"Fiscal policy in a growth model with bequest-as-consumption"

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Abstract
This paper analyses a growth model wherein saving results from bequest-as-consumption. It first looks at the market equilibrium and at the optimal solution. Then it turns to the issue of decentralizing the optimal solution with various taxes and transfers. Depending on the available instruments, either a first-best or a second-best optimum can be achieved. Throughout the paper the results are contrasted with those obtained in the standard altruistic (dynastic) model and in the overlapping generation model without intergenerational transfers.


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Abstract
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Keywords: bequests, fiscal policy, optimal growth.

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1 Introduction

Intertemporal macroeconomics and fiscal policy are generally studied within two competing models: the Ramsey model and the overlapping generations model. In the Ramsey model people live infinitely or alternatively, they have finite lives but are linked across generations by altruistic bequests. Barro (1974)\(^1\) used this dynastic interpretation of the Ramsey model to obtain the neutrality of public debt and the efficiency of the market equilibrium. In the overlapping generations model developed by Diamond (1965), people live finitely. In such a model, dynamic inefficiency is possible and public debt matters. In general Diamond’s model is analyzed without any bequest motives.

The belief is, however, that most of the results so obtained can be extended with bequests provided they are not based on pure altruism, that is, made by parents concerned with the welfare of their offspring, as is the case in Barro. There are a number of models of bequeathing behavior, besides the one based on pure altruism.\(^2\) Indeed, there is the accidental bequest model, wherein parents may leave unintended bequests because of premature death combined with having not annuitized their saving. There is also the bequest-as-exchange model, wherein parents give bequests to their children for some attention or assistance they have received from them. Finally, there is the bequest-as-consumption model, wherein parents give bequests to their children for some utility they receive from the act of giving. In their utility function bequests appear then as a sort of final consumption. They provide the parents with a feeling of virtue or sacrifice, or some other form of benefit for having helped their children. Andreoni (1990) refers to this phenomenon as “warm glow” giving; one can also call it paternalistic altruism. The main implication of this type of bequest, as opposed to the one based on pure altruism, is that it does not depend on other sources of consumption for the children. A number of authors have studied the implications of these alternative bequest motives on estate taxation, and more generally on the taxation of inheritance, capital and labor income.\(^3\) However, focusing on partial equilibrium effects, and on a rather static model consisting (quite often) of just two periods, does not give a clear picture of some of the differences.

The purpose of this paper is to study the optimal taxation of inheritance,

\(^1\)See also Arrow and Kurz (1970).
\(^2\)For a survey see Masson and Pestieau (1997) and Ihori (1995).
\(^3\)See, e.g., Kaplow (2000) and Gale and Perosek (2000).
capital and labor income in an overlapping generation model with paternalistic altruism. We believe that it is an important and relevant question for three reasons. First, a clarification is in order. At first sight, in an overlapping generations model that type of paternalistic bequest enters the utility function like a second period consumption, and it is tempting to think that it should be taxed as a regular consumption good. We show that this is not the case in a dynamic setting where saving for bequest is the source of capital accumulation. Second, models with paternalistic bequests are widely used in the literature, particularly in two types of works: endogenous growth models, wherein education is provided by "altruistic" parents (Glomm and Ravikumar, 1992), and dynamic models of wealth distribution (Bevan and Stiglitz, 1975). While these are positive models, it is interesting to explore their normative implications notably regarding the introduction of tax instruments. Finally, there is an empirical reason. It seems that besides accidental bequests, an important fraction of observed inheritance is motivated by some kind of ad hoc altruism much closer to paternalistic than to pure altruism.  

Our objective in this paper is to provide some rule of optimal taxation that can be contrasted with those obtained for the two canonical models of Diamond (1965) and Barro (1974). For the overlapping generation model, we know from Atkinson and Sandmo (1980) that there is a case for a non zero tax (or transfer) on capital income whereas there is the result of a zero tax on capital income in the infinitely lived individuals model (Chamley, 1986, Lucas, 1990). As it will appear, we obtain a result that is related to that of Atkinson and Sandmo, for whom the second-best taxation of saving depends on the revenue requirement and on the level of capital accumulation.

In our paper, we adopt a utilitarian objective for the social planner, that is, a discounted sum of generational utilities, the discount factor reflecting social time preference. When there is altruism, one can face the issue of whether or not the social planner should ignore this dimension in designing its social objective. In other words, the question is whether or not the social planner may forgo the warm-glow form of giving, and simply adopt as its objective the discounted sum of generational utilities, each purged of its altruistic component. In work on pure altruism, the social objective only includes the selfish component of the generational utility. If this were not

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4For a survey, see Arrondel et al. (1997).
5See also Stiglitz (1985).
6In an economy with pure altruists and non altruists, Michel and Pestieau (1998, 1999) show that the zero taxation result does not hold.
the case, there would be double counting and the social weights would in- 
crease with time. The same approach can be adopted with other types of 
altruism, thus following Harsanyi (1995) who wants to "exclude all external 
preferences, even benevolent ones, from our social utility function". Harsanyi 
defines external preferences as preferences for assignments of goods to oth-
ers, which includes the type of paternalistic altruism used here. Hammond 
(1988) is also in favor of "excluding altruistic preferences from welfare to 
avoid undesirable double counting" and he advocates to just add selfish util-
ities. Since we don’t want to take sides in this paper, we adopt a general 
form that combines the two extreme cases. What clearly appear is that, 
without washing out individual preferences, the first- and the second-best 
level of capital accumulation are higher than when we wash them out. We 
can thus obtain a level of investment that is higher than the one implied by 
the traditional modified golden rule. Hence the case for a lower tax (or even 
a subsidy) on inheritance becomes stronger. Our view is that the two ap-
proaches deserve to be explored. We realize that by washing out individual 
utilities we avoid double counting, but at the same time we get a result that 
is not Pareto efficient. For these reasons, we study the two possibilities.

The rest of the paper is organized as follows. Section 2 presents the model 
and the market solution. Section 3 derives the first-best optimum. The 
decentralization question is analyzed in Section 4, and Section 5 provides the 
second-best tax formula. A final section concludes.

2 The market solution

We consider an economy wherein identical individuals live one period. Indi-
viduals belonging to generation $t$ work for $\ell_t$ years, consume $c_t$ and leave a 
bequest of $x_t$ to each of their $(1 + n)$ offsprings. Population size in period $t$ is 
$N_t$ with $N_t = (1 + n) N_{t-1}$ and labor supply is given by $N_t \ell_t$. This is a simpli-
fied version of the traditional two overlapping generations economy wherein 
people work in the first period and save for their second period consumption

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Hammond (1988) argues however that "concepts of Pareto efficiency which include 
altruism in individual welfare have little normative significance". He adds: "altruistic 
behavior often help to promote social welfare, but it may not if the altruism happens 
to be directed toward those whose income should receive only small weight in the social 
welfare function." We here face the distinction between the positive and the normative 
significance of Pareto efficiency.
and for bequeathing \((1 + n) x_{t+1}\). Such a model is surely more realistic but also analytically extremely complex. Hence we opted for a model in which the only source of saving is bequest.

### 2.1 Consumers

The individuals’ resources come from two sources: inheritance and work. We denote them by \(\Omega_t\) and write:

\[
\Omega_t = \tilde{R}_t x_{t-1} + \tilde{w}_t - a_t,
\]

where \(\tilde{R}_t = R_t - \tau^r_t\) is the gross rate of return on saving minus tax, \(\tilde{w}_t = w_t - \tau^w_t\) is the wage level minus tax and \(a_t\) is a lump-sum tax. These resources are allocated to consumption, \(c_t\), leisure, \(1 - \ell_t\), and bequests to the \((1 + n)\) children including inheritance taxation \(\tau^x_t\). That is:

\[
\Omega_t = c_t + \tilde{w}_t (1 - \ell_t) + (1 + n)(1 + \tau^x_t) x_t. \tag{1}
\]

Individuals’ preferences are given by a quasi-concave utility function written as:

\[
u_t = \nu(c_t, 1 - \ell_t) + h(x_t). \tag{2}\]

Maximizing (2) subject to (1) we obtain the following interior condition:

\[
\frac{\partial u_t}{\partial c_t} = u'_1(c_t, 1 - \ell_t) = \pi_t; \tag{3}
\]

\[
\frac{\partial u_t}{\partial (1 - \ell_t)} = u'_2(c_t, 1 - \ell_t) = \pi_t \tilde{w}_t; \tag{3.1}
\]

\[
\frac{\partial u_t}{\partial x_t} = h'(x_t) = \pi_t (1 + n)(1 + \tau^x_t). \tag{3.2}
\]

where \(\pi_t\) is Lagrange multiplier associated with (1).

### 2.2 Production

Turning now to the production sector, we assume that at each period, a representative firm produces a homogeneous good \(Y_t\) with capital, \(K_t\), and
labor, $L_t$. We use a strictly concave constant return to scale production function, $F(K_t, L_t)$, where $L_t = N_t \ell_t$ and

$$K_t = N_t x_{t-1}.$$  \hfill (4)

This says that capital stock installed in period $t$ results from bequests made in the previous period. In the competitive setting assumed here, profit maximization implies the equality between factor prices and marginal productivity:

$$w_t = F'_L(K_t, L_t) = F'_L(k_t, 1) = F'_L(x_{t-1}, \ell_t)$$

$$R_t = F'_K(K_t, L_t) = F'_K(k_t, 1) = F'_K(x_{t-1}, \ell_t)$$

with $k_t = K_t / L_t$.

### 2.3 The government

The government budget constraint can now be expressed in *per capita* terms:

$$g = \tau^w_t \ell_t + \tau^r_t x_{t-1} + a_t + \tau^r_t (1 + n) x_t$$ \hfill (5)

where $g$ is a given amount of *per capita* public spending. One can easily show that this constraint is equivalent to the overall resource constraint:

$$F(x_{t-1}, \ell_t) = c_t + (1 + n) x_t + g.$$ \hfill (6)

Combining the above conditions and particularly (4), one obtains a dynamic equation:

$$k_{t+1} = \frac{x_t(\cdot)}{\ell_t(\cdot)}$$ \hfill (7)

where $(\cdot) = (x_{t-1}, \tau^w_t, \tau^r_t, \tau^r_t, a_t)$.

### 3 Social optimum

As it is standard in optimal growth we adopt as a social criterion the discounted sum of generational utilities, with a discount factor reflecting social
time preference. Such an approach is straightforward with the standard overlapping generational model. In infinitely lived individual model, it is also simple: the social objective is the individuals’ utility.

With the dynastic model à la Barro, we use as social objective the dynastic utility if the individuals’ rate of time preference coincide with the social one. If no, the social rate of time preference is used to discount generational utilities.

The difficulty arises when we consider the type of altruism used here, namely when individual derive utility not only from their own consumption but also from transfers to children. These can be a certain amount of bequest as here or a certain level of education as it is the case in endogenous growth models. As we argue in the introduction, one can make a case of washing out utilities or of keeping them as they are even though it involves giving a lot of weight to capital accumulation.

Given that both views have their own advocates, we decide to express the social objective as a linear combination of the two possibilities. We thus write the social objective as:

\[ \sum_{t=0}^{\infty} \gamma^t U_t^\epsilon \]  

(8)

where \(0 < \gamma < 1\) is the time preference factor and

\[ U_t^\epsilon = u(c_t, 1 - \ell_t) + \epsilon h(x_t), \]  

(9)

with \(0 \leq \epsilon \leq 1\). For \(\epsilon = 0\), we have the case where individuals’ utilities are purged from their “joy of giving” component and for \(\epsilon = 1\), we have the case where individuals’ utilities are unaltered. Combining (8) and (9) we write after some rearrangements:

\[ \sum_{t=0}^{\infty} \gamma^t \left[ u(c_t, 1 - \ell_t) + \frac{\epsilon}{\gamma} h(x_{t-1}) \right], \]

where \(x_{-1} = K_0/N_0\) is given.

To find the optimal conditions, we write for each period the following Lagrangean:

\[ \mathcal{L}_t = u(c_t, 1 - \ell_t) + \frac{\epsilon}{\gamma} h(x_{t-1}) + \frac{\gamma q_t}{1 + n} [F(x_{t-1}, \ell_t) - c_t - g] - q_{t-1} x_{t-1}, \]  

(10)
where the $q_t$ are the shadow prices of $x_t$.

Maximizing (10) with respect to $c_t$ and $\ell_t$ yields:

$$\frac{\partial u (c_t, 1 - \ell_t)}{\partial c_t} = \frac{\gamma q_t}{1 + n} \quad \text{and} \quad \frac{\partial u (c_t, 1 - \ell_t)}{\partial (1 - \ell_t)} = \frac{\gamma q_t}{1 + n} F'_L (x_{t-1}, \ell_t). \quad (11)$$

Furthermore, for the dynamic of $q_t$ we differentiate $\mathcal{L}$ with respect to $x_{t-1}$ and use the transversality condition:

$$\frac{\varepsilon}{\gamma} h' (x_{t-1}) + \frac{\gamma q_t}{1 + n} F'_K (x_{t-1}, \ell_t) = q_{t-1}, \quad (12)$$

$$\lim_{t \to \infty} \gamma^t q_t \; x_t = 0. \quad (13)$$

We now turn to the steady-state conditions. They can derived from (5), (11) and (12):

$$\frac{\partial u (c, 1 - \ell)}{\partial c} = \frac{\gamma q}{1 + n}; \quad \frac{\partial u (c, 1 - \ell)}{\partial (1 - \ell)} = \frac{\gamma q}{1 + n} F'_L (x, \ell);$$

$$\frac{\varepsilon}{\gamma} h' (x) = q \left(1 - \frac{\gamma}{1 + n} F'_K (x, \ell)\right); \quad (14)$$

and

$$F (x, \ell) = c + (1 + n) x + g.$$

With $\gamma < 1$, the transversality condition is verified for $q_t = q$ and $x_t = x$.

From (14), one observes that for $\varepsilon > 0$, $\gamma F'_K (x, \ell) < 1 + n$. In other words, the steady-state capital stock $k = x/\ell$ is higher that the one verifying the modified golden rule. This is not surprising. The same result is obtained when the individuals’ utility depends on current consumption but also on the level of the capital stock. In that case when there is a preference for wealth per se and no just as a mean of getting consumption, capital accumulation is seeked for two reasons and is higher than when it is seeked just as a mean of producing consumption goods. Note however that in the case when $\varepsilon = 0$, then the outcome is consistent with the modified rule.

At this point, it is interesting to adopt a simple illustration with a Cobb-Douglas production function $F (K_t, L_t) = K_t^\alpha L_t^{1-\alpha}$ and a loglinear utility

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8See on this Kurz (1968).
function \( u_t = \ln c_t + \lambda \ln (1 - \ell_t) + \mu \ln x_t \), with \( 0 < \alpha < 1 \), \( \lambda > 0 \) and \( \mu > 0 \). The parameter \( \lambda \) reflects the individual’s time preference and \( \mu \), his level of altruism.

With such a specification, the steady-state optimal value is simply:

\[
\hat{k}_{\gamma \epsilon} = \left[ \frac{\gamma \alpha + \epsilon \mu}{(1 + n)(1 + \epsilon \mu)} \right] \frac{1}{1 - \alpha}.
\]

Not surprisingly \( \hat{k}_{\gamma \epsilon} \) increases with \( \gamma \) and \( \epsilon \). It can be contrasted with the standard golden rule capital stock \( \hat{k}_{10} = \left( \frac{\alpha}{1 + n} \right) \frac{1}{1 - \alpha} \). And thus:

\[
\hat{k}_{\gamma \epsilon} > \hat{k}_{10} \quad \text{if} \quad \epsilon \mu (1 - \alpha) > \alpha (1 - \gamma),
\]

which necessarily occurs for \( \gamma \to 1 \) and \( \epsilon > 0 \). It is interesting to contrast this value of \( \hat{k}_{\gamma \epsilon} \) with that obtained in a competitive equilibrium with given taxes. When all taxes are 0, the latter can be found to be equal to:

\[
k^* = \left( \frac{\mu}{(1 + n)(1 + \mu)} \right) \frac{1}{1 - \alpha}.
\]

This implies that:

\[
k^* \geq \hat{k}_{10} \quad \text{if} \quad \mu \geq \frac{\alpha}{1 - \alpha}.
\]

In the absence of tax, the \textit{laisser faire} capital stock can be higher than the optimal capital stock for a sufficiently high level of altruism.

From this simple example, we see that the desirability and the direction of tax policy depends on the degree of altruism, \( \mu \), the social rate of time preference, \( \gamma \), and the issue of laundering individual utilities parameterized by \( \epsilon \). We now turn to optimal tax policy in two alternative settings: a first best setting, this is what we call the decentralization problem, and a second best setting wherein some tax instruments are missing.
4 The decentralization problem

Let us \((\hat{c}_t, \hat{\ell}_t, \hat{x}_t)\) \(t \geq 0\) be the optimum assumed to exist with shadow prices \(\hat{q}_t\) \((t \geq 0)\). It depends on the values of \(\epsilon\) and \(\gamma\) chosen. Such an optimum satisfies the conditions (11), (12) and (13). For the market solution to coincide with this optimum, the Lagrange multipliers of the consumer’s maximization should verify the following equations:

\[
\pi_t = \frac{\partial u(\hat{c}_t, 1 - \hat{\ell}_t)}{\partial \hat{c}_t} = \frac{\gamma \hat{q}_t}{1 + n}; \tag{15}
\]

\[
\pi_t \hat{w}_t = \frac{\partial u(\hat{c}_t, 1 - \hat{\ell}_t)}{\partial (1 - \hat{\ell}_t)} = \frac{\gamma \hat{q}_t}{1 + n} F_L(\hat{x}_{t-1}, \hat{\ell}_t); \tag{16}
\]

\[
\pi_t (1 + n) (1 + \tau^r_t) = h'(\hat{x}_t) \tag{17}
\]
as well as the budget constraint:

\[
\hat{R}_t \hat{x}_{t-1} + \hat{w}_t - a_t = \hat{c}_t + \hat{\ell}_t (1 - \hat{\ell}_t) + (1 + n) (1 + \tau^r_t) \hat{x}_t. \tag{18}
\]

Let us define \(\hat{\pi}_t = \frac{\gamma \hat{q}_t}{1 + n}, \hat{w}_t = F'_L(\hat{x}_{t-1}, \hat{\ell}_t)\) and \(\hat{R}_t = F'_K(\hat{x}_{t-1}, \hat{\ell}_t)\). One then sees right away that (15) is satisfied. So is (16) for \(\tau^w_t = 0\). Equation (17) is satisfied with:

\[
1 + \tau^r_t = \frac{h'(\hat{x}_t)}{(1 + n) \frac{\partial u(\hat{c}_t, 1 - \hat{\ell}_t)}{\partial \hat{c}_t}}.
\]

We are left with equation (18) that can be satisfied by choosing \(\tau^r_t\) for a given \(a_t\) or by choosing \(a_t\) for a given \(\tau^r_t\). This equation can indeed be rewritten as:

\[
-a_t - \tau^r_t \hat{x}_{t-1} = \hat{c}_t + (1 + n) (1 + \tau^r_t) \hat{x}_t - \left(\hat{R}_t \hat{x}_{t-1} + \hat{w}_t \hat{\ell}_t\right) = (1 + n) \tau^w_t \hat{x}_t - g.
\]

It clearly appears that the tax on interest income, that is the return from inherited wealth, here acts as a lump-sum tax in the same way as \(a_t\). We can now state a proposition.
Proposition 1  The optimal path \( \left( \hat{c}_t, \hat{\ell}_t, \hat{x}_t \right) \) can be achieved in a competitive economy with no wage taxation \( (\tau^w_t = 0) \), with a unique tax on bequests defined by \( \tau^x_t = \frac{\mu h'(\hat{x}_t)}{(1 + n) \frac{\partial u}{\partial \hat{c}_t}(\hat{c}_t, 1 - \hat{\ell}_t)} \), and a unique lump-sum tax consisting of a mix of \( \tau^r_t \) and \( a_t \), the tax on capital income and the lump-sum transfer, given by \( \tau^r_t \hat{x}_{t-1} + a_t = g - (1 + n) \tau^x_t \hat{x}_t \).

Using the above example with Cobb–Douglas production function and loglinear utility functions, it is possible to find explicit values for the tax parameters when \( g = 0 \).

\[
1 + \tau^x_t = \frac{\mu (1 - \gamma \alpha)}{\gamma \alpha + \epsilon \mu}.
\]

It is interesting to note that \( \tau^x \) is constant and it increases with \( \mu \) and decreases with \( \gamma \) and \( \epsilon \). This is pretty intuitive.

Bequest has to be encouraged (low or even negative \( \tau^x \)) if the rate of time preference is high or if the social planner takes into account the altruistic component of the individuals’ utilities. Bequest has to be discouraged (high \( \tau^x \)) when individuals have a strong ”joy of giving”. Note that this optimal bequest tax is financed by a lump-sum transfer. We now turn to the case where this possibility is not open, that is, when the available transfer/tax instruments are insufficient.

5  Second-best taxation

We thus consider now the problem raised when the only available instruments are taxes on wages and savings, namely \( \tau^w \) and \( \tau^x \). We know that in general this problem, which is quite standard in public finance, results is a second-best allocation. There are two objectives at work here: financing the public good \( g \) and achieving optimal capital accumulation. We know that \( \tau^w \) implies a deadweight loss given the endogeneity of labor supply. If this loss were infinitesimal, \( \tau^w \) would be used to finance \( g \) and the value of \( \tau^x \), which would lead to optimal value of \( k \). Conversely, if there were a perfect compatibility between the value of \( \tau^x \) to finance \( g \) and to achieve the optimal \( k \), there would be no use for \( \tau^w \). In these particular cases, the first-best solution results.
In general, however, both instruments are needed and some efficiency loss cannot be avoided.\(^9\)

Note that we don’t use public debt here as it is often the case when dealing with overlapping generations models. In the particular setting adopted here, debt could play an unexpected and unusual role. As long as the social planner takes into account the joy of giving effect \(\epsilon > 0\), it would be tempting to have households saving a huge amount with the government neutralizing private saving by an as huge public debt. This point could be used to strengthen the case of washing out individual utilities.

Let us first look at the problem of the consumer. We focus on the choice of \(c_t\) and \(\ell_t\) given an endowment \(z_t\) defined as:

\[
z_t = x_{t-1} F'_K \left( x_{t-1}, \ell_t \right) - (1 + n) (1 + \tau_t^x) x_t
\]

(19)

Subject to that constraint, the consumer maximizes

\[
u(c_t, 1 - \ell_t) = u(z_t + (w_t - \tau_t^w) \ell_t, 1 - \ell_t),
\]

where we take \(z_t\) as given.

This yields standard demand and supply functions:

\[c_t = c(\tilde{w}_t, z_t)\] and \[\ell_t = \ell(\tilde{w}_t, z_t)\]

where \(\tilde{w}_t = w_t - \tau_t^w\).\(^{10}\)

The social planner is going to choose \(\tau_w\) (or \(\tilde{w}\)) and \(\tau^x\) so as to maximize the discounted sum of utilities (8). In so doing he is subject to three constraints: (i) the above definition of \(z_t\), (ii) the FOC for the optimal choice of \(x_t\) by the consumer: \(h'(x_t) \equiv (1 + n) (1 + \tau_t^x) \frac{\partial u(c_t, 1 - \ell_t)}{\partial c_t};\) (iii) the resource constraint:

\[
F(x_{t-1}, \ell(\cdot)) = c(\cdot) + (1 + n) x_t + g.
\]

This gives the following Lagrangean:

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\(^9\)We focus on the steady-state solution as it is often the case in this type of work. We don’t know whether this steady-state is unique, whether the economy converges to it and what the transition path looks like. Only in the first-best and with simple functions such as those used in our example can we say something on these questions.

\(^{10}\)In solving this second-best problem, we use the direct approach used in dynamic macroeconomics more than in public finance wherein the individuals’ optimality conditions are used as constraints of the government’s optimization.
\[ L = \sum \gamma \left\{ (u(c, 1 - \ell) + \epsilon h(x_t)) + p_t (F(x_{t-1}, \ell) - c(1 + n) x_t - g) \\
+ \psi_t (x_{t-1} F'_K(x_{t-1}, \ell) - z_t - (1 + n)(1 + \tau_t^x) x_t) \\
+ \phi_t (h'(x_t) - (1 + n)(1 + \tau_t^x) u'_c) \right\}, \]

where \( p_t, \psi_t \) and \( \varphi_t \) are Lagrange multipliers associated with the three respective constraints and \( u'_c \equiv \frac{\partial u(c(\tilde{w}_t, z_t), 1 - \ell(\tilde{w}_t, z_t))}{\partial c}. \)

Note that by combining (19) and (20) and using the equality \( c_t = \tilde{w}_t \ell_t + z_t \), we obtain the revenue constraint of the government:

\[ g = \tau^w \ell_t + \tau^x (1 + n) x_t. \]

Besides the tax instruments \( \tau^x \) and \( \tau^w \) or \( (\tilde{w}) \), \( L \) is differentiated with respect to the state variables \( x_t \) and \( z_t \) (which can be viewed as adjustment variables). We thus obtain the following first order conditions:

\[ \frac{\partial L}{\partial \tilde{w}_t} = u'_c (\frac{\partial c}{\partial \tilde{w}_t} - \frac{\partial \ell}{\partial \tilde{w}_t} \tilde{w}_t) + p_t \left( F'_L \frac{\partial \ell}{\partial \tilde{w}_t} - \frac{\partial c}{\partial \tilde{w}_t} \right) \\
- \varphi_t (1 + n)(1 + \tau^x_t) \frac{\partial u'_c}{\partial \tilde{w}_t} + \psi_t x_{t-1} F''_{KL} \frac{\partial \ell}{\partial \tilde{w}_t} = 0, \tag{21} \]

\[ \frac{\partial L}{\partial z_t} = u'_c (\frac{\partial c}{\partial z_t} - \frac{\partial \ell}{\partial z_t} \tilde{w}_t) + p_t \left( F'_L \frac{\partial \ell}{\partial z_t} - \frac{\partial c}{\partial z_t} \right) \\
- \varphi_t (1 + n)(1 + \tau^x_t) \frac{\partial u'_c}{\partial z_t} + \psi_t x_{t-1} F''_{KL} \frac{\partial \ell}{\partial z_t} - \psi_t = 0, \tag{22} \]

\[ \frac{\partial L}{\partial \tau^x_t} = - (\varphi_t u'_c + \psi_t x_t)(1 + n) = 0, \tag{23} \]

\[ \frac{\partial L}{\partial x_t} = \epsilon h'(x_t) - p_t (1 + n) + \varphi_t \mu h''(x_t) - \psi_t (1 + n)(1 + \tau^x_t) \\
+ \gamma p_t F'_K + \gamma \psi_{t+1} (F'_K + F''_{KK} x_t) = 0. \tag{24} \]

The multiplier associated with the resource constraint, \( p_t \), is clearly positive. As to the two others, \( q_t \) and \( \psi_t \), they are of opposite sign from (23). The multiplier \( \varphi_t \) measures the tax distortion. At the steady-state, it is equal to zero when a lump sum transfer is available and there is no distortion. It
is also equal to zero when by chance the tax revenue \( \tau^x (1 + n) \) is sufficient to finance \( g \) and the tax rate \( \tau^x \) generates a level of bequest just sufficient to fulfill optimal amount of capital accumulation \( k_{\gamma c} \). Actually, there is a value of \( g \) or a specification of \( h(x) \) that leads to this coincidence.

We now try to interpret the above conditions in the steady-state. First let us take \( \frac{\partial L}{\partial \tilde{w}} - \frac{\partial L}{\partial z} \ell (\cdot) \). After some manipulation, we obtain:

\[
\frac{\partial L^c}{\partial \tilde{w}} = p (w - \tilde{w}) \frac{\partial \ell^c}{\partial \tilde{w}} + \psi \left[ x F''_{KL} \frac{\partial \ell^c}{\partial \tilde{w}} + \ell + \frac{x}{u''_c} (1 + \tau^x) (1 + n) \frac{\partial \ell^c}{\partial \tilde{w}} (u''_{12} + u''_{11} \tilde{w}) \right] = 0
\]

(25)

where the superscript \( c \) denotes compensated price derivatives and \( u''_{12} = \frac{\partial^2 u(c, 1 - \ell)}{\partial c \partial (1 - \ell)} \), \( u''_{11} = \frac{\partial^2 u(c, 1 - \ell)}{\partial c^2} \).

With loglinear utility and Cobb Douglas production functions, (25) becomes:

\[
p (w - \tilde{w}) + \psi [\alpha w - \mu \tilde{w} + \tilde{w} / \eta] = 0
\]

where \( \eta = \frac{\partial \ell^c \tilde{w}}{\partial \tilde{w} \ell} \) is the compensated labor supply elasticity. This can also be written as follows:

\[
\frac{\tau^w}{w} = \frac{w - \tilde{w}}{\tilde{w}} = \frac{\varphi (1/\eta + \alpha - \mu)}{p + \alpha \psi} CE
\]

(26)

When there is no distortion, \( \varphi = 0 \) and then \( \tau^w = 0 \). When \( \varphi > 0 \), there is a wage and when \( \varphi < 0 \) there is a wage tax subsidy. We take the case when \( \varphi > 0 \) : the tax rate decreases with \( \eta \) and with \( \mu \). This is quite intuitive. The elasticity \( \eta \) measures the degree of distortion implied by the tax. The lower \( \eta \) is, the higher the tax. The parameter \( \mu \) reflects altruism; when it is sufficiently low there is a clear need for subsidizing bequests \( (\tau^x < 0) \), and hence for taxing earnings.

We can also rewrite (24) for the steady-state:

\[
\frac{\partial L}{\partial x} = \epsilon \ell' (x) + p (\gamma F'_K - (1 + n)) \\
+ \psi \left[ \frac{x}{u'_c} \left( \left( h'' (x) + \frac{h'(x)}{x} \right) + \gamma (F'_K + h''_{KK} xx) \right) \right] = 0
\]

(27)

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Again, with the particular functions used in this paper, (27) becomes:

\[
\frac{\epsilon \mu}{px \gamma F'_K} + \frac{\gamma F'_K - (1+n)}{\gamma F'_K} = -\frac{\alpha \psi}{p} = \frac{\varphi \alpha}{pcx}.
\]  

When there is no distortion, \( \varphi = 0 \), we have the first-best optimal capital accumulation rule, that reduces to the standard modified golden rule for \( \epsilon = 0 \). When \( \varphi > 0 \), there is underaccumulation with respect to that optimal rule. This calls for a subsidy on bequests (\( \tau_x < 0 \)) and a tax on earnings (see (26)).

One clearly sees that the sign of \( \tau^w \) and that of \( \frac{\epsilon \mu}{px} + \gamma F'_K - (1+n) \) are the same. In other words, when there is underaccumulation and thus a need for subsidizing bequests, the tax on earnings is positive. Naturally, this occurrence depends in part on the value of \( \mu \).

The main lesson one can draw from this is that the optimal (second best) tax structure in a standard overlapping generations model à la Diamond with endogenous labor supply and that in a model with bequest-as-consumption are not the same. And yet, at first sight, the two models look similar. In one case saving is used for second period consumption, and in the other case it is used for a "joy of giving" type of bequest. The difference is that in the latter case the saving of one generation becomes the endowment of the next generation.

6 Conclusion

In the traditional overlapping generation model, the optimal tax structure on capital and labor income implies positive or negative rates depending on demand elasticities, revenue requirement and level of capital accumulation. In contrast, in the infinite horizon model à la Ramsey or à la Barro, there is a strong case for not taxing capital income in the long run. In this paper, we have considered a model with bequests based not on pure altruism like in Barro but on the mere joy of giving. Within such a model, there is a case for taxing (or subsidizing) bequests and earnings. The tax rates depend mainly on four factors: the revenue requirement, the labor supply elasticity, the level of capital accumulation and also on whether or not the social planner adopt different preferences than the society he plans for. The case for not taxing or even subsidizing bequests is stronger when the social planner incorporates
the warm-glow from giving in its objective function.

References


