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Equilibrium vertical differentiation in a Bertrand model with capacity precommitment

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A B S T R A C T

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1. Introduction

It is well-known since Gabszewicz and Thisse’s (1979) seminal contribution that quality differentiation offers a powerful way out of the Bertrand paradox. Many scholars have elaborated on their pioneering work and today a robust “principle of differentiation” prevails in the literature on vertically differentiated industries. As nicely summarized in Shaked and Sutton (1982), firms are indeed likely to “relax price competition through product differentiation”.

Interestingly, capacity commitment also has the virtue of relaxing price competition. The seminal contribution in this area is given by Kreps and Scheinkman (1983). They show how capacity commitment may be instrumental in sustaining Cournot outcomes in pricing games. Since Kreps and Scheinkman (1983), the strategic value of capacities has been widely studied, though almost exclusively in markets for non-differentiated goods. For instance, Brock and Scheinkman (1985), Lambson (1994), Compte et al. (2002), Davidson and Deneckere (1990) and Benoît and Krishna (1987) study the role of limited capacities in a repeated game of price competition. Deneckere and Kovenock (1992) rely on capacity constraints to provide a model where, in equilibrium, the dominant firm chooses to be the price leader. More recently, Allen et al. (2000) show that capacity precommitment may act as a barrier to entry when price competition takes place post-entry.

Casual observation suggests that in many industries firms sell products differing by quality while being limited by their production capacities. In those industries, it is hard to see a priori whether strategic behavior at the price competition stage is mainly determined by the quality dimension, the capacity restrictions or both. More generally, to what extent are firms’ quality choices dependent on the possibility of committing to capacities? How does the strategic value of capacities depend on the degree of differentiation? Despite their relevance, these questions do not seem to have been addressed in the literature, either theoretically or empirically. Our paper takes a first step in this direction.

We study a three-stage game of complete information where firms first decide on quality. Then, when the specifications of the product are known, firms build production capacities and, finally, they compete in price on the consumer market. The possibility of committing to capacities before price competition takes place sheds new light on vertical differentiation issues. We show indeed that within the standard model of vertical differentiation, capacity commitment may supplant quality differentiation in relaxing price competition. The possibility of committing to capacities before price competition tends to destroy much of the incentive to choose different qualities in the first stage. In particular, if quality costs are sufficiently low, firms sell homogeneous products in equilibrium.

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This “no-differentiation” result may seem surprising at first sight because it runs against the well-established “principle of differentiation”. According to this principle, firms always differentiate their products in order to relax price competition. In fact our finding is quite intuitive. Eaton and Harrald (1992) have already shown that under quantity competition, firms are not inclined to differentiate in quality unless this allows a reduction in sunk costs. In particular, under quantity competition, choosing the best available quality is a dominant strategy for all firms when there are no costs to quality upgrading. In the present paper, we show how capacity commitment may transform the initial pricing game into a quantity game. More specifically, in a duopoly game, when production costs are symmetric and products are differentiated, the reduced forms of firms’ payoffs at the quality stage are the Cournot payoffs. The no-differentiation outcome then naturally follows if quality costs are low.

Our result does not invalidate vertical differentiation as such; instead, it underlines that in a duopoly framework, quality differentiation is more crucially rooted in asymmetries of costs than in a desire to relax competition. In this last respect, indeed, quality differentiation is supplanted by capacity commitment.

Like the present analysis, the literature on multidimensional differentiation can be regarded as dealing with models where firms are endowed with multiple commitment tools, aimed at relaxing competition. In a setting of multidimensional horizontal differentiation, Irmen and Thissen (1998) show that firms always differentiate in equilibrium, but along one dimension only. Neven and Thissen (1990) deal with a two-dimensional model where firms may differentiate their products by quality and (or) variety. They also show that firms differentiate along one dimension only. Furthermore, maximal differentiation obtains (in equilibrium) either in quality or variety depending on the distribution of consumers’ tastes. Even closer to our present analysis is Economides’ (1989) setting where quality and variety can be combined. He shows that minimal quality differentiation and maximal variety differentiation are likely outcomes. However, unlike the current paper, he does not consider a population of consumers whose preferences are heterogeneous with respect to quality. All in all, these papers suggest that firms tend to concentrate on one instrument (one dimension of differentiation) in order to relax competition. Our paper does so as well, with the difference that it is differentiation itself which turns out not to be retained as an equilibrium strategy.

The paper is organized as follows. Section 2 introduces the model and review properties of the equilibrium of a quality–price game when production capacities are assumed to be arbitrarily large. In Section 3, we analyze the class of subgames where products are differentiated and show that these subgames cannot belong to the equilibrium path. We then turn in Section 4 to the class of subgames where firms sell homogeneous products and establish the existence of a subgame perfect equilibrium in which firms enjoy equilibrium payoffs equal to the collusive ones. Section 5 concludes.

2. Quality, capacity, price: a three-stage game

2.1. Model

Consumers’ preferences are set according to the simplified framework of Mussa and Rosen (1978), as popularized by Tirole (1988). The good with label 1 has a quality $s_1$, drawn from the interval $[0,1].$ Consumers have unit demand for the good and are characterized by a “taste for quality” $x$ uniformly distributed on $[0,1].$ The indirect utility function of a consumer with taste for quality $x$ is $u_i(x) = x s_i - p_i$ for $i = 1, 2.$ Not consuming yields a normalized nil utility. In case of a tie among the two products, the consumer randomly chooses among the two with equal probability.

We consider the three-stage game $G$ developing as follows. In stage 1, firms $i = 1, 2$ simultaneously choose quality levels $s_i.$ Since we are essentially interested in analyzing the implications of capacity commitment on the intensity of competition, we concentrate on the cases where quality costs are negligible. This way, the presence of quality differentiation must result from strategic concerns and not from costs saving concerns (we address positive cost for quality in Boccard and Wauthy (2009)). In stage 2, the subgame is denoted $G(s_1, s_2).$ Firms have the opportunity to simultaneously commit to capacities $k_1$ and $k_2$ at a nearly zero positive unit cost $\delta.$ In stage 3, the subgame is denoted $G(s_1, s_2, k_1, k_2).$ Firms simultaneously compete in price. The analysis will be conducted with the concept of subgame perfect equilibrium (hereafter SPE).

In $G(s_1, s_2, k_1, k_2),$ the installed capacity $k_i$ allows firm $i = 1, 2$ to produce up to $k_i$ units at constant unit cost $c;$ producing beyond capacity is feasible but at a constant unit cost $c + \theta,$ with $\theta > 0.$ The marginal cost function is therefore discontinuous at $k_i.$ This cost framework was originally proposed by Dixit (1985) within a quantity competition model and was used by Bulow et al. (1985) and Maggi (1996) under price competition. We assume in the following that $c = 0$ and $\theta = 1$ to capture the notion of limited production capacity; under this assumption, there exist no prices for which it is profitable to produce beyond capacity.

Given costs, firms produce to satisfy demand, i.e. firms cannot turn consumers away once they have named their prices. We follow in this respect the definition of Bertrand competition used for instance by Bulow et al. (1985), Vives (1989, 1990), Kuhn (1994), Dastidar (1995, 1996) under price competition. In our framework was originally proposed by Dixit (1980) within a quantity competition model and was used by Bulow et al. (1985) and Maggi (1996) under price competition. We assume in the following that $c = 0$ and $\theta = 1$ to capture the notion of limited production capacity; under this assumption, there exist no prices for which it is profitable to produce beyond capacity.

2.2. Pure Bertrand competition

Having defined our game completely, we now review the standard quality–price game where capacity commitment is not possible. This will provide a suitable benchmark for the analysis of the full game. We denote $G_0$ the benchmark game where firms cannot commit to capacities. Formally, we restrict the analysis to the class of subgames $G(s_1, s_2, k_1, k_2)$ with $k_1, k_2 \geq 1.$ When $s_1 \neq s_2$ we may relabel firm $l$ for low quality and $h$ for high quality with $s_h > s_l.$

**Lemma 1.** Whatever the quality choices, $G_0(s_1, s_2)$ has a unique price equilibrium.

- If firms sell homogeneous products, the equilibrium is $p_l^* = p_h^* = 0.$
- If firms sell different qualities, the equilibrium is $p_h^* = \frac{s_h (s_h - s_l)}{4s_h - s_l}.$

\[ p_l^* = \frac{s_l (s_h - s_l)}{4s_h - s_l} \]

\[ p_h^* = \frac{s_h (s_h - s_l)}{4s_h - s_l} \]

\[ p_l^* = \frac{s_l (s_h - s_l)}{4s_h - s_l} \]

\[ p_h^* = \frac{s_h (s_h - s_l)}{4s_h - s_l} \]

\[ p_l^* = \frac{s_l (s_h - s_l)}{4s_h - s_l} \]

\[ p_h^* = \frac{s_h (s_h - s_l)}{4s_h - s_l} \]
Proof. The first part of Lemma 1 follows directly from the fact that at the no-differentiation limit, our model corresponds to a standard Bertrand model with a linear demand function and zero production costs. Therefore the unique equilibrium is \( p_i^* = p_j^* = 0 \).

For \( s_h > s_i \), at the price stage the demands resulting from consumers’ choices, given prices \( p_h \) and \( p_i \), are

\[
D_j(p_i, p_h) = \begin{cases} 
1 - \frac{p_i}{s_i} & \text{if } p_i \leq p_h - s_h + s_i \\
\frac{p_h s_h - p_h s_i}{s_i(s_i - s_h)} & \text{if } p_h - s_h + s_i \leq p_i \leq p_h \frac{s_i}{s_h} \\
0 & \text{if } p_h \geq p_h \frac{s_i}{s_h} 
\end{cases} 
\]

and that the closed form equilibrium payoffs are

\[
\Pi_i^b(s_i, s_j) = \frac{4s_i^2(s_i - s_j)}{(4s_i - s_j)^2} \quad \Pi_h^b(s_i, s_j) = \frac{s_h s_i(s_i - s_j)}{(4s_i - s_j)^2}. \tag{5}
\]

We now turn to the first stage of the game where qualities are chosen.

Lemma 2. Up to a permutation of players, there is a unique SPE of GB in which chosen qualities are \( s_i^* = 1 \) and \( s_j^* = \frac{4}{7} \).

Proof. Note first that \( s_1 = s_2 \) cannot be part of an equilibrium because it yields zero profits to both firms while any deviation in quality leads to a price subgame where products are differentiated, so that payoffs resulting from this deviation are strictly positive. Therefore, product differentiation must prevail in any SPE. Standard computations using Eqs. (6) and (7) enable to show the existence of a unique SPE (up to a permutation of players) where one firm chooses the best available quality \( s_h = 1 \) and the other one optimally differentiates to the lower quality \( s_i = \frac{4}{7} \) \( \square \).

3. Differentiated goods

A key assumption of our Bertrand competition model is that firms are not allowed to ration consumers. Therefore, raising one’s price in order to increase the competitor’s demand beyond installed capacity is not profitable since it does not generate spillovers. In other words the lack of quasi-concavity associated with Bertrand–Edgeworth competition is not present in this model. On the other hand, since the extra marginal cost of producing beyond capacity is \( \theta = 1 \), no firm will find it profitable to name a price such that its demand exceeds capacity.

We build on these observations to identify the nature of the set of equilibria in \( G(s_1, s_2, k_i, k_j) \) with \( s_1 \neq s_2 \). We then go backward to the capacity stage and characterize equilibrium capacity levels. Last, we establish the non-existence of a subgame perfect equilibrium displaying product differentiation.

3.1. Price competition under capacity commitment

The best response of firm \( i \) to price \( p_j \) is the “classical” best response \( \psi(p_j) \) as defined in Eqs. (3) and (4), provided the corresponding demand does not exceed capacity i.e., whenever \( D_i(\psi(p_j), p_j) \leq k_i \). Solving this equation for equality defines a critical level \( \beta_i(k_i) \), above which firm \( i \) would face a demand that exceeds its capacity if it were to play along \( \psi(p_j) \). When \( p_j \geq \beta_i(k_i) \), firm \( i \) prefers to respond by selling its capacity at the highest possible price, i.e. the price \( p_j \) that solves \( D_i(p_j, p_j) = k_i \). Let us denote this price \( p_j^c(p_j) \). The best response functions are therefore piecewise linear with a “classical” branch, \( \psi(p_j) \), where firms fight for market shares and a strategic branch, \( p_j^c(p_j) \) where they exactly sell their capacity. In equilibrium it may be the case that two, one or zero firms are capacity-constrained. The formal analysis (developed in Appendix A) reveals that there are four possible equilibrium configurations in the space of capacities, as displayed on Fig. 1.

