref}} = \frac{1}{3p\psi_0} T_{\text{ref}}
\]
\[
i_{d2,\text{ref}} = 0
\]
\[
i_{q3,\text{ref}} = \frac{4}{9p\psi_0} T_{\text{ref}}
\]
\[
i_{d3,\text{ref}} = \frac{-\sqrt{3}}{9p\psi_0} T_{\text{ref}}
\]  

(7.11)

\[
i_{q4,\text{ref}} = 0
\]
\[
i_{d4,\text{ref}} = 0
\]
\[
i_{q5,\text{ref}} = \frac{4}{9p\psi_0} T_{\text{ref}}
\]
\[
i_{d5,\text{ref}} = \frac{\sqrt{3}}{9p\psi_0} T_{\text{ref}}
\]
\[
i_{q6,\text{ref}} = \frac{1}{3p\psi_0} T_{\text{ref}}
\]
\[
i_{d6,\text{ref}} = 0
\]

Figure 7.19 shows the reference currents of the q-axis of the 6 phases, and superposes to them the measured \(i_q\) currents. We can see the modification of amplitude of the reference currents in fault-tolerant mode, and that it is easy to detect that the faulted phase is phase 4 on the basis of the waveform of the measured \(i_q\) currents.
Figure 7.20: Results for the control with a Park reference frame per phase for the case of the loss of the current sensor of phase 4
Simulation of the torque control with one Park frame per phase

Figure 7.21: Measured and reference $i_q$ currents in the Park frames
7.4.2.2 Case of the loss of one current sensor

Figure 7.20 shows the results for the case of the loss of the current sensor of phase 4. In the fault operation mode, the torque ripple is far more important than in the case of the loss of the phase in open circuit, as the current in phase 4 adds a pulsating torque to the useful torque generated by the other phases. In comparison with the previous control strategy using an extended Park transform, the torque ripple is also more important. This is due to the independence of the phases in terms of control. In the previous strategy, the total torque is controlled, and the action of the faulted phase is limited by the other, due to the star connection.

As shown in Figure 7.21, the measured current $i_q$ in phase 4 has a behavior similar in the case of the loss of a phase than in the case of the loss of a current sensor. In both cases, the measured current is equal to 0 in phase 4, even if the real current is not equal to zero in the second case. Therefore, the same detection strategy works for the two cases.

7.5 Experimental validation of the torque control strategy with one Park reference frame per phase

For the validation of the control strategy using an extended Park frame, we used a 6-phase motor fed by a 6-leg inverter. In order to use the same material for the experimental validation of the control strategy with one Park reference frame per phase, we connected the 6-leg inverter and the 6-phase motor of the test bench as shown in Figure 7.22.

Two phases in opposition are connected in series, and these two phases in series are connected to two legs of the 6-leg inverter. This is scheme is equivalent to a 3-phase machine connected to 3 H-bridges fed by the same DC bus.

As the phase are controlled independently, the control strategy with one reference Park frame per phase can be validated with this reconfigured 3-phase machine as well as with a 6-phase machine. The latter would have required 6 H-bridges, or a 12-leg inverter.

Simulation and experimental results obtained with the three phase machine have been published in [36] and are shown in Figure 7.23 and
7.24. The increase in the torque ripple after a fault occurrence and control reconfiguration in the simulation comes from the fact that in normal operation mode the sum of the currents in the phases is equal to zero so that in this case the harmonics of rank 3 and multiple of 3 do not contribute to the torque ripple. In the experimental results, the harmonics of the motor are compensated with feedforward terms in the control.
In Figure 7.24, we can see that the detection system detects rapidly the phase that has been manually disconnected by using a mechanical switch. The detection strategy that has been implemented is the simplest one and consists in the following one: each computation step, the detection system computes the differences between each of the measured currents $i_q$ (shown in Figure 7.24(a)) and the others. If one of them has a difference with the other currents higher than a fixed threshold $\Delta i_{thr}$, $x$ times successively, then the phase is considered as faulted and the control is reconfigured.

Of course, threshold $\Delta i_{thr}$ and number of computations $x$ before an action is taken, are entirely dependent of the application, the quality of the measurements, and the electrical transient behavior of the machine. This point has not been deeply investigated in this thesis and could be deepened in a post doctoral research.

7.6 Conclusions

In this chapter we have validated the two control strategies presented in chapter 6 by considering a 6-phase machine. Both of them keep the same $dq$ control in the fault-tolerant operation mode than in the normal operation mode. However it needs to be coupled with a reliable fault detection strategy.

The control using an extended Concordia-Park transform is quite simple to implement, and requires a minimum of reconfiguration for going into fault-tolerant operation. Indeed, the reconfiguration of the vector control in an extended Park reference frame does not imply any modification of the core of the controller. It only introduces a correction of the measured currents and a proper combination of the controller output signals.

With the control with a Park reference frame per phase, the reconfiguration in fault-tolerant operation mode and the fault detection of the faulted phase are easier to perform than with the control based on the extended Park transform. But the independence of the phases in terms of control makes the torque generation more sensitive to any fault, as, in fault operation mode, the remaining healthy phases will not compensate any part of the loss.

With the control in the extended Park transform, the fault-tolerant operation mode allows to highly reduce the Joule’s losses in comparison
Conclusions

with the fault operation mode, in which the controller with no reconfiguration tries alone to compensate the effect of the loss of one phase. With the control using one reference frame per phase, as the controller does not try to compensate the loss of one phase, we have no increase of the Joule’s losses but a reduction of the average torque and higher torque pulsations.

The vector control in the extended Park reference frame developed in section 6.2 requires 5 PI controllers. The torque control with one Park reference frame per phase requires 12 PI. The control with one Park reference frame per phase cannot be used with an isolated star connection of the windings. On the contrary, the extended Park control not only can be used with the isolated star connection, but also with a separate feeding of the phases. In that case it requires 6 PI controllers as the sum of the currents is no more automatically kept to zero.

The vector control in the extended Park reference frame seems less expensive, as it requires less PI controllers, but if we count the number of basic operations, we get:

- \((3n^2+5n+20)\) multiplications, \((3n^2+4n+9)\) additions/subtractions for the vector control in the extended Park reference frame used without an isolated star connection;
- \((3n^2+4n+17)\) multiplications, \((3n^2+3n+5)\) additions/subtractions for the vector control in the extended Park reference frame used with an isolated star connection;
- \(25n\) multiplications, \(17n\) additions/subtractions for the vector control with one Park reference frame per phase.

with \(n\) the number of motor phases. The vector control in the extended Park reference frame is in \(\mathcal{O}(n^2)\) for the additions, subtractions and multiplications, while the vector control with one Park reference frame per phase is in \(\mathcal{O}(n)\), which means that the latter is more efficient with a high number of phases (since 6 phases for the additions/subtractions and 4 phases for the multiplications).

However, the vector control in the extended Park reference frame requires only one \(\sin\) operation and one \(\cos\) operation. The vector control with one Park reference frame per phase needs one angle per independent control, and hence \(n\ \sin\) operations and \(n\ \cos\) operations.
Implementation and validation of the torque control strategies

Figure 7.24: Experimental results of a phase disconnection and control reconfiguration: (a) currents $i_{q,k}$, (b) torque
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Design issues
Motor modeling in view of optimal design

8.1 Design considerations

Classically an electrical motor is designed in order to be able to deliver permanently its nominal torque at its nominal speed when fed at its nominal voltage under well defined ambiance conditions (air at sea-level pressure and at 25°C).

For the design of the motor driving an actuator used for the thrust vector control of a space launcher, this approach is not relevant. The motor specifications are defined in terms of a mission profile, i.e. the time evolution of the motor torque and speed envelopes during the mission. The envelopes define the maximum amplitudes of the torque and of the speed at every time. In the example shown in Figure 8.1, we can see periods of time in which the motor must be able to produce various torque values at standstill and other periods in which the motor must develop a given torque for a given speed.

Figure 8.2 shows a possible integration of the motor in the actuation system. The mission time is very short (ranging from few minutes to few tens of minute) and ambiance conditions (low pressure in a closed enclosure) make that the motor temperature rise is almost corresponding
to an adiabatic process with an initial motor temperature. Possibly, the motor can exchange heat with the mechanical parts of the actuator acting as a heat storage mass.

The thermal constraints, which are probably the strongest as their non respect may damage the motor, are mainly:

- The maximum temperature the motor coils may reach: this temperature depends on the insulating material of the wire used for making the stator windings. This temperature typically ranges from $105^\circ$ (class A) to $180^\circ$ (class H).

- The maximum temperature the stator yoke may reach.

- The maximum temperature the aluminum housing may reach (this temperature is about $100^\circ$).

- The maximum temperature the rotor magnet may reach, which must remain sufficiently below their Curie point (about $320^\circ$ for neodymium magnets and about $800^\circ$ for samarium-cobalt magnets).

To these constraints, we must add dimensional constraints in order to integrate the motor in the actuation system:
Design considerations

Figure 8.2: Actuation system

- a minimum rotor yoke inner diameter fixed by the constraint of the minimum diameter of the ball-screw transmission and the rotor shaft,

- a maximum motor outer diameter, fixed by the constraint of the maximum size of the housing,

- and a maximum motor length.

Eventually, other design constraints may be added as:

- a maximum harmonic content for the motor EMFs,

- the maximum value of the magnetic induction in the stator and rotor yokes

- a maximum value of the phase self and mutual inductances,
Motor modeling in view of optimal design

- a maximum value of the cogging torque,
- a maximum value of the mass of the motor.

These last constraints are less critical. They just make the motor a little less efficient as scheduled. For instance the maximum harmonic content of the EMFs may be exceeded without making the motor unable to perform the mission.

In addition to the above mentioned constraints, some optimization criteria will be added. Their minimization will produce the best possible motor. Depending on the optimization criteria and their number, it may arise that it is not possible to find one best motor design but a set of solutions, each of them more optimal for some optimization criteria but less optimal for other ones. The designer has then to choose among these solutions the one that fits the most its criteria. Among the optimization criteria, the weight of the motor will be one of the most important for a aerospace application.

Eventually as the reliability is also a fundamental criteria and as this criteria can be met by making the motor and its power electronics fault tolerant, the motor must be designed in order to be able to operate in fault tolerant mode during the whole mission. Indeed in the worst case a fault can occur from the start of the mission.

The design process will comprise the following steps:

- the first step consists in selecting a motor type and to fix the main parameters of the motor as the number of phases, the number of coils per phase, the number of pole pairs, etc. This step allows to define the structure on which the optimization of the design must be performed by acting on the values of the dimensional parameters defining the structure.

- the second step consists in building an analytical model allowing to evaluate the performances of a given design (ability to perform the mission or not, values of the optimization criteria, etc.).

- the third step consists in developing an optimization process which modify the design parameters until an optimum is reached in terms of ability to perform the mission and of minimization of the optimization criteria.
8.2 Motor structure and main dimensional parameters

The selection of the motor type, a PM segment synchronous motor, has been validated in the first section on the basis of a survey of the literature devoted to high performance fault-tolerant drives and on the basis of a thorough analysis of the main characteristics of the segment motor.

Considering the simplest structure corresponding to one coil per phase and one magnet per pole\(^1\), we just need to choose the number \(n\) of phase and the number \(n + 1\) or \(n - 1\) of the rotor pole pairs to completely define the motor structure. This structure is shown in Figure 8.3, and the definition of the main dimensional parameters is listed in Table 8.1.

We assume that the teeth widths are equal. The airgap height in-\(^1\)

\(^1\)The choice of one coil per phase is made in order to minimize the number of connections inside the machine and hence the risk of failure due to a broken connection. The choice of one magnet per pole is made in order to maintain each magnets inside a closed space as a filling materials is added in the space between the magnets.
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Some of the design parameters are not free parameters, they are dependent of the others and defined for convenience and the readiness of equations:

\[
\begin{align*}
R_{ry} &= R_{ri} + h_r \\
R_{ro} &= R_{ry} + h_m \\
R_{si} &= R_{ro} + h_a \\
R_{so} &= R_{si} + h_{c2} + h_t + h_s
\end{align*}
\] (8.1)

So only 11 dimensional parameters must be defined by the design process.

### 8.3 Analytical model for design

#### 8.3.1 Introduction

This model aims to determine successively on the basis of a set of values of the design parameters:

<table>
<thead>
<tr>
<th>Names</th>
<th>Parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{so}$</td>
<td>Stator outer radius</td>
</tr>
<tr>
<td>$R_{si}$</td>
<td>Stator inner radius</td>
</tr>
<tr>
<td>$R_{ro}$</td>
<td>Rotor outer radius</td>
</tr>
<tr>
<td>$R_{ry}$</td>
<td>Rotor yoke outer radius</td>
</tr>
<tr>
<td>$R_{ri}$</td>
<td>Rotor inner radius</td>
</tr>
<tr>
<td>$b_0$</td>
<td>Slot opening</td>
</tr>
<tr>
<td>$h_t$</td>
<td>Teeth height</td>
</tr>
<tr>
<td>$w_t$</td>
<td>Teeth width</td>
</tr>
<tr>
<td>$h_{c2}$</td>
<td>Teeth closing maximum height</td>
</tr>
<tr>
<td>$h_{c1}$</td>
<td>Teeth closing minimum height</td>
</tr>
<tr>
<td>$h_a$</td>
<td>Airgap (physical + can) height</td>
</tr>
<tr>
<td>$h_m$</td>
<td>Magnet height</td>
</tr>
<tr>
<td>$h_r$</td>
<td>Rotor yoke height</td>
</tr>
<tr>
<td>$h_s$</td>
<td>Stator yoke height</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Half the magnet angular width</td>
</tr>
<tr>
<td>$L_e$</td>
<td>Active length of the motor</td>
</tr>
</tbody>
</table>

Table 8.1: List of the design parameters

Includes the rotor jacket (non illustrated in Figure 8.3).
Analytical model for design

- the magnetic field produced by the magnets at the stator periphery of the airgap and in the various iron parts (rotor yoke, teeth, etc.);
- the flux induced by the magnets in the stator coils, taking into account the leakage flux in the slot opening;
- the armature reaction field produced by the currents flowing in the stator coils for the maximum torque and the most critical position of the rotor and the magnetic field components they add in the stator and rotor yokes to the field components produced by the magnets, taking into account the leakage flux in the slot opening;
- the self and mutual inductances of the coils;
- the electromagnetic torque (i.e. including the cogging torque) and the time evolution of the current waveforms deduced from the mission torque and speed profiles;
- the Joule’s losses on the basis of the time evolution of the current waveforms;
- the thermal behavior of the motor on the basis of the time evolution of the Joule’s losses during the mission;
- the number of turns per coil on the basis of the value of the feeding voltages and the electrical lumped parameters of the motor.

The choice of using an analytical approach for evaluating the performance of a given design is linked to the fact that such an approach reduces strongly the computation time needed when compared with a FEM analysis. This is important as the optimization process will rely on a comparative study of an important number of designs.

8.3.2 No load airgap field

The computation is done in two steps. The first step consists in determining the field distribution that the magnets produce at the airgap periphery on the stator side by assuming a smooth airgap (i.e. by neglecting the slot effects) and with the assumption of an infinite permeability of the iron parts. This computation is made for two type of magnets (Figure 8.4):
radially shaped magnets with a radial or parallel magnetization,

and magnets of constant width with a parallel magnetization.

The second step consists in correcting the airgap field at the stator periphery by an iterative computation of the permeability of the iron parts (i.e. stator teeth, stator and rotor yokes), using their $B$-$H$ curves.

### 8.3.2.1 Airgap field with radially shaped magnets

For radially shaped magnets, we use the analytical solution given by the authors of [3, 4]. Assuming an infinite permeability of the iron yokes; they have developed the analytical solution for the radial magnetization (Figure 8.4(a)), and for the parallel magnetization (Figure 8.4(b)).

Smoothing the airgap reduces to four the number of dimensional parameters needed for computing the no load airgap field: the magnets inner radius $R_{mi}$, the magnet outer radius $R_{ro}$, the stator inner radius $R_{si}$, and the magnet angular width, equal to $2\xi$.

If we select the center of a magnet as origin for determining the angular position $\zeta$ of a point along the airgap (Figure 8.5), we get the general expression of the airgap field at the stator inner border, for both parallel and radial magnetization, by [4]:

$$B(\zeta) = \sum_{2m+1}^{\infty} B_{2m+1} \cos (\nu \zeta)$$

(8.2)

with $\nu = (2m+1)p$ and $m$ a nonnegative integer. $B_{2m+1}$ is the amplitude
Figure 8.5: $\theta_m$ the rotor position with respect to a reference slot and $\zeta$ the position of a point along the air gap

of the $(2m + 1)$th harmonic, given by:

$$B_{2m+1} = \frac{pB_r}{\pi \mu_0} \left( \frac{R_{cy}}{R_{cr}} \right)^{\nu+1} \frac{(\nu' - 1) + 2 \left( \frac{R_{cy}}{R_{cr}} \right)^{\nu+1} - (\nu' + 1) \left( \frac{R_{cy}}{R_{cr}} \right)^{2\nu}}{\frac{\mu_a + 1}{\mu_a} \left[ 1 - \left( \frac{R_{cy}}{R_{cr}} \right)^{2\nu} \right] - \frac{\mu_a - 1}{\mu_a} \left[ \left( \frac{R_{cy}}{R_{si}} \right)^{2\nu} - \left( \frac{R_{ro}}{R_{si}} \right)^{2\nu} \right]} K_{2m+1}$$

(8.3)

with $B_r$ the remanent field.

$K_{2m+1}$ and $\nu'$ are two parameters depending on the type of magnetization. For a radial magnetization, they are equal to:

$$K_{2m+1} = \frac{8 \sin \nu \xi}{\nu^2 - 1}$$

(8.4)

$$\nu' = \nu$$

(8.5)

and for a parallel magnetization:

$$K_{2m+1} = 4\nu \frac{\sin((\nu + 1)\xi) + \sin((\nu - 1)\xi)}{\nu^2 - 1}$$

(8.6)

$$\nu' = \frac{\sin((\nu + 1)\xi) + \sin((\nu - 1)\xi)}{\sin((\nu + 1)\xi) - \sin((\nu - 1)\xi)}$$

(8.7)
Note that expression (8.3) is not valid for computing the fundamental field components for machines with one pole pair, as in that case $\nu = 1$ yields an indetermination of the equation. However, in the frame of our design process, machines with one pole pairs are not considered. As said in the previous section, we focus on polyphase machine ($n > 4$) and a number of pole pairs equal to $n - 1$ or $n + 1$.

Expressions (8.4) and (8.6) confirm the fact the magnet width is a parameter that can be used to delete an harmonic family\(^2\). Indeed it is possible in either case to find a value of $\xi$ that set $K_{2m+1}$ equal to 0 for an harmonic $2m + 1$. If we want for instance to delete harmonic 3, we must set $\xi$ to:

- $\frac{2\pi}{3p}$ for a radial magnetization,
- a value such that $\sin((3p+1)\xi) + \sin((3p-1)\xi) = 0$ for a parallel magnetization.

It is important to notice that this is independent of any other design parameters, such as the rotor outer diameter, the airgap height, etc.

### 8.3.2.2 Airgap field with magnets of constant width

For magnet with a constant width, we use the method presented in [7] which consists in using a Gaussian representation of the magnets, i.e. replacing magnets polarization by equivalent magnetic charge densities $dq_M$ at their borders. In the case shown in Figure 8.6(a), the Gaussian equivalent model needs only charge densities on the outer edge of the magnets. The other borders have no contribution. Note that, as the magnet angular width is not constant, $\xi$ is defined at the outer border of the magnet.

The equivalent magnetic charges of the other magnets are easily found by taking into account the fact that the center of the two successive magnets are shifted by $\pi/p$. Amplitudes of the airgap field harmonics at the stator inner border are hence given:

$$B_{2m+1} = \frac{pB_p}{\pi} \frac{R_{ro}^{2\nu} - R_{ry}^{2\nu}}{R_{si}^{2\nu} - R_{ry}^{2\nu}} R_{ro}^{\nu-1} R_{si}^{\nu-1} \left( \frac{\sin((\nu-1)\xi)}{\nu-1} + \frac{\sin((\nu+1)\xi)}{\nu+1} \right)$$  \hspace{1cm} (8.8)

\(^2\)as it has been already shown in section 3.4.1.1 by considering a simplified airgap field distribution
with $\nu = (2m + 1)p$.

Again, we can suppress an harmonic by choosing correctly the value of $\xi$, independently of the other parameters.

### 8.3.2.3 Magnetic field distribution in the iron parts

The magnetic field distribution in the various parts of the stator yoke is obtained by using a network of equivalent permeance. The definition of the network depends on the angular position of the rotor. The computation is made at the most critical position, such that the center of a magnet is aligned with the center of a tooth. In that position of the rotor, the flux is maximum in the considered tooth. This gives the network as shown in Figure 8.7.

The computation of the various permeances is straightforward but the computation of the permeance in the slot opening. Indeed it is difficult to predict the directions of the flux lines, especially at the location where the slot opening expand itself (at the location of the dashed permeance in Figure 8.7). A comparison with several FEM computations has shown that the best way to compute that permeance is to compute analytically only the permeance where the slot opening is equal to $b_0$ (before it expands itself) and to multiply the value by a factor 2. The current source in Figure 8.7 represents the fluxes computed by integrating the airgap field on an half teeth periphery.

The network allows us to determine the fluxes in the various part, and therefore the mean field corresponding the each fluxes.

On the rotor yoke the field with the maximum value is located at the junction between two rotor pole. This field is obtained by conservation of the flux at the rotor yoke outer periphery.
On the basis of the permeance network and the $B-H$ curve, the values of the relative permeabilities in all the iron parts are determined. By the Ampere’s law, these values are then used to correct the airgap field, computed previously by assuming an infinite permeability of the iron. Then, in an iterative process, new values of the permeabilities are computed, aiming to a new correction of the airgap field.

The final values of the permeabilities are used to compute the mean value of the field in the different iron parts. The optimization process will therefore be able to compare the values of the fields in the different iron parts with the maximum allowed values defined by the designer.

### 8.3.2.4 Fluxes induced by the magnets in the stator coils

The permeance network also allows to distinguish the useful flux flowing into the teeth from the leakage flux in the slot opening. The part of the flux flowing into the teeth is the flux induced by the magnets in the windings of the stator. The computation is made for one turn per phase, as adapting at the end of the design process the number of turns to the feeding voltage is a simple rule of three. From the design point of view, only the fundamental component $\psi_0$ of the flux is useful for the
torque production as we consider a feeding by sinusoidal currents. But the whole content is necessary to determine the relative permeability of the iron parts.

8.3.3 Armature reaction field and phase inductances

An analytical solution for the armature reaction field is determined on the basis of [8]. As for the no load airgap field, we assume a smoothed airgap. The coils are replaced by a distributed current sheet representation on the stator side of the airgap. For each slot the current sheet is distributed such that the current density is uniform along an arc whose length is equal to the slot opening $b_0$. Figure 8.8 shows the distributed current sheet representation of one turn with a current $I$ flowing into it. The two current densities are respectively equal to $I/b_0$ and $-I/b_0$ at the location of the two openings of the slots containing the turn.

$$B_{ar}(\zeta) = \sum_{m} B_{ar,m} \cos(m\zeta) \tag{8.9}$$
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$B_{ar,m}$ is the amplitude of the $m$th harmonic, given by:

$$B_{ar,m} = \frac{2\mu_0 I}{\pi R_{si}} \sin \left( \frac{m b_0}{2 R_{si}} \right) \frac{R_{si}^2 + R_{ry}^2}{R_{si}^2 - R_{ry}^2} \sin \left( \frac{m \pi}{2n} \right)$$  \hspace{1cm} (8.10)

The armature reaction field obtained with equation (8.10) looks like shown in Figure 8.9.

![Figure 8.9: Typical waveform of the armature reaction field at the stator side of the airgap, when one coil is fed by a current $I$](image)

The main part of the self inductance is obtained by integrating the field on the surface between the two current sheets and by dividing the result by $I$:

$$L_{main} = R_{si} L_e \int_{-\frac{\pi}{2n} + \arctan \left( \frac{b_0}{2 R_{si}} \right)}^{\frac{\pi}{2n} - \arctan \left( \frac{b_0}{2 R_{si}} \right)} B_{ar}(\theta_m) d\theta_m$$

$$= R_{si} L_e \sum_{m=1}^{\infty} \frac{B_{ar,m}}{I} \frac{2}{m} \sin \left( \frac{\pi}{2n} m - \arctan \left( \frac{b_0}{2 R_{si} m} \right) \right)$$  \hspace{1cm} (8.11)

with $n$ the number of phases and $L_e$ the active length of the machine.
Inserting equation (8.10) in (8.11), we get:

\[
L_{\text{main}} = \sum_{m=1}^{\infty} \mu_0 \frac{\sin\left(\frac{mb_0}{2R_{si}}\right) \sin\left(\frac{m \pi}{2n}\right) \sin\left(\frac{m \pi}{n} - \arctan\left(\frac{mb_0}{2R_{si}}\right)\right)}{m^2 \frac{b_0 \pi}{8R_{si} L_e}} R_{st}^{-2m} + R_{cy}^{-2m} \frac{R_{st}^{2m} - R_{cy}^{2m}}{m^2 \frac{b_0 \pi}{8R_{si} L_e}}
\]

This expression depends only on the design parameters defined in the previous section.

To be complete we must add the leakage inductance \( l \) in the slot opening. The leakage inductance is equal to the permeance of the slot opening shown in Figure 8.7. Adding it to the inductance given by the armature reaction field in the airgap gives the total self inductance:

\[
L = L_{\text{main}} + l
\]

The example of Figure 8.9 shows that the value of the armature reaction field at the location of the other phases is nearly constant. The mutual inductances with each of the other phases may therefore be considered to have all the same value \( M \). The mutual inductance is obtained by integrating on an interval of \( \frac{2\pi}{2n} \) the field at a mechanical angular shift of 180° with the two current sheets:

\[
M = \int_{\frac{\pi}{2n} - \arctan\left(\frac{b_0}{2R_{si}}\right) + \pi}^{\frac{\pi}{2n} + \arctan\left(\frac{b_0}{2R_{si}}\right) + \pi} B_{ar}(\zeta) d\zeta
\]

\[
= \sum_{m=1}^{\infty} \frac{B_{ar,m}}{I} \frac{2}{m} \sin\left(\frac{m \pi}{2n}\right) \sin\left(\frac{m \pi}{m} - \arctan\left(\frac{mb_0}{2R_{si}}\right) + \pi m\right) R_{st}^{-2m} + R_{cy}^{-2m} \frac{R_{st}^{2m} - R_{cy}^{2m}}{m^2 \frac{b_0 \pi}{8R_{si} L_e}}
\]

\[
(8.14)
\]

### 8.3.4 Electromagnetic torque

As we consider a machine without any saliency on the rotor side, the values of the self and mutual inductances of the coils are constant and do not vary with the rotor position. Hence, we find only two sources of torque: the electrodynamic torque, which is the useful torque, and the cogging torque, which results from the effect of the slot-teeth alternation on the no load magnetic energy (the magnetic energy associated to the sole magnets).
### 8.3.4.1 Electrodynamic torque

The useful component of the electrodynamic torque (i.e. the component due to the fundamental component $\psi_{k0}$ of the fluxes $\psi_k$ induced by the magnets) is given by:

$$T_{em} = \sum_{k=1}^{n} \frac{\partial \psi_{k0}(\theta_m)}{\partial \theta_m} i_k$$  \hspace{1cm} (8.15)$$

Indeed, with sinusoidal phase currents, as said before (see section 5.3.7.1), the harmonics of fluxes $\psi_k$ only produce a pulsating torque component.

Using the torque reference given by the mission profile, we can determine on the basis of the results of chapter 5 the currents needed to produce a torque equal to the torque reference.

### 8.3.4.2 Cogging torque

We presented an analytical expression for the contribution of one slot to the cogging torque in [9]:

$$T_{cog} = -\frac{b_0^2 L_e}{4\pi \mu_0} \frac{\delta B^2(\theta_m)}{\delta \theta_m}$$  \hspace{1cm} (8.16)$$

with $b_0$ the slot opening, $L_e$ the active length of the machine, and $B$ the amplitude of the magnetic field produced by the magnets at the center of the location of the slot for a position $\theta_m$ of the rotor, the magnetic field being computed by assuming a smoothed airgap.

This formula is valid only if the effect of the neighboring slots on the energy variation associated to one slot is negligible. In other words, we assume that the slot perturbs the airgap field locally. One of the main advantage of that expression is that it only requires the computation of the field with a smoothed airgap, which has been determined accurately as a function of the magnets shape in section 8.3.2.

The series expansion of the square of $B$ comes then as:

$$B^2 = \sum_{m} A_m \cos(mp\zeta) \hspace{1cm} m = 0, 2, 4, 6, 8, ...$$  \hspace{1cm} (8.17)$$

with

$$A_m = \frac{1}{2} \sum_{\nu=\mu=m} B_{\nu} B_{\mu} + \frac{1}{2} \sum_{|\nu-\mu|=m} B_{\nu} B_{\mu}$$  \hspace{1cm} (8.18)$$
where $B_\nu$ and $B_\mu$ are respectively the amplitude of the harmonics of rank $\nu$ and $\mu$ ($\nu$ and $\mu \in [1, 3, 5, 7, ...]$) of the series expansion of the airgap field $B$ computed as explained in section 8.3.2.

Replacing $\zeta$ by $-\theta_m$ in equation (8.17) gives the value of $B^2$ at the center of the slot. Introducing expression (8.17) in (8.16) yields the contribution of the slot to the cogging torque as a function of the rotor position:

$$T_{cog} = -\frac{b_0^2 L_e}{4\pi\mu_0} \sum_m \delta \theta_m \sum_m m p A_m \cos(mp\theta_m)$$

where $p = \frac{n_s}{2}$ or $\frac{n_s}{2} \pm 1$. Hence the cogging torque is now given by:

$$T_{cog} = \left(\frac{n_s}{2} \pm 1\right) \frac{b_0^2 L_e}{4\pi\mu_0} \sum_m \sum_k \left[ n_s A_m \sin\left(m\left(\frac{n_s}{2} \pm 1\right)\left(\theta_m - \frac{k-1}{n_s}2\pi\right)\right) \right]$$

Only terms for which $m\left(\frac{n_s}{2} \pm 1\right)$ is multiple of $n_s$ contribute to the cogging torque. The first and most important component of the cogging torque is thus found for $m = n_s$ and is equal to:

$$T_{cog,n_s} = \frac{b_0^2 L_e}{4\pi\mu_0} n_s A_{n_s} \sin\left(n_s\left(\frac{n_s}{2} \pm 1\right)\theta_m\right)$$

To limit the cogging torque, we can of course reduce the slot opening, but we are limited to a minimal value to be able to insert the copper wires in the slot, and reducing the slot opening has also the effect to increase the leakage fluxes, what reduces the flux induced in the windings and increases the saturation of the slot closings.
8.3.5 Joule’s losses

The Joule’s losses have to be computed in function not only of the mission torque profile, but also of the mission speed profile. Indeed in fault-tolerant operation mode, the Joule’s losses vary as a function of the rotor position. During the time the machine is stopped, and must maintain a torque, we assume the worst case scenario (i.e. the position in which the Joule’s losses are maximum). During the time in which the machine is rotating, we takes the mean Joule’s losses on an electrical cycle.

To estimate the Joule’s losses, we assume that the currents flowing into the machine are equal to the reference currents (i.e. we assume a perfect control of the torque). Therefore, referring to equation (5.54), we get the following total Joule’s losses in fault-tolerant mode:

\[
P'_J = R \frac{2 T_{ref}^2 n - 2 - \cos \left(2p\theta_m - \frac{j-1}{2}\pi n \right)}{n - 3} \tag{8.22}
\]

with the constraint of a zero valued sum of the currents.

Refering to equation (5.60), we get:

\[
P'_J = R \frac{2 T_{ref}^2 n - 1 - \cos \left(2p\theta_m - \frac{j-1}{2}\pi n \right)}{n - 2} \tag{8.23}
\]

without the constraint of a zero valued sum of the currents.

In both expressions \( j \) is the index of the lost phase and \( R \) is the phase resistance. For an accurate analysis of the temperature rise of the various coils, we must compute the losses in each phase, as the amplitude of the various phase currents are not equal. We have to consider two different cases: when the motor is running and when it is at standstill.

8.3.5.1 Joule’s losses at standstill

When the mission profile defines a torque at standstill, we consider the worst case scenario in which the electrical angular position yields the maximum Joule’s losses. If we consider that phase 1 is lost \((j = 1)\), and referring to equations (8.22) and (8.23) the maximum Joule’s losses are obtained for \( \theta_m = \pi/(2p) \). This angle corresponds to the position where current in phase 1 would have normally contributed the most
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to the torque production in normal mode. The Joule’s losses in each
remaining phase \( k \) is different from each others. Referring to equations
\((5.48)\), they are given by:

\[
P_{jk}'(\theta_m=\pi/(2p)) = R \left[ i_k' |_{\theta_m=\pi/(2p)} \right]^2
\]
\(= R \left[ \frac{2 T_{ref}}{n \psi_0} \cos \left( \frac{k-1}{n} \frac{2\pi}{2} + c_k \right) \right]^2 \) \( (8.24) \)

Without the constraint of a zero valued sum of the currents, coefficient \( c_k \) is given by equation \((5.24)\), and the Joule’s losses of phase \( k \) becomes equal to:

\[
P_{jk}'(\theta_m=\pi/(2p)) = R \left[ \frac{2}{n-2} \frac{T_{ref}}{\psi_0} \cos \left( \frac{k-1}{n} \frac{2\pi}{2} \right) \right]^2 \) \( (8.26) \)

With the constraint, coefficient \( c_k \) is given by equation \((5.27)\), and
the Joule’s losses are equal to:

\[
P_{jk}'(\theta_m=\pi/(2p)) = R \left[ \frac{1}{n-3} \frac{T_{ref}}{\psi_0} \left( \frac{1}{n-3} + \frac{n-1}{n-3} \cos \left( \frac{k-1}{n} \frac{2\pi}{2} \right) \right) \right]^2 \) \( (8.27) \)

8.3.5.2 Joule’s losses at constant speed and constant torque

When the mission profile defines a speed, we use the mean value on an
electrical period of the Joule’s losses in each phase. The amplitudes
of the fault-tolerant operation mode currents are given by equation \((5.19)\).
The Joule’s losses in each phase is therefore given by:

\[
\langle P_{jk}' \rangle = \frac{RI_k^2}{2}
\]
\(= \frac{RI^2}{2} \left( 1 + 2c_k \cos \left( \frac{k-1}{n} \frac{2\pi}{2} \right) + c_k^2 \right) \) \( (8.28) \)

\[
= 2R \left[ \frac{1}{n \psi_0} T_{ref} \right]^2 \left( 1 + 2c_k \cos \left( \frac{k-1}{n} \frac{2\pi}{2} \right) + c_k^2 \right) \) \( (8.30) \)

Without the constraint of a zero valued sum of the currents, coefficient \( c_k \) is given by equation \((5.24)\), and the Joule’s losses of phase \( k \)
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becomes equal to:

\[
< P'_{Jk} > = 2R \left[ \frac{1}{n} \frac{T_{ref}}{p\psi_0} \right]^2 \left( 1 + \frac{4(n - 1) \cos \left( \frac{k - 1}{n} \cdot \frac{2\pi}{2} \right)}{(n-2)^2} \right)
\]

\[
= 2R \left[ \frac{1}{n} \frac{T_{ref}}{p\psi_0} \right]^2 \left( 1 + \frac{2(n - 1) \cos \left( \frac{k - 1}{n} \cdot \frac{2\pi}{2} \right) \cdot 2 \cos \left( \frac{k - 1}{n} \cdot \frac{2\pi}{2} \right)}{n-2} \right)
\]

\begin{align*}
(8.31)
\end{align*}

With the constraint, coefficient \( c_k \) is given by equation (5.27), and the Joule's losses are equal to:

\[
< P'_{Jk} > = 2R \left[ \frac{1}{n} \frac{T_{ref}}{p\psi_0} \right]^2 \left( 1 + \frac{1 + 2(n - 1) \cos \left( \frac{k - 1}{n} \cdot \frac{2\pi}{2} \right) + 4(n - 2) \cos^2 \left( \frac{k - 1}{n} \cdot \frac{2\pi}{2} \right)}{(n-3)^2} \right)
\]

\[
= 2R \left[ \frac{1}{n} \frac{T_{ref}}{p\psi_0} \right]^2 \left( 1 + \frac{1 + 2(n - 2) \cos \left( \frac{k - 1}{n} \cdot \frac{2\pi}{2} \right) + 2 \cos \left( \frac{k - 1}{n} \cdot \frac{2\pi}{2} \right)}{n-3} \right)
\]

\begin{align*}
(8.32)
\end{align*}

We may verify that summing the Joule's losses of all the phases (from 2 to \( n \)) by using expressions (8.31) or (8.32) give the total Joule's losses, respectively given by expressions (5.54) or (5.60).

In all the expressions above, the phase resistance \( R \) is computed for one turn per coils:

\[
R = \rho_{co} \frac{L_{\text{turn}}}{f_j S_{\text{slot}}}
\]

where \( \rho_{co} \) is the resistivity of the copper, \( S_{\text{slot}} \) is the cross section of the turn\(^3\), \( L_{\text{turn}} \) is the length of the turn, taking into account the winding heads and \( f_j \) is the filling factor, as shown in Figure 8.10.

Turn heads is supposed to be formed of three sections: one section with a length equal to the tooth width, and two sections following quarters of circles for the connexion with the active part of the turn, and whose radius is given by:

\[
g = \pi \frac{R_{so} - h_s - h_t/2}{2n} - \frac{w_t}{2}
\]

\begin{align*}
(8.34)
\end{align*}

\(^3\)after subtracting the cross section of the tape isolating the windings from the stator.
8.3.6 Thermal model

As we have determined that the losses in the rotor yoke and in the magnets are negligible (see section 3.3.9 and reference [2]), we focus on the thermal behavior of the stator. Besides, we suppose that, due to the short mission time, there is no heat exchange between the stator and the rotor, and no exchange with the environment. The two most critical limitations in terms of temperatures are the temperature of the windings, and the temperature of the outer housing.

The thermal evolution of the various parts of the motor as a function of the mission profile are determined on the basis of a thermal model of the stator of the machine (Figure 8.11). This model takes into consideration the thermal transient behavior, the possible exchange with mechanical parts of the actuator and determines the temperature of the windings as well as of the stator yoke and of the housing. Due to the unbalance of the fault-tolerant operation mode, we must take into account the phases independently.

The thermal model determines an equivalent thermal capacitance of the windings ($C_1$ to $C_n$), the stator yoke ($C_s$), and the aluminum housing.

\footnote{There is barely no convection as we are rapidly in a space environment.}
of the motor \((C_h)\), and the thermal resistances between the windings and the stator \((R_1 \text{ to } R_n)\), between the stator yoke and the housing \((R_s)\), and between the housing and the possible mechanical parts \((R_{hmp})\), which is represented by a thermal capacitance \((C_{mp})\). The thermal capacitance of the mechanical parts is not determined by the model, it is a parameter given by the designer. Each thermal resistance of the model shown in Figure 8.11 is an equivalent resistance computed from a network of resistances corresponding to a more detailed model (not represented). For example, for each resistance between the windings and the stator yoke, we use the model shown in Figure 8.12. The resistances of the windings are computed by taking into account an equivalent homogenized material corresponding to the copper, its insulating varnish and the
8.4 Optimization strategy

This section aims to explain how the mission profile, the ambiance constraints and the analytical models described in the previous sections have been integrated in an optimization process. Figure 8.13 shows that the design process is the interaction of two blocks: the optimization algorithm and the solution builder which are completely independent from each other.

8.4.1 Principle

To reach a complete independence between the optimization algorithm and the builder, the interactions between these two blocks are defined as follow:

- For all the dimensional parameters that need to be optimized, the optimization algorithm proposes values between 0 and 1, and give them in the form of vector \( x \) to the builder. The builder translates these variables and interprets them as values for some of dimensional parameters listed in table 8.1. Then, on the basis of these parameters, it builds the solution and establishes the magnetic,
thermal and electrical models presented in section 8.3, and the thermal behaviour as a function of the mission profile.

- In return, in order to improve the solution, the optimization algorithm requires an information about the quality of the solution. That information takes the form of a vector of constraints $c$ and of a vector $f$ of evaluations defined by the designer.

  - **Vector of constraints $c$** : Using the models, the builder determines if the motor respects the constraint defined by the practical specifications given by the considered application. For each solution it defines a vector $c$ with a number of entries equal to the number of constraints. If the $k$th constraint is met, the $k$th entry of $c$ is set to 0. Otherwise the entry is set to a positive value defined by the user: the greater the value, the more important the constraint that is not satisfied. We usually classify the constraints into two types: weak ("the 3th harmonic of the EMFs is too high") and strong ("the temperature of the windings is higher than the insulator thermal limit"). So, the optimizer will first eliminate the solutions that does not satisfy the strong constraints.

  - **Vector of evaluations $f$** : The builder also rates the solutions according to evaluation functions to be minimized. Each score obtained by the solution for each evaluation function is
an entry of vector $f$. The lower the score for a criterion, the better the solution according to that criterion.

These evaluations and constraints allows the optimizer to rank the solutions. By seeking to satisfy the constraints and to minimize the evaluations functions, the optimizer will use that information to find at the end not one but several optimal sets of parameters $x$.

Indeed the optimizer has to deal with evaluation functions which are most of the time in contradiction: improving one means degrading another (for example reducing the mass while keeping the same performances implies an increase of the temperature rise of the windings). When comparing the final solutions given by the optimizer during of the optimization process with one criterion in $x$-axis and another in $y$-axis, the solutions have a repartition similar to the one shown Figure 8.14. The optimal solutions are the ones linked by the dashed dot, which is called a Pareto front.

Of course with $n$ evaluation criteria, the Pareto front lies in a $n$-dimension space. The solutions that forms the Pareto front are defined
as the rank 1 solutions. If we remove them, we can define a new Pareto and a rank 2 set of solutions, and so on to classify the solutions. At the end of the optimization process, the users will have to chose the rank 1 solution that, for him, gives the best trade off between the evaluation functions.

The whole interest of this structure is its flexibility. The builder and the optimizer are totally decoupled. The optimizer does not know that it optimizes an electrical machine and the builder does not know how the optimizer determines the design parameters. It is therefore simple to modify a model, add a parameter or a constraint, without redefining the whole design process. For example, we may seek to eliminate the third harmonic family by choosing the adequate value of the magnet width $\xi$ (see end of section 8.3.2.1). In that case, it is not necessary to let $\xi$ be a free parameter to be optimized, as it is possible to compute directly its value. In another case, if we seeks a low harmonic content of the EMFs, but not necessarily eliminating the third harmonic family, we let $\xi$ be a free parameter that will be added in vector $x$ of the parameters to be optimized, and we define either a constraint on the harmonic content, or we define the harmonic content as an evaluation criterion. In the first case, the optimizer will preferentially reject the solutions that does not respect the constraint. In the latter, the harmonic content participates to the ranking of the solutions.

When a strong constraint is not respected (e.g. a negative slot cross section), the builder stops the computation of the analytical model to save computation time, and simply sets the evaluation results and all the constraints to a high value. Automatically the optimizer will reject that solution in the selection process.

8.4.2 The builder

The builder is shown in Figure 8.15, and includes three actions:

- the geometric modelization of the solution, which, among others, translates of the dimensionless parameters $x$ given by the optimization algorithm into dimensional parameters,

- the physical modelization of the solution, on the basis of the analytical models presented in section 8.3;
Optimization strategy

8.4.2.1 Geometric modelization

As said above, the parameters given by the optimizer are dimensionless and bounded between 0 and 1. For each parameter \( x_i \), the builder makes the correspondence with a dimensional parameter \( \chi_i \). For that, the designer has to define a lower limit \( \chi_{i,min} \) and a higher limit \( \chi_{i,max} \). The builder is then able to interpret the dimensionless parameter given by the optimizer, by using the following correspondence law:

\[
\chi_i = \chi_{i,min} + (\chi_{i,max} - \chi_{i,min}) x_i
\]  

The designer must choose adequately the limits of the parameters to be optimized. If he chooses an interval too great, a lot of solutions will be rejected because they do not respect the constraints (for example if

- the evaluation of the solution and the satisfaction of the constraints.

Figure 8.15: Schematic view of the builder
we allow the teeth height to be greater than the maximum radius of the outer periphery of the stator). On the contrary if he choose an interval too small, the parameter’s optimum value may be outside the window allowed by the designer, and the optimizer will stop in a suboptimal solution. In that case, in most of the time the parameter $x_i$ will tend to 0 or 1, and will show that the window must be increased, but it is not necessarily the case as the optimizer may be trapped in a suboptimal solutions.

In addition, he must also give to the builder some predetermined parameters, such as the number of phase, the number of pole pairs, but also the dimensional parameters that are predetermined by mechanical constraints, such as the thickness of the isolating tape, the thickness of the jacket around the magnets, etc. On the basis of these dimensional parameters, a geometrical modelization the motor is performed, required for the physical modelization.

### 8.4.2.2 Physical modelization

The builder then determine the analytical models associated to the machine, by using the analytical models presented in section 8.3.

The designer must give to the models physical characteristics, such as the $B$-$H$ curve of the iron, the resistivity of the copper, as well as the thermal conductivities, specific heats, densities of the materials, the initial temperature of the system, etc. The mission profile, required to compute the thermal behavior, is also included in the characteristics to provide. The designer must also indicate if the sum of the currents is constraint to be equal to zero, as it determines the choice of the current law among the ones presented in section 5.3 of chapter 5.

The creation of the solution follows the step below:

- computation of the no load airgap field, and of the maximum values of the no load fields in the various iron parts, as explained in section 8.3.2;

- computation of the phase currents in fault tolerant operation mode to meet the mission profile, using the sinusoidal currents determined in section 5.3;

- computation of the Joule’s losses according to the mission speed profile and of the mission torque profile, as explained in section
8.3.5;

- determination of the thermal behavior of the motor using the thermal network depicted in section 8.3.6;

- computation of the armature reaction field in the airgap and in the various parts of the motor, and determination of the inductances, and of the total saturation level of each iron part of the motor;

- computation of the mutual and self inductance for one turn;

- and computation of the cogging torque.

8.4.2.3 Constraints satisfaction and evaluations

The designer must define the constraints that the motor has to satisfy, and evaluation functions required by the optimization algorithm.

The greatest the number of evaluation functions we define, the harder the optimization algorithm will converge, as it yields a lot of solutions in the Pareto front. Usually we define two evaluation functions: a main one and a secondary. The main one is:

- the mass of the actuator (the sum of the mass of each modelized part);

- or the length of the motor.

The secondary one may be:

- the harmonic content of the EMFs;

- the cogging torque;

- the inductance value;

- the maximum winding voltage required for one turn;

- the rotor inertia;

- or the temperature rise.
The criteria that are not taken into account in evaluation functions are considered with the vector of constraints. We define a maximum allowed value for the cogging torque, the amplitude of the main harmonics, etc. We can also combine an evaluation function with a constraint. For example, a constraint defines the maximum allowed temperature in the windings, and the secondary evaluation function is the difference between this limit and the maximum temperature reached during the mission profile.

Other considered constraints are physical constraints, as the maximum field amplitude in the various iron parts of the motor, but also constraints that helps the optimization algorithm to put aside not interesting solutions, as solutions with too great motor length or motor weight.

8.4.3 The optimization algorithm

The optimizer lies on a genetic algorithm developed in the lab [11, 12]. This algorithm uses a population of solutions and makes that population evolve with the aim of finding the most fitted solutions to satisfy the constraints and minimize the evaluation functions. The process of optimization is iterative, and is shown in more details in Figure 8.16. The optimization algorithm is described through one step of the iterative process. It is decomposed in three actions: genetic manipulation, classification and selection.

8.4.3.1 Genetic manipulations

At step \(k\) of the optimization process, the optimizer receives from the previous step a population of \(n\) solutions.\(^5\) Each solution \(i\) consists in its vector \(x_i\) of the free parameters to be optimized, its vector \(c_i\) of constraints and its vector \(f_i\) of the evaluations. The population \(k\) is therefore defined by three matrices:

- \(X_k\): a \(n \times n_x\) matrix, with \(n_x\) the number of free parameters;
- \(C_k\): a \(n \times n_c\) matrix, with \(n_c\) the number of constraints;
- \(F_k\): a \(n \times n_f\) matrix, with \(n_f\) the number of evaluation functions.

\(^5\)At the first step, the solutions are built with random parameters.
Figure 8.16: Schematic view of the optimization process
First, the optimizer creates a new matrix $X_c$ by mixing and modifying matrix $X_k$ of the free parameters. This creation reproduces genetic crossovers and genetic mutations of the life evolution. The crossover in the optimization process is the following:

- Two solutions are randomly chosen. The one with the best ranking\(^6\) is kept. The other is put aside. This tournament is done a second time to select a second solution.

- A new vector $x_c$ is created for the childs. Each parameter of the vector $x_c$ is randomly the parameter of first solution, the parameter of second one or a random linear combination of the two.

This process is iterated $n$ times in order to get a $n \times n_2$ matrix $X_C$. Then the mutation process modify matrix $X_C$ simply by replacing some parameters by a random number between 0 and 1.

The two manipulations are complementary. The crossover manipulation seeks the convergence by looking for solutions close to the current optimal solutions. The mutation manipulation has an objective of exploration of the parameter space. While the convergence is necessary along the whole optimization process, the exploration is more useful at the beginning of the iterative process, in order to explore a maximum the parameter space, and not at the end, when we want to refine the solutions. Therefore, we decrease the probability of mutation during the optimization process, from a value of about 20% at the first iteration, and 0% at the last one.

The childs are then given to the builder, which builds the solutions associated to each set of parameters, evaluates them and return to the optimization algorithm matrix $F_C$ and matrix $C_c$, respectively giving the values returned by the evaluation functions and the constraint satisfaction, as explained in section 8.4.1.

### 8.4.3.2 Classification and selection

The optimization algorithm has now $2n$ solutions, $n$ from the initial population, and $n$ from the one generated by the genetic manipulations. The last action of the optimization algorithm is to rank these solutions and to select $n$ among them, as input of the next iteration.

\(^6\)The method of ranking is explained below in the next section Classification and selection
The solutions are first ranked using the information given by the constraints. For each solution, we compute the sum of the entries of the vector of constraints, and we define one set of solution for each value of the sum.

In each set, the solution are then compared using their vectors of evaluations:

- if all the values of the vector of solution \( a \) are lower than the values of solution \( b \), we say that \( a \) dominates \( b \) (and obviously, if all the values of \( b \) are lower than the values of \( a \), \( b \) dominates \( a \));

- else, we say that \( a \) and \( b \) are in non-dominance.

By computing the relation between all the solutions in each set, we can establish the ranks of the solution. The solutions that are not dominated by any other are the rank 1, the ones that are only dominated by rank 1 are rank 2, and so on.

After computing this classification of the solutions, the optimization algorithm keeps the \( n \) best ones. When it comes to choose between solutions that have the same value of the sum of the entries of the vector of constraints, and having the same rank given by the evaluated functions, the optimization algorithm choose the ones that are the most distant from the others in the space of the evaluations, in order to maximize the diversity among them.

If the iterative process lasts long enough, and if the constraints are not too hard to satisfy (all of them), the optimization process goes through two mode. In the first one, the convergence is mainly determined by the vectors of constraints. In the second mode, the iterations eventually ends with solutions satisfying all the constraints. Their classification depends only on the ranking of the evaluations functions.

### 8.4.3.3 End of the process

The stop condition is simply an number of iteration fixed by the designer. Indeed it is not easy to find an automatic stop condition of the optimization process, as the process is stochastic. Besides, there is no way to prove that the optimization algorithm found global optima. Some of the iterations do not make any substantial progress, others marks big steps by introducing some new optimal solutions.
The best method to ensure that the solution is the optimal one is to run the optimization process several times and to check that each process gives the same final results.

8.4.3.4 Number of turns per phase and electrical characteristics of the machine

In the design process, we considered that each phase is made of one turn and we determined the characteristics of the machine independently of that parameter. To complete the design, the designer must define a DC bus voltage value $U_{DC}$, and, depending on the power electronic considered to feed the machine, must precise if it is the line bus voltage or the phase bus voltage.

At the end of the process, the builder computes the maximum phase/line voltage, named $U_{turn}$, considering one turn per phase. The number of turns is then determined by the equation:

$$n_t = \left[ \frac{U_{DC}}{U_{turn}} \right]$$ (8.36)

Each phase of the machine is then wound with $n_t$ turns. The cross section of the turns is equal to:

$$S_{turn} = f_f S_{slot} / n_t$$ (8.37)

and the length $L_{winding}$ of the winding is equal to:

$$L_{winding} = n_t L_{turn}$$ (8.38)

The final electrical variables of a phase are then equal to:

$$R_{phase} = n_t^2 R$$

$$L_{phase} = n_t^2 L$$

$$M_{phase} = n_t^2 M$$

$$\psi_{0,phase} = n_t \psi_0$$ (8.39)

The phase current amplitude is given by:

$$I_{phase} = \frac{I}{n_t}$$ (8.40)
The determination of $U_{\text{turn}}$ is not straightforward because it depends on the number of phases, of the constraint of keeping the sum of the currents equal to zero or not (what defines the current law), and of the value of the phase resistance and the phase inductance in regards with the phase back EMF coefficient. Therefore we compute the maximum phase voltage of each phase, using equation (3.35), considering one turn, as the electrical variables have been computed for one turn. Then we determine numerically the maximum global phase/line voltage $U_{\text{turn}}$ in order to determine $n_t$ and then recompute the electrical variables.

### 8.4.4 Visualization of the optimization process

To get an idea of the application of the optimization process, Figure 8.17 shows an example of evolution of the solutions during the optimization process, at iterations 1, 5, 10, 20, 50 and 100. In each Figure, we can see:

- on the left: the cross section of the best solution of the motor (we chose the solution that has the minimal mass).

- on the top right: the Pareto front with the mass on the x-axis, and the current on the y-axis. These two criterion are represented in per unit, divided by the expected value by a pre-dimensional design. It is expected that the optimization shows values lower than 1 in both case. The solutions that does not satisfy the strong and weak constraints (such as a too high temperature) are plotted in red and green respectively. The solutions that satisfy all the constraints are plotted in blue. As the optimization process goes, the red solution gives way the green ones, which in turn disappear in favor of the blue ones.

- on the bottom right, the 8 free parameters that are optimized, from the optimizer point of view (i.e. between 0 and 1). The values are represented on an polygon, what shows that one of them is not close to 0 or 1, and that we do not need to start again the optimization process with a modified value of the bounding of that parameter.

In that example, the process of optimization took half an hour to converge to a set of optimal solutions, with 100 iterations, a population
of 100 individuals, 2 evaluations criterions, 12 constraints and 8 design parameters.
Optimization strategy

Figure 8.17: Example of an optimisation process
Figure 8.17: Example of an optimisation process (con’t)
Figure 8.17: Example of an optimisation process (con’t)
Figure 8.18 shows the thermal evolution result of one of the optimal solutions in another optimization for a four phase motor. We can see in that example that one phase (phase 2) reaches a higher temperature. As we suppose that we run in fault tolerant mode since the beginning of the mission, with phase 4 as the lost phase. The phase in opposition with the lost phase has the highest current amplitude, and therefore the highest Joule’s losses. Phase 4 does not produce losses, and is nearly at the same temperature that the stator yoke.

![Figure 8.18: Temperature evolution of a 4-phase motor as a function of the mission profile](image-url)
CHAPTER 9

Validation of the model used for sizing: FEM analysis and tests on a prototype

9.1 Introduction

In this chapter, we validate the analytical models detailed in the previous chapter, both numerically and by experimental measurements. In order to do that, we built a 6-phase motor prototype. The interest is that we know every materials and therefore their characteristics, such as the remanent field of the magnets, the $B$-$H$ curve of the iron yokes, ... necessary to build the analytical and numerical models.

We first modelize the prototype in the FEM software COMSOL, and we validate the analytical magnetic models, including inter alia the no load airgap field, the fields in the various iron parts, as well as the cogging torque and the electrodynamic torque. Then we compare the values of the electrical parameters predicted by the FEM and analytical models with the value of these parameters measured on the prototype.
9.2 The prototype

The prototype is a 6-phase 5 pole pairs segment motor. The dimensional parameters are bound by confidentiality with our industrial partner.

The geometry used for the analytical modelization is shown in Figure 9.1(a). The geometry of the FEM model is shown in Figure 9.1(b), and takes into account the rounded corners of the slots of the prototype. In the FEM model, phase 1 is is the right-most one.

9.3 Validations of the magnetic models

9.3.1 No load airgap field

The machine holds magnets with parallel magnetization and constant width. Hence we use equation (8.8) to modelize the field generated by the magnets in the airgap.

Figure 9.2 shows on top the map of the no load field given by the FEM software\textsuperscript{1} for the model of the prototype.

On bottom we can see the field at the stator periphery computed analytically as explained in section 8.3.2 and numerically using the FEM

\textsuperscript{1}COMSOL multi-physics 3.5a
Validations of the magnetic models

software. The FEM computation and the analytical modelization are in good agreement, despite the fact that the analytical modelization does not take into account the slots effects.
9.3.2 Armature reaction field

Figure 9.3 shows on top the map of the field generated by one phase (without the magnet field) at the nominal current amplitude. On bottom we can see the field at the stator periphery computed analytically as explained in section 8.3.2 and numerically using the FEM software.

Once again, the FEM computation and the analytical modelization are in good agreement. The analytical model takes into account the slot opening of the phase generating the field, but not the slot opening of the other phases. The FEM result shows small perturbations due to these slot openings, but the value of the field is merely constant, as expected by the analytical model.

Figures 9.4 and 9.5 show respectively the armature reaction fields in normal operation mode and in fault-tolerant operation mode with the constraint of the zero valued sum of the currents.

In normal operation mode, the currents expression is given by equation (5.10). In the case shown in the two Figures, the rotor position relatively to the stator phases yields an electrical angle of $p\theta = -\pi/2$. Therefore, the currents amplitudes are given by:

\begin{align}
I_1 &= I_{nom} \\
I_2 &= \frac{1}{2}I_{nom} \\
I_3 &= -\frac{1}{2}I_{nom} \\
I_4 &= -I_{nom} \\
I_5 &= -\frac{1}{2}I_{nom} \\
I_6 &= \frac{1}{2}I_{nom}
\end{align}

(9.1)

In fault-tolerant operation mode with a zero valued sum of the currents, the currents waveforms are shown in Figure 5.17. With the electrical angle equal to $p\theta = -\pi/2$, the currents amplitudes are given by the
Figure 9.3: No magnets field - nominal current in phase 1
Validation of the model

Figure 9.4: No magnets field - nominal currents in the 6 phases
projection on the $y$-axis of the vector representation of Figure 5.17(b):

\begin{align*}
I'_1 &= 0 \\
I'_2 &= I_2 + \frac{2}{3}I_1 = \frac{7}{6}I_{\text{nom}} \\
I'_3 &= I_3 = -\frac{1}{2}I_{\text{nom}} \\
I'_4 &= I_4 - \frac{1}{3}I_1 = -\frac{4}{3}I_{\text{nom}} \\
I'_5 &= I_5 = -\frac{1}{2}I_{\text{nom}} \\
I'_6 &= I_6 + \frac{2}{3}I_1 = \frac{7}{6}I_{\text{nom}}
\end{align*}

In both case, we can see that the field is equal to zero at the periphery of non-wound teeth. As the sum of the currents is equal to zero in normal and in fault-tolerant operation mode, the sum of the contributions of each phases to the field under non-wound teeth is equal to zero. In fault-tolerant mode, this is also true under the tooth of the lost phase.

### 9.3.3 Airgap field at load

Figures 9.6 and 9.7 show the airgap field mapping at the nominal torque respectively in normal and in fault-tolerant operation mode. In the analytical model, the field is obtained by summing the no load airgap field with the armature reaction field.

### 9.4 Electromagnetic torque

As said in section 8.3.4 the electromagnetic torque has only two components: the cogging torque due to the interaction of the magnets field with the slot-teeth alternation, and the electrodynamic torque, due to the interaction of the magnets field with the armature reaction field. Appendix 9.7.1 sums up the method used for computing both the cogging torque and the electrodynamic torque.

#### 9.4.1 Electrodynamic torque

Figures 9.8(a) and 9.8(b) show the torque computed by the FEM model and by the analytical model, for various angular position $\theta_m$ of the rotor,
Validation of the model

Figure 9.5: No magnets field - fault-tolerant currents in the 6 phases
Figure 9.6: Magnets field - nominal current in the 6 phases
Figure 9.7: Magnets field - fault-tolerant currents in the 6 phases
Electromagnetic torque

from 0 to \(18^\circ\) with steps of 0.5\(^\circ\). The analytical model takes into account the harmonic content of the EMFs up to the 15th harmonic.

The currents injected in the phases are the sinusoidal ones, defined to keep the torque constant if the EMFs are sinusoidal. In normal operation mode (Figure 9.8(a)), they are equal to:

\[
i_k(\theta_m) = -I_{\text{nom}} \sin \left(5\theta_m - 5\frac{k - 1}{3}\pi\right)
\]  

(9.3)

and in fault tolerant operation mode (Figure 9.8(b)):

\[
\begin{align*}
i_1(\theta_m) &= 0 \\
i_2(\theta_m) &= -I_{\text{nom}} \left(\sin \left(5\theta_m - \frac{5}{3}\pi\right) + \frac{2}{3} \sin \left(5\theta_m\right)\right) \\
i_3(\theta_m) &= -I_{\text{nom}} \left(\sin \left(5\theta_m - \frac{4}{3}\pi\right)\right) \\
i_2(\theta_m) &= -I_{\text{nom}} \left(\sin \left(5\theta_m - \pi\right) + \frac{1}{3} \sin \left(5\theta_m\right)\right) \\
i_3(\theta_m) &= -I_{\text{nom}} \left(\sin \left(5\theta_m - \frac{2}{3}\pi\right)\right) \\
i_2(\theta_m) &= -I_{\text{nom}} \left(\sin \left(5\theta_m - \frac{1}{3}\pi\right) + \frac{2}{3} \sin \left(5\theta_m\right)\right)
\end{align*}
\]  

(9.4)

The torque obtained from the FEM model shows a pulsating component of about \(\pm7\%\) around the nominal value, due to the harmonic content of the EMFs. It shows a good concordance with the torque obtained by the analytical model. The small divergences come mainly from the fact that the analytical model does not take into account the influence of the slot openings on the harmonic content of the EMFs.

The mean value obtained by the FEM model is exactly 1.00 p.u. in normal operation mode, and 0.98 p.u. in fault-tolerant operation mode. The slight decrease of the mean torque in fault-tolerant mode is due to the higher level of saturation of the iron parts of the motor.

9.4.2 Cogging torque

Figure 9.9 allows to compare the value of the cogging torque in function of the rotor position computed analytically with the values given by a FEM analysis, which takes into account the finite permeance and the saturation of the iron parts. The analytical result of the cogging torque is obtained by taking into account the fundamental term of the cogging
Validation of the model

Electromagnetic torque [p.u.]

rotor angular position [°]

(a) normal operation mode

(b) fault-tolerant operation mode

Figure 9.8: Torque at nominal current
torque, given by equation (8.21). Concordance between the analytical prediction and the values given by the FEM analysis is quite satisfying, and could be improved by taking into account the main harmonics with the analytical expression given by (8.20). But this is not mandatory for the design process, which needs only the maximum value of the cogging torque. The comparison shows that the amplitude of the analytical prediction is 2% greater than the maximum value obtained by the FEM modelization. Hence the amplitude of the analytical prediction is a good prediction of the maximum value of the cogging torque.

To complete the validation, some experimental measurements made on the prototype, on the stable part of the curve giving the cogging torque, confirm that the predicted values of the cogging torque are also close to the experimental ones.

### 9.5 Validation of the electrical parameters

Table 9.1 shows the parameters obtained with the analytical model, the FEM model, and eventually the values measured on the prototype. The FEM computation of the $\psi_0$, $L$ and $M$ is detailed in appendix 9.7.

The parameters obtained analytically and numerically are in very good agreement. As the analytical and FEM models are 2D models, the small divergences with the measurements on the prototype mainly come from border effects. These effects could be taken into account, but would almost surely increase the computation time. These deviation are sufficiently small to validate the use of the analytical model in an optimization process.
Validation of the model

<table>
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<th>analytical values</th>
<th>FEM values</th>
<th>measurements</th>
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<td>44.3</td>
<td>39.8</td>
</tr>
<tr>
<td>phase resistance $R$ [mOhm]</td>
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<td>/</td>
<td>106.7</td>
</tr>
<tr>
<td>self inductance $L$ [$\mu$H]</td>
<td>709.8</td>
<td>705.6</td>
<td>560</td>
</tr>
<tr>
<td>mutual inductance $M$ [$\mu$H]</td>
<td>28.7</td>
<td>28.5 ± 0.7</td>
<td>30</td>
</tr>
</tbody>
</table>

Table 9.1: Comparison of the parameters

9.6 Conclusions

The analytical model has been validated by comparison with a 2D FEM modelization and measurements on a prototype. The validation also shows the limitations of the model:

- The prototype has a low armature reaction field in comparison with the magnets field, and the saturation level is not critical. Therefore the analytical model, which suppose that the load field is the addition of the no load airgap field with the armature reaction field computed independently from each other, has good agreement with the FEM model and the prototype. But we can see that the torque with the fault-tolerant currents, is slightly smaller than with currents in normal operation mode. To improve the model for machines with a more important armature reaction field, the load field should be computed at once, taking into account the field due to the magnets and the field due to the currents, and determines recursively the level of saturation.

- It is unfortunately difficult to modelize the magnetic model of the slot closing and their saturation level with a simple analytical model, or with an equivalent network of permeance. For machines with an acceptable saturation level in the closing, the empirical coefficient 2 works fine, and all the comparison of analytical models with FEM models have shown a good agreement. But with machines with a saturation of the closing much higher than in the teeth, the FEM and analytical models diverge. In order to obtain a more precise model allowing for a finer optimization of the slot shape, it is most likely having to go through a shape optimization by finite element.
The analytical and FEM models are both 2D models, and does not take into account border effects.

9.7 Appendices

9.7.1 Computation of the torque in the FEM model via the Maxwell’s stress tensor

The torque is determined numerically by integrating the component of the Maxwell’s stress tensor tangential to a surface in the airgap. The FEM software gives the value of the $x$ and $y$ components of force density $\sigma$ computed from the Maxwell’s tensor:

$$\sigma = (\sigma_x, \sigma_y)$$  \hspace{1cm} (9.5)

By defining $r_a$ the distance between the center of the rotor and a point in the airgap, the formulation of the torque $T_{FEM}$ is hence given by:

$$T_{FEM} = 2\pi L_e r_a \int_0^{2\pi} y\sigma_{\theta,x} - x\sigma_{\theta,y} d\theta$$  \hspace{1cm} (9.6)

where $\sigma_{\theta,x}$ and $\sigma_{\theta,y}$ are the $x$ and $y$ components of the tangential component of the Maxwell’s stress tensor, as shown in Figure 9.10. They are linked to the $x$ and $y$ components of the Maxwell’s stress tensor by using:

$$\sigma_{\theta,x} = -\sigma_\theta \frac{y}{\sqrt{x^2 + y^2}}$$

$$= -\sigma_y \frac{xy}{x^2 + y^2} + \sigma_x \frac{y^2}{x^2 + y^2}$$

$$\sigma_{\theta,y} = \sigma_\theta \frac{x}{\sqrt{x^2 + y^2}}$$

$$= -\sigma_x \frac{xy}{x^2 + y^2} + \sigma_y \frac{x^2}{x^2 + y^2}$$  \hspace{1cm} (9.7)

With no current density in the windings, this method computes the cogging torque. With the adequate current densities in the windings, the method computes the electromagnetic torque (i.e. the sum of the cogging torque and the electrodynamic torque).

The electrodynamic torque is therefore computed by subtracting the cogging torque from the electromagnetic torque.
9.7.2 Computation of the flux induced in a phase of the FEM model

To compute the flux induced in a phase of the motor, we use the following equation:

$$\psi = \int_V N \cdot A dV$$  \hspace{1cm} (9.8)

where $N$ is the density of conductors of the phase, $A$ is the potential vector, directly given by the FEM model, and $V$ the volume of integra-
In our case, the density of conductor is equal to zero everywhere except for the two slots of the phase, where the density is simply equal to plus or minus the number of turns divided by the cross section of the slot, as shown in Figure 9.11.

The volume of integration is therefore reduced to the volume of these two slots. As we consider only the active part of the machine, these two volumes are equal to $L_e S_{\text{slot}}$, with $L_e$ the active length of the machine and $S_{\text{slot}}$ the cross section of a slot. The equation of the flux eventually becomes:

$$\psi = L_e \frac{n_{sp}}{S_{\text{slot}}} \left( \int_{S_{\text{slot},1}} \text{AdS} - \int_{S_{\text{slot},2}} \text{AdS} \right)$$

Without current densities in the windings, this method determines the flux induced by the magnets. By computing the flux for different
positions of the rotor, it is possible to deduce the EMFs waveforms. By defining a current density in the phase with the conductor density, one can determine the self inductance, and by defining a current density in another phase, the mutual inductance between the two phases.
References


Validation of the model


Conclusions

This thesis is a contribution to the development of high performances high reliability electromechanical actuation systems for aerospace applications.

In the first part of the thesis, based on an analysis of the literature on the topic, we have investigated why polyphase permanent magnet synchronous segment motors are considered as the most suited candidate for aerospace actuation systems in terms of ease of torque control, torque to mass ratio and reliability. By a proper motor and control electronics design, they can still correctly operate in the case of the loss of feeding of one phase.

The second part of the thesis deals with control issues. In the first section of this part, we have developed a general formulation of the problem of finding in function of the rotor position the expressions of the phase currents allowing to develop, in normal operation mode or after a fault (the loss of feeding of one phase), a given torque while minimizing the Joule’s losses. This formulation based on a Lagrangian principle has allowed us to find the optimal currents for any number of phases with or without the constraint of having a sum of the currents equal to zero. Furthermore, for machines with sinusoidal EMFs, we have developed a method allowing to constraint the currents to vary sinusoidally with the rotor position for both modes of operation. We have also shown that in fault-tolerant operation mode the constraint of imposing to the currents to remain sinusoidal increases somewhat the Joule’s losses, but in a way which remains acceptable. In return, the motor is much more easier to control with reference currents varying according to sinusoidal reference values.

In the second section of this part we have developed two control
Validation of the model

strategies allowing to control the currents in the rotor reference frame when their reference values vary sinusoidally in function of the rotor position. The main advantage of making the control in the rotor frame is that in this frame the dependency of the reference and measured currents on the rotor position disappears in steady state and that the current controller has only to track constant reference values in steady state. This is still more interesting for polyphase segment motors than for classic three phase ones since, due to their high number of rotor pole pairs, segment motors have a much higher electrical frequency.

The first control strategy relies on generalized Concordia and Park transforms. It differs from the solution found in the literature by the fact that we use the same transforms in normal and fault-tolerant operation modes, so that we do not have to modify the core of the control algorithm when moving from normal operation mode to fault-tolerant operation mode. Switching to fault-tolerant operation mode only implies some simple actions on the measured values of the currents and on the phase voltages generated by the controller.

The second control strategy developed is based on the use of one Park reference frame per phase and allows to control independently each current in the rotor frame. With this control strategy the switching to fault-tolerant operation mode only implies to modify the values of the reference currents in the Park frame.

The third section of this second part presents the validation of the theoretical results both by simulation and experiments on a laboratory 6-phase segment prototype of a synchronous machine.

The third part of the thesis has been devoted to the optimal design of the motor by considering a typical mission profile: the thrust vector control of a space launcher. In this part we have built a design program aiming at optimizing the motor size under given constraints. The main originality of the solution we have developed is the segregation between the optimization process and the modelization process.

The optimization process relies on a genetic algorithm and makes evolve a population of solutions by modifying the design parameters. The modelization process uses the design parameters given by the optimization algorithm to build the solutions, and evaluates them in function of evaluation functions and of constraints satisfactions. The optimization process uses that information to modify the design parameters and, eventually, converge to optimal solutions. This performance evaluation
of the various designs is made on the basis of an analytical modeling in order to fasten the optimization. The modeling includes both electromagnetic and thermal aspects. At the end of the optimization a FEM analysis is used for validating the final design.

The main contributions developed in this thesis have already been summarized in 12 conference papers (6 referred in the IEEE database) and 2 journal papers referred in the science citation index.
The first part of this thesis has covered the technological choice. It comes to the conclusion that to provide fault tolerant abilities to the actuator, the segment motor is certainly the most fitted motor technology. About the power electronics, a qualitative overview has described some architectures more or less fitted to be integrated in a fault tolerant actuation system. We arrived to the conclusion that the ability to pursue the mission after the loss of one phase in open circuit has the benefit to reduce the search field to power electronics architecture tolerant to short circuit failure only (or that are able to transform a short circuit failure into an open circuit failure). But we did not determine the best architecture, as it depends on manufacturing technologies, and on which fault has to be considered or not, with which level of criticality. On the basis of that, a rigorous study of the reliability of a set of architectures could be performed, using Markovian chains for example, in order to determine the best suited for a given application.

Without dismissing any power architectures, the control laws and control strategies of fault-tolerant segment motors have been deeply investigated, and validated, in the second part of this thesis. For every control laws that we developed, we have chosen to minimize the Joule’s losses, which impact directly the sizing of the motor. We saw that it leads to different current amplitudes in each phases. Other criterions could be investigated, such as the minimization of the radial forces due to the unbalance that appears when a phase is lost, or the minimization of the maximum current amplitude, which impacts directly the sizing of the power electronics.

About the control strategies, the fault detection and diagnosis has just been summed up, as it has already been investigated by a lot of
people. The next step is to fusion on line fault detection systems with a control reconfiguration, and to elaborate the software taking the decision of reconfigure the control from the normal operation mode to the faulted one. At that moment, we could compare more precisely optimized implementations of the two control strategies, including the fault detection systems.

Also, as the fault-tolerant strategies that we developed do not modify the core of the controllers developed for the normal operation mode, it is easy to adapt them to other types of controllers developed for the normal operation mode, such as a Sigma-Delta controller type.

The possibilities of flux weakening in fault-tolerant mode could be investigated. It has not been done in this thesis, because it was not relevant in the context of the considered application case, as the maximum torque may be needed at the same time that the maximum speed.

In the last part of this thesis dedicated to the design issues, we have introduced some pre-design assessment in order to limit the number of design parameters. But the segregation between the optimizer and the builder offers a great modularity and the possibility to add design parameters defining more complex structures of segment motors, as detailed in section 3.4 of the first part of this thesis:

- the use of unequal widths for wound teeth and non-wound teeth;
- the use of more than one coil per phase (and the type of slot subdivision);
- the use of more elaborate magnets shapes and configurations, such as Halbach arrays;

The analytical model has been developed to be used in an optimization process. There is hence a trade off between the computation time and the level of accuracy of the computed characteristics. The validation by comparison with a FEM modelization and measurements on a prototype have shown that the analytical model is very accurate but two effects not taken into consideration affects the results: the end effects and the saturation level in the slot lips. These two effects affects mainly the value of the self inductance. A perspective is to find a way to take into account these effects by a modelization that does not increase significantly the computation time.
We could also integrate the number of phases and the number of pole pairs in the design process. This has not been done as, from the motor point of view only, the more the number of phases the smaller the importance of the loss of one phase. To insert the number of pole pairs and of phases in the optimization process, we must perform the optimization from the actuator optimization point of view. This yields a lot of other parameters to take into account, such as the available power electronics packages, the weight of the cables between the motor and the power electronics, ...

The genetic algorithm used in the optimization process is efficient for the exploration, and is fitted to mix integer parameters (such as the number of phases, the number of coils per phase, ...) with real parameters. But it is not adequate for refining the parameters obtained at the end of the optimization. The optimization process should consist of two steps:

1. A global optimization using the genetic algorithm explores the space of solutions, with possibly some integer parameters.

2. A local optimization process refining precisely the real parameters to their optimal values without modifying the integer parameters given by the global optimization.

In the end, this thesis has investigated paths of improving the reliability of permanent magnet drives, with an application case in the aerospace application field. But other application fields require high reliability actuation systems. The wind power sector for example, could have great interest in fault-tolerant abilities of the generators used in the wind turbines (specially off shore wind turbines) to drastically decrease the maintenance cost. The traction sector also follows the “More Electric” trends, and, with hybrid and electric vehicles, will require actuation systems with a high level of reliability. The fact that control and design aspects have been developed with a generic phylosophy (the reconfiguration strategies outside the core of the controller, the optimization process independent of the analytical modelization) let see interesting opportunities to be used in these applications fields.
“As I previously noted, I have designed in the days when I was a university student, a flying machine of a different nature from those currently circulating in the air.”

Nikola Tesla,
free translation from "Mes inventions : Autobiographie d’un génie"