"COD and VFA's control in a two-phase anaerobic digestion process"

Aguilar Garnica, Efrén ; Dochain, Denis ; Alcaraz-Gonzalez, Victor ; Drame, Abdou ; Gonzalez-Alvarez, Victor

Document type: Communication à un colloque (Conference Paper)

Référence bibliographique

Aguilar Garnica, Efrén ; Dochain, Denis ; Alcaraz-Gonzalez, Victor ; Drame, Abdou ; Gonzalez-Alvarez, Victor. COD and VFA's control in a two-phase anaerobic digestion process. Proc. CAB'07 (Cancun, Mexico, June).
COD AND VFA’S CONTROL IN A TWO-PHASE ANAEROBIC DIGESTION PROCESS


* Department of Chemistry, Autonomous University of Guadalajara
Av. Patria 1201, Lomas del Valle, Guadalajara, México
** CESAME, Université Catholique de Louvain
4–6 Avenue G. Lemaître 1348, Louvain-la-Neuve, Belgium
*** Department of Chemical Engineering, University of Guadalajara
Blvd. Marcelino García Barragán 1451, Guadalajara, México

Abstract: In this paper, a two-phase anaerobic digestion system is considered to control the outlet concentration of both the Volatile Fatty Acids (VFA’s) and the organic matter (other than the VFA’s) characterized by its Chemical Oxygen Demand. The two-phase system consists of two fixed bed reactors whose dynamics are described by Partial Differential Equations. The recycle flow rates in each reactor are used as the manipulated variables to reach the control objective. The control laws designed here are decoupled and they are tested by means of simulation runs obtaining satisfactory results in spite of system input disturbances and changes on the set points. Copyright © 2007 IFAC

Keywords: Control laws, distributed parameter systems, partial differential equations.

1. INTRODUCTION

The Anaerobic Digestion (AD) is an organic matter degradation process used for wastewater treatment. Normally, it is carried out as a single phase system; i.e., the AD takes place in a single reactor which contains a mixed culture of acidogenic and methanogenic microorganisms to degrade the organic matter (Genovesi, et al., 1999; Bernard, et al., 2001; Alcaraz-González, et al., 2002; Steyer, et al., 2002; Simeonov and Queinnec, 2006). However, in most of the AD processes, the operating conditions may impose certain limitations on the physiological and growth properties of both microorganisms (Ince, 1998; Demirer and Chen, 2005). For example, low pH values favor the growth of the acidogenic bacteria but inhibit the methanogenic bacteria growth. In fact, this was one of the reasons why Pohland and Ghosh (1971) proposed the separation of the acidogenic and the methanogenic phases of typical AD processes. They showed that a two-stage AD process offers some advantages over the conventional single-phase process: a) the use of different types of bacteria for each phase, b) the prevention of methanogens overloads; c) the improvement of the stability of the whole process by controlling the acidification phase, and d) the reduction of the reactors volume which leads to a more cost efficient process (Demirer and Chen, 2005). Although the two-phase AD process has been applied in wastewater treatment sludge (Ghosh, et al, 1995), brewery wastewater (Ahn, et al., 2001), dairy municipal wastewater (Yu and Fang, 2002) and even in human night soil (Kunte, et al., 2004), only a few of works have been focused on the control of the methanogenic phase of such a process (e.g., von Sachs, et al., 2003). In this paper, it is proposed to consider a two-stage AD process in order to regulate, at a certain set point, the outlet concentration of both the Volatile Fatty Acids (VFA’s) and the Chemical Oxygen Demand (COD) which is defined throughout the paper as the organic matter (other than the VFA’s) characterized by its COD. The AD process under consideration consists of two fixed-bed reactors whose dynamic behavior is described by a Partial Differential Equation (PDE) model since it is assumed that a biomass concentration gradient exists in each one of these phases. The acidogenic phase is carried out in the first reactor while the methanogenic phase takes
place in the second reactor. The recycle flow rates in each one of these phases are selected as manipulated variables. By using the Salmi-Romanainen boundary conditions (Babary, et al., 1999), it is shown that both control laws are conveniently decoupled, easy to implement and they reach the regulation objective previously defined. The performance of the control laws is illustrated by means of simulation runs that provide satisfactory control results.

The paper is organized as follows. In Section 2, the configuration of the two-phase AD process and its mathematical model is described. The design of the control laws is illustrated in Section 3 whereas the mathematical model is described. The design of the observer–based estimators which are related to the implementation of the control laws are discussed in Section 5. Finally, the performance of these laws is illustrated by means of simulation experiments in Section 5 before to establish the conclusions in Section 6.

2. THE TWO-PHASE AD PROCESS

Let us consider a two-phase AD process with two fixed-bed reactors (see Fig. 1).

![Fig.1. Two-phase AD process](image)

Regarding this figure, the acidogenic phase is carried out in the first reactor of the system (tank A). In this phase, acidogetic microorganisms degrade a fraction of the organic matter, converting it to VFA’s. The effluent of the first reactor is then fed in the second reactor (tank M). In this stage, methane-producing bacteria (methanogens) use the VFA’s for methane and carbon dioxide production. It is assumed that the degradation of the COD by methanogens is negligible and as a consequence, only diffusion-convection phenomena affect the COD in this phase. Furthermore, in practice, the concentration of the bulk components may be assumed uniform within the reactors, but fixed biomass is spatially distributed and a biomass gradient can take place (Schoefs, et al., 2004). These assumptions lead to the following model of the two-phase AD process:

Tank A
\[ \frac{dX}{dt} = (\mu_1(S_1, X_1) - dD)X_1 \]  
\[ \frac{dS_1}{dt} = E_1 \frac{\partial^2 S_1}{\partial z^2} - \frac{Q_{in}}{\eta_1} \frac{dS_1}{\partial z} - k_1 \mu_1(S_1, X_1)X_1 \]  
\[ \frac{dS_2}{dt} = E_2 \frac{\partial^2 S_2}{\partial z^2} - \frac{Q_{in}}{\eta_2} \frac{dS_2}{\partial z} + k_2 \mu_2(S_2, X_2)X_2 \]  

Boundary conditions
\[ \frac{\partial S_1(0,t)}{\partial z} = P_S(S_1(0,t) - S_{in}) \]  
\[ \frac{\partial S_1(1,t)}{\partial z} = -\frac{Q_{in}}{\eta_1} \frac{dS_1(1,t)}{\partial z} - f_1(P_S)k_1 \mu_1(S_1, X_1)X_1 \]  

Tank M
\[ \frac{dX_2}{dt} = (\mu_2(S_2, X_2) - dD)X_2 \]  
\[ \frac{dS_2}{dt} = E_2 \frac{\partial^2 S_2}{\partial z^2} - \frac{Q_{in}}{\eta_2} \frac{dS_2}{\partial z} - k_2 \mu_2(S_2, X_2)X_2 \]  

Boundary conditions
\[ \frac{\partial S_2(0,t)}{\partial z} = 0 \]  
\[ \frac{\partial S_2(1,t)}{\partial z} = -\frac{Q_{in}}{\eta_2} \frac{dS_2(1,t)}{\partial z} - f_2(P_S)k_2 \mu_2(S_2, X_2)X_2 \]  

Initial conditions
\[ X_1(z,0) = X_1(z) \]  
\[ X_2(x,0) = X_2(x) \]  
\[ S_1(z,0) = S_1(z) \]  
\[ \zeta_1(x,0) = \zeta_1(x) \]  
\[ S_2(z,0) = S_2(z) \]  
\[ \zeta_2(x,0) = \zeta_2(x) \]  

In the above equations, \( X_1 \) (gVSS/L), \( S_1 \) (g/L), \( S_2 \) (mmol/L) denote, respectively, the concentrations of acidogenic bacteria, COD and VFA’s in the tank A, while \( X_2 \) (gVSS/L), \( \zeta_1 \) (g/L), \( \zeta_2 \) (mmol/L) represent, respectively, the concentrations of methanogetic bacteria, COD and VFA’s, in tank M. Furthermore, \( t \) (d) is the time variable whereas \( z \) (-) and \( x \) (-) \((z, x \in [0,1])\) are dimensionless scaled space variables in tank A and tank M, respectively. It is straightforward to deduce from the mass balance that \( Q_m + Q_{rec} = Q_{EA} \) and \( Q_m + Q_{rec} = Q_{EM} \) where \( Q_m, Q_{rec}, Q_{EA}, Q_{EM} \) are the volumetric flow rates at the inlet of the system, the recycle rate in tank A, the inlet rate to tank A, the recycle rate in tank M and the inlet flow rate to tank M, respectively. Moreover, the mass balances for the species in the bulk phase at the inlet of both reactors yields \( Q_mS_{in} + Q_{rec}S_{1in} = Q_{EA}S_{EM} \) and \( Q_mS_1(1,t) = Q_{rec}S_{EM} \) which leads to \( S_{EM} = g_1S_{in} + \gamma_1S_1(1,t) \) and \( S_{in} = g_2S_{1in} + \gamma_2S_1(1,t) \) with \( \gamma_1 = Q_m/Q_{EA}, \gamma_2 = Q_m/Q_{EA} \) and \( \gamma_2 = Q_{rec}/Q_{EM} \) \((k=1,2)\) which describe the inlet concentration of COD and VFA’s in tank A \( (S_{in,EM}) \) and the inlet concentration of COD and VFA’s in tank M \( (S_{EM}) \). In addition, \( \mu \) \((S_1, X_1) \) represents the acidogenic bacteria kinetics given by a Monod-Contois model: \( \mu(S_1, X_1) = \mu_{max}S_1/(K_{C_s}X_1 + S_1) \) whereas \( \mu_2(S_2, X_2) \) denotes the methanogetic bacteria kinetics which is given by the following Haldane-Contois model: \( \mu_2(S_2, X_2) = \mu_2\zeta_2/(K_{C_s}\zeta_2 + \zeta_2 + \zeta_2^2/K_2) \)
where $\mu_\text{tot}$, $\mu_\text{str}$, $K_c$, $K_c^2$ and $K_y$ are bio-kinetic parameters. On the other hand, $k_t$, $k_h$ and $k_3$ are the yield coefficients. $E$, $\eta$, $H_A$ and $A_A$ denote the dispersion coefficient, the porosity, the height and the cross sectional area in tank A respectively whereas $E_\varepsilon$, $\eta_M$ and $A_M$ represent these same parameters but in tank M. $D_{\text{EM}} = Q_{\text{EM}} / \eta A_M H_M$ is the dilution rate in tank A and $D_{\text{EM}} = Q_{\text{EM}^2} / \eta M A_M H_M$ is the dilution rate in tank M. Besides, $\varepsilon$ and $\varepsilon_M$ are the bacteria fraction in the liquid phase of tank A and tank M respectively whereas $P_e$ and $P_e$ denote the Peclent number in tank A and tank M and are defined by: $P_e = D_{\text{EM}} H_e / E \varepsilon$, $P_e = D_{\text{EM}} H_e^2 / E \varepsilon_M$. Finally, $f_1(P_e)$ and $f_2(P_e)$ are semi-empirical continuous functions represented by: $f_1(P_e) = (1 + 1/P_e)^{n_1}$; $f_2(P_e) = (1 + 1/P_e)^{n_2}$ where $r_i$ ($i = 1, 2, 3$) is a constant (see Salmi and Romaninen, 1995). Notice that the so-called Salmi-Romaninen Boundary Condition (SRBC) (Babary, et al., 1999) is selected to describe the output dynamical behavior of the species that are consumed or produced in tank A and tank M (see (5), (6) and (12)) while the Danckwerts Boundary Condition (DBC) is chosen to represent the behavior of the COD at the output of reactor M. The selection of these boundary conditions is motivated by the following arguments: on the one hand, for the non-reacting species it seems obvious that a concentration gradient can take place at the output of the fixed-bed. Nevertheless, this gradient is considerably smaller than the one presented for reacting species and therefore the output behavior of non-reacting species can be satisfactorily represented by a DBC. On the other hand, the reacting species can even react at the output of the fixed-bed and as a consequence this may produce a concentration gradient at the output of the reactor. The SRBC can describe not only such phenomenon but also the two extreme modes of the reactor operating: the plug flow and the complete mixing (Babary, et al., 1999).

### 3. The Control Problem

The control objective considered in this paper is to regulate the value of the concentration of COD ($\xi_1(t)$) and VFA ($\xi_2(t)$) at the system’s output at certain desired values ($S_{1,p}$) and ($S_{2,p}$) manipulating the recycle rates $Q_{\varepsilon\varepsilon_1}$ and $Q_{\varepsilon\varepsilon_2}$ respectively. As a matter of fact, the recycle rate $Q_{\varepsilon\varepsilon_1}$ is the input control that will allow regulate the concentration of COD at the output of the tank A ($S_1(t)$). Nevertheless, it will be demonstrated by means of numerical simulations, that $\xi_1(t)$ tends to $S_1(t)$ which is mainly due to the assumption that the COD dynamical behavior in tank M is only affected by transport phenomena. Besides, the recycle rate $Q_{\varepsilon\varepsilon_2}$ is considered as the manipulated variable to regulate $\xi_2(t)$. In other words, $Q_{\varepsilon\varepsilon_2}$ will manipulate $\xi_2(t)$ in an indirect form whereas $Q_{\varepsilon\varepsilon_2}$ will manipulate $\xi_2(t)$ in a direct form. It is assumed that it is possible to measure the concentration of both the COD and the VFA’s at certain positions (including at the outlet). The first of these measurements is taken in tank A whereas the second measuring procedure is carried out in tank B. From these measurements, both $\partial S_{1}(t)/\partial \varepsilon$ and $\partial S_{2}(t)/\partial \varepsilon$ can be computed (i.e., they are assumed as known). Then, equations (5) and (12) can be represented as:

$$\frac{dS_{1}(t)}{dt} = \frac{Q_{\text{EM}}}{\eta A_M H_M} - \frac{r_{\text{COD}}}{\varepsilon}$$

$$\frac{d\xi_2(t)}{dt} = \frac{Q_{\text{EM}}}{\eta A_M H_M} - \frac{r_{\text{VFA}}}{\varepsilon}$$

where $r_{\text{COD}}$ and $r_{\text{VFA}}$ are the COD and the VFA consumption rates at the outlet of tank A and tank M, respectively, and are given by:

$$r_{\text{COD}} = f_1(P_e) k_1 \mu_x X_{\text{out}}$$

$$r_{\text{VFA}} = f_2(P_e) k_2 \mu_x X_{\text{out}}$$

Thus, the following decoupled regulation laws are proposed

$$Q_{\varepsilon\varepsilon_1} = -\left(\frac{r_{\text{COD}} + \lambda_1 (S_{2,p} - S_1(t))}{\eta A_M H_M} - Q_{\alpha}\right)$$

$$Q_{\varepsilon\varepsilon_2} = -\left(\frac{r_{\text{VFA}} + \lambda_2 (S_{1,p} - \xi_2(t))}{\eta A_M H_M} - Q_{\alpha}\right)$$

which reduce (13) and (14) into

$$\frac{d\phi_1(t)}{dt} = -\lambda_1 \phi_1 + p_1(t)$$

$$\frac{d\phi_2(t)}{dt} = -\lambda_2 \phi_2 + p_2(t)$$

where $\hat{r}_{\text{COD}}$ and $\hat{r}_{\text{AGV}}$ denote the estimates, that can be computed by means of an observer of $r_{\text{COD}}$ and $r_{\text{AGV}}$ respectively. Besides, $p_1(t) = \hat{r}_{\text{COD}} - r_{\text{COD}}$, $p_2(t) = \hat{r}_{\text{AGV}} - r_{\text{AGV}}$ $\phi_1 = S_1(t) - S_{2,p}$ and $\phi_2 = \xi_2(t) - S_{1,p}$ ($\phi_1(t), p_1(t), \phi_2(t), p_2(t) \in \mathbb{R}^5$). It is important to remark that biomass concentration sensors are either very expensive or not readily available. This explains why both terms $r_{\text{COD}}$ and $r_{\text{VFA}}$ are assumed as unknown. Moreover, notice that both (17) and (18) have the same mathematical structure. For this class of systems, it is well known that the dependent variable (in this case $\phi_1$ or $\phi_2$) remains bounded if the disturbance element (i.e., $p_1(t)$ or $p_2(t)$) is bounded and if the unperturbed system is asymptotically stable. Furthermore, consider that if $\lim_{t \to \infty} p_1(t) = \xi_1^*$ and $\lim_{t \to \infty} p_2(t) = \xi_2^*$ then, it can be demonstrated that under certain assumptions $\lim_{t \to \infty} \phi_1(t) = \xi_1^*/\lambda_1$ and $\lim_{t \to \infty} \phi_2(t) = \xi_2^*/\lambda_2$ with $(\lambda_1, \lambda_2 > 0)$. This implies that at the limit, the values $\phi_1$ and $\phi_2$ are proportional to the values $p_1(t)$ and $p_2(t)$, respectively. To analyze the limit of these elements it is necessary to study the structure of the observer that provides the estimates of $r_{\text{COD}}$ and $r_{\text{VFA}}$ which is described in the following section.
4. THE ESTIMATION ALGORITHM

The COD and VFA consumption rates can be written as:

\[ r_{\text{COD}} = \sigma_1(l,t)S_1(l,t) \]  
\[ r_{\text{VFA}} = \sigma_2(l,t)\zeta(t, l) \]

with

\[ \sigma_1(l,t) = f_1(\mu_{\text{max}} X_1(l,t)) \]  
\[ \sigma_2(l,t) = f_2(\mu_{\text{max}} X_2(l,t)) \]

where \( f_1 \) and \( f_2 \) are tuning parameters. The COD and VFA consumption rates are differentiable and therefore the second requirement is fulfilled. Finally, it can be considered that the disturbance element is differentiable due to assumption (a2). Thus, \( \hat{\epsilon} = \Gamma \epsilon + \theta \) is stable. Under assumptions (a1) and (a2), it can be demonstrated that \( e_1 \) and \( e_2 \) are asymptotically bounded and besides, both can be made asymptotically arbitrarily small by choosing a convenient value for \( \omega_1 \).

Moreover, notice that the disturbance element \( p(t) \) can be written as \( p(t) = e_2 S_1(l,t) \) and due to the previous result, the disturbance element tends to a value (e.g., \( \xi_1 \)) close to zero. Afterwards, since (19) has a mathematical structure which is identical to the one of (20), it is possible to deduce that the disturbance element \( p(t) \) will also tends to a number (e.g., \( \xi_2 \)) near to zero. Then, according to the previous section, the variables \( \varphi_1 \) and \( \varphi_2 \) may tend to a scalar number that is close to zero which implies not only that \( \xi_1(l,t) \) will reach a value near to \( S_{1p} \) but also that \( \xi_2(l,t) \) will tends to a number close to \( S_{2p} \). In other words, the controllers that are proposed here are stable but they will produce an offset whose magnitude will depend on the performance of the OB estimators.

5. SIMULATION RUNS

Before presenting the performance of the control laws, it is important to remark that the computation of \( \partial S(x,t) / \partial x \) and \( \partial^2 S(x,t) / \partial x^2 \) plays a fundamental role in the implementation of the such laws since both derivative terms appear in their denominators (see (15) and (16)). In this work, it is proposed to approximate these gradients by using an approximation in terms of Backward Finite Differences (BFD) as follows:

\[ \frac{\partial S(x,t)}{\partial x} = \frac{S_1(x,t) - S_1(x^*, t)}{\Delta x} \]  
\[ \frac{\partial^2 S(x,t)}{\partial x^2} = \frac{\zeta_2(x^*, t) - \zeta_2(x, t)}{\Delta x} \]

where \( \Delta x = 1 - x^* \) and \( \Delta x = 1 - x \). The selection of the BFD approximation is explained as follows: physically, there exists a substrate concentration gradient in both, tank A and tank M (inclusively at the beginning of the process) which is due by a biomass concentration gradient. It can be reasonably assumed that \( \partial S(z,t) / \partial z \) and \( \partial^2 S(z,x,t) / \partial x \) fulfills with \( \partial S(z,t) / \partial z < 0 \) and \( \partial^2 S(x,t) / \partial x > 0 \) \( \forall t \geq 0 \). Therefore, from this assumption, there will be no crossings by zero in the denominator of both (15) and (16) when a BFD approximation is used. In addition, BFD are cost efficient since they require only two sensors in each tank in order to make the computation of the output gradients. The performance of the control laws proposed here is...
tested by means of simulations runs. These simulations are carried out using the Orthogonal Collocation Method (OCM) which allows the reduction of the PDE model into a set of Ordinary Differential Equations (ODE) that are valid at some specific axial positions along the reactors. The numerical values of these positions are obtained as a result of selecting the reduction parameters of the OCM: $N$, $\alpha$ and $\beta$. The first of these parameters denotes the number of internal collocation points (all except the boundaries) whereas $\alpha$ and $\beta$ modify the structure of the Jacobi Polynomials depending on their values. Specifically, the roots of such polynomials are taken as the numerical values of the axial positions (Finlayson, 1980). In this paper, the reduction parameters were selected as $N=3$ and $\alpha=\beta=2$ which provides the following distribution of axial positions in tank A and tank M (i.e., $x=z$): $z_1=0$ (the inlet); $z_2=0.21$; $z_3=0.5$; $z_4=0.79$ and $z_5=1$ (the outlet). The simulation runs are carried out for a period of 100 $d$, with the inlet flow rate ($Q_{in}$) maintained in a constant value of 12.5 L/h and using the parameters (Schoefs, et al., 2004): $\mu_{max}=1.2d^{-1}$; $K_{C1}=50.5$(g/g); $\mu_2=0.74d^{-1}$; $k_1=42.12(-)$; $k_2=250$(mmol/g); $k_3=40$(mmol/g); $K_{c}=256$(mmol/L); $K_{c2}=16.6$(mmolS$_2$/gVSS); $c=\tilde{E}=0.5(-)$; $S_1(0)=10$gL$^{-1}$ and $S_2(0)=40$mmolL$^{-1}$ whereas $\eta_1=\eta_2=\eta_4=\eta_5=\eta_7=0.9m^2$; $\tilde{E}_z=E_z=1$m$^3d^{-1}$ and $H_A=H_M=3.5m$. In addition, the tuning parameters of the observer-based estimator are considered as $\omega_1$ = $\omega_2$ = 20 both positive and constant (as it is required). The closed loop dynamics are chosen such that $\lambda_1=\lambda_2=1d^{-1}$ in the presence of the following disturbances: the inlet COD concentration increases from 10 to 12 g/L at time $t=50d$ whereas the inlet VFA concentration increases from 40 to 50 mmol/L at time $t=75d$. Besides, $S_{1sp}$ increases from 1.5 to 2 g/L at time $t=20d$ and $S_{2sp}$ increases from 10 to 20 mmol/L at time $t=40d$. It is important to remark that the simulation results were obtained assuming that it is possible to measure $S_1(1,t)$, $S_2(x*,t)$, $\zeta_1(1,t)$ and $\zeta_2(x*,t)$. In this work, these measurements were taken from the model. Fig. 2 and Fig. 3 describe the evolution of the regulation laws. The effect of the first of these laws on the behavior of the COD is shown in Fig 4.

Notice that the regulated variable follows the change in the set point and remains at such a point in spite of an increment in the inlet concentration of COD. In
Fig. 5 it is described the behavior of the COD but in the tank where the methanogenesis is taking place. As it was specified, only diffusion and convection phenomena affect the COD when it flows through tank M. This is the reason why the behavior of the regulated variable in tank A is similar to the one obtained in tank M. Afterwards, the evolution of the second regulation law is shown in Fig. 3. The effect of this law on the behavior of the concentration of VFA’s is described in Fig. 6. This figure shows that the recycle rate in tank M is capable to drive the regulated variable towards certain set point and to keep it despite the disturbances in the inlet concentration of the VFA’s. Finally, it is necessary to mention that a convenient selection of both tuning parameters and initial conditions allowed a satisfactory performance of the OB estimators. The behavior of these estimators is not shown here due to the lack of space but certainly it contributes to drastically reduce the offset. Obviously, if an OB estimator is not correctly initialized then it will take more time to converge and therefore, the regulation laws may not yield satisfactory results at least at the beginning of the estimation procedure. This fact may happen in practice. In such a case, it is recommended to initialize the OB estimators in an open-loop scheme for a reasonable time before to apply them in closed-loop schemes.

6. CONCLUSIONS AND PERSPECTIVES

In this paper, it was proposed a configuration for the anaerobic digestion process that facilitates the control of both the concentration of VFA’s and the concentration of the organic matter. The control laws here obtained are fully decoupled and they were tested by means of numerical simulations. The simulation results show that both control laws perform adequately not only to drive the controlled variables towards a different reference value but also to keep these values despite the disturbances. Even when the simulation runs demonstrates that \( \hat{z}(x,t) \) converges towards \( S_0(1,t) \), it is necessary to analytically formalize such result. It is also convenient to analyze the effect of noisy measurements in the computation of \( \hat{c}_S(1,t)/\hat{c}_z \) and \( \hat{c}_S(1,t)/\hat{c}_x \). Both topics will be the subject of our future work.

ACKNOWLEDGEMENTS

The authors gratefully acknowledge CONACyT, PROMEP and the Catholic University of Louvain for the support that made this study possible.

REFERENCES