In region \( A \), installed capacities are sufficiently large to sustain the Nash equilibrium in prices characterized in Lemma 1; hence the lower left-hand corner of region \( A \) is the pair of quantities \( (D_i^r, D_h^r) \) sold at the equilibrium of \( G^r \) given by Eq. (5). For a smaller \( k_i \), we move to area \( B \) where firm \( h \) is capacity-constrained in the price equilibrium. Likewise if \( k_i \) is smaller we pass from area \( A \) to \( C \) where firm \( i \) is capacity-constrained in the price equilibrium. Finally, in region \( D \), the Nash equilibrium is the pair of prices which equate each firm’s demand to its capacity. We prove the following theorem in Appendix A.
Theorem 1. Consider \( k_1, k_2 \leq 1 \) and \( s_1 \neq s_2 \), \( G(s_1, s_2, k_1, k_2) \) has a unique price equilibrium. Four different formulas apply according to the combination of capacities:

\[ \frac{1}{2} \frac{A}{C} \]

\[ \frac{1}{2} \frac{B}{C} \]

\[ \frac{1}{2} \frac{C}{D} \]

\[ \frac{1}{2} \frac{D}{E} \]

\[ \frac{1}{2} \frac{E}{F} \]

\[ \frac{1}{2} \frac{F}{G} \]

\[ \frac{1}{2} \frac{G}{H} \]

\[ \frac{1}{2} \frac{H}{I} \]

\[ \frac{1}{2} \frac{I}{J} \]

\[ \frac{1}{2} \frac{J}{K} \]

\[ \frac{1}{2} \frac{K}{L} \]

\[ \frac{1}{2} \frac{L}{M} \]

\[ \frac{1}{2} \frac{M}{N} \]

\[ \frac{1}{2} \frac{N}{O} \]

\[ \frac{1}{2} \frac{O}{P} \]

\[ \frac{1}{2} \frac{P}{Q} \]

\[ \frac{1}{2} \frac{Q}{R} \]

\[ \frac{1}{2} \frac{R}{S} \]

\[ \frac{1}{2} \frac{S}{T} \]

\[ \frac{1}{2} \frac{T}{U} \]

\[ \frac{1}{2} \frac{U}{V} \]

\[ \frac{1}{2} \frac{V}{W} \]

\[ \frac{1}{2} \frac{W}{X} \]

\[ \frac{1}{2} \frac{X}{Y} \]

\[ \frac{1}{2} \frac{Y}{Z} \]

\[ \frac{1}{2} \frac{Z}{A} \]

3.2. Capacity choice

Going backward in the game tree, we analyze firms’ strategic incentives with respect to capacity levels. The intuition is captured by referring to Fig. 2 and by relying on the intermediate results established in Theorem 1.

If the pair \((k_h, k_l)\) lies in area A or B, then firm \( L \) is never capacity-constrained i.e., neither the price nor its demand depend on its capacity. As capacity is costly, it is in the interest of firm \( L \) to reduce it. Thus, her capacity best response cannot lie in the interior of areas \( A \) or \( B \). In region \( B \), firm \( h \)'s payoffs depend only on its own capacity; it is thus possible to identify a constant capacity best response in the interior of this region. Likewise, firm \( h \) avoids areas \( A \) and \( C \) by reducing its capacity, and firm \( l \) has a constant best response candidate in region \( C \). Accordingly, there exists no equilibrium in the interior of regions \( A \), \( B \) or \( C \).
In area $D$, both firms have “small” capacities and sell their full capacity in the corresponding price equilibrium. If an equilibrium exists, it must lie in region $D$. Using the characterization of equilibrium prices in $G(s_1,s_2,k_1,k_2)$ given in Eq. (8), we formally define payoffs in region $D$ by $k_{l1}^D$ and $k_{l2}^D$ for firm $l$ and $h$ respectively. The best responses candidates in region $D$ are easily computed as $\pi_l^k(k_l) = \frac{1-k_l}{2}$ and $\pi_h^k(k_h) = \frac{1}{2s_h-2k_h}$. Since the two best reply lines are obtained from the frontier lines by rotation at their common axis point, their intersection

$$k_l^* = \frac{s_h}{4s_h-s_l} \quad \text{and} \quad k_h^* = \frac{2s_h-s_l}{4s_h-s_l} \quad (9)$$

lies within area $D$. By comparing each firm’s best response candidate in region $D$ and $B$ or $C$ respectively, we can characterize capacity best responses; they are shown in bold face on Fig. 2. Both correspondences jump up when facing a competitor with a large capacity. These jumps occur for capacity levels which exceed the candidate equilibrium values so that the existence of a pure strategy equilibrium is not called into question. This is why $(k_l^*,k_h^*)$ is the unique SPE of $G(s_h,s_l)$.

To see that the capacity equilibrium replicates Cournot outcomes, observe that the demand system (1)–(2) is invertible from quantities to prices and yields exactly the market clearing prices obtained in Eq. (8-D): $p_l^D$ and $p_h^D$. The payoffs in the corresponding quantity game are

$$\hat{\Pi}_l(q_l,q_h) = q_l p_l^D = q_l(1-q_h-q_l)s_l \quad (10)$$

$$\hat{\Pi}_h(q_h,q_l) = q_h p_h^D = q_h(1-q_h) s_h-q_l s_l \quad (11)$$

The best responses (in this quantity game) are easily characterized as $q_l = \frac{1-q_h}{2}$ and $q_h = \frac{s_h-s_hq_l}{2s_h}$. Solving for a Nash equilibrium, we immediately obtain $q_l^* = k_l^*$ and $q_h^* = k_h^*$, as given by Eq. (9). We can therefore claim that in our duopoly framework, under Bertrand competition and vertical differentiation, capacity precommitment yields Cournot outcomes. This claim is summarized in the next theorem which is formally proved in Appendix A.

**Theorem 2.** For $s_1 \neq s_2$, $G(s_1,s_2)$ has a unique SPE, replicating the Cournot outcome of the corresponding quantity setting game with product differentiation.

Some comments are in order at this step. Theorem 2 states that Cournot outcomes are subgame perfect equilibrium outcomes of a game where capacity commitment precedes price competition in vertically differentiated markets. This is strongly reminiscent of the Kreps and Scheinkman (1983) result. Let us stress however that the present analysis cannot be viewed as an extension of their analysis to the case of a differentiated market. Indeed, the rules of the pricing game are quite different, since we consider Bertrand competition whereas they deal with Bertrand–Edgeworth competition. Studying the behavior of our model at the no-differentiation limit unambiguously reveals this difference. While Kreps and Scheinkman (1983) obtain a unique subgame perfect equilibrium (replicating the Cournot outcome), we indeed obtain multiple subgame perfect equilibria which may entail lower installed aggregate capacity than the Cournot ones, and therefore higher prices.

### 3.3. Quality competition

Recall that in the first stage of $G$, qualities are chosen at no cost. We limit ourselves in this section to quality choices $s_1 \neq s_2$. Thanks to Theorem 2, we can compute the firms’ gross payoffs arising from the subgame perfect capacity equilibrium. Using capacities $s_1 = s_1$ and the price equilibrium associated to region $D$ as defined in Eq. (8), we obtain:

$$\Pi_l(s_h,s_l) \equiv \pi_l^D(k_l^*,k_h^*) = \frac{\frac{s_h(2s_h-s_l)}{2}}{(4s_h-s_l)^2} \quad (12)$$

$$\Pi_h(s_h,s_l) \equiv \pi_h^D(k_l^*,k_h^*) = \frac{s_h^2}{(4s_h-s_l)^2} \quad (13)$$

In the benchmark case $G^\infty$ where firms have unlimited capacities (so called Bertrand competition), Lemma 2 shows that the best response of the low quality firm is to set $s_1 = \frac{4s_h}{4s_h-s_l}$ and thus remain the low quality firm. The ability to commit in capacity alters the price competition landscape. In $G$, the low quality firm’s payoff is monotonically increasing in its own quality (as is the case under Cournot competition). Therefore, the low quality firm tends to imitate the high quality one and we reach the no-differentiation limit.

**Proposition 1.** In game $G$, there exists no SPE in pure strategies where firms choose different qualities.

**Proof.** Observe that $\frac{\partial \Pi_l}{\partial s_h} = \frac{(2s_h-s_l)}{(4s_h-s_l)} \frac{7s_l^2}{(4s_h-s_l)^2} + \frac{(4s_h-s_l)q_l^2}{(4s_h-s_l)^2} > 0$ and $\frac{\partial \Pi_l}{\partial s_l} = \frac{(4s_h-s_l)q_l^2}{(4s_h-s_l)^2} > 0$. We restrict our attention to pure strategies. If $s_1 = s_h = s_2$ was true in a SPE, then the high quality firm would surely choose the highest possible quality $s_h = 1$. Then no choice $s_l < 1$ can be optimal since $S_1 = \frac{1}{2} + \frac{1}{2} = \pi_l^D(s_l,1)$ would be a better choice than $s_l$. \[\square\]
4. Homogeneous goods

Having ruled out the presence of product differentiation in a SPE of the overall game G, we must tackle the case of identical qualities s₁ = s₂ = s. Our model then simplifies to a linear demand D(p) = max {0, 1 − p/5}. In the subgame G(ς,k₁,k₂), firms simultaneously name prices and produce to satisfy demand. We assume that demand is split equally between the two firms in case of a tie.

In the presence of capacity constraints, a firm may typically end up facing a demand level which exceeds installed capacity. Since rationing is not allowed, this firm meets demand even if it exceeds capacity and therefore sells at a loss those units beyond capacity. When prices are low, individual demand addressed to each firm may exceed capacity even when the firms share the market. These two configurations add to those usually prevailing under Bertrand competition. Accordingly, the profit function for i = 1,2 in G(ς,k₁,k₂), assuming prices are chosen in [0,5], is defined by relying on five branches.

\[ \Pi_i(p_i, p_j) = \begin{cases} 
- \left(1 - \frac{p_i}{5}\right) \left(1 - \frac{p_j}{5}\right) & \text{if } p_i < p_j \text{ and } p_i < (1-k)s \quad (a) \\
- \left(1 - \frac{p_i}{5}\right) & \text{if } (1-k)s \leq p_i < p_j \quad (b) \\
- \frac{s}{2} \left(1 - \frac{p_i}{5}\right) & \text{if } p_i = p_j \geq (1-2k)s \quad (c) \\
- \left(1 - \frac{p_i}{5}\right) \left(1 - \frac{p_j}{5}\right) & \text{if } p_i = p_j < (1-2k)s \quad (d) \\
0 & \text{if } p_i < p_j \quad (e)
\end{cases} \]

Branch (a) defines the firm’s payoff when firm i is a price leader which faces a demand exceeding installed capacity. Branch (b) corresponds to the standard Bertrand price leader. Branch (c) defines payoffs in case of tie where the firm is unconstrained. Branch (d) corresponds to a tie at a low constraining price. Lastly, branch (e) corresponds to the case where firm i’s price is strictly larger than j’s.

The equilibrium analysis starts by observing that three different strategy profiles are relevant: undercutting, pricing above and matching the other firm’s price. Introducing quantitative restrictions while preventing rationing has two direct effects. Because the “no-rationing” rule prevents the existence of demand spillovers, the kind of high price strategic deviation that generates price instability in Bertrand–Edgeworth models is not at work here. However “pricing above” may be a relevant strategy because it allows a firm to avoid losses by securing zero sales. Since demand is discontinuous (goods are homogeneous), undercutting the other firm’s price may lead to losses if one’s capacity is low relative to the demand that has to be served (recall indeed that \( p_i \leq s \leq 1 \) implies that the second term in Eq. (14:a) is negative).\(^{10}\)10 Responses then conform to intuition i.e., one should price above an aggressive price, match an intermediate one and undercut a large one.

Theorem 3 in Appendix A shows there is a multiplicity of equilibria in the pricing subgames. If capacities are not too dissimilar, matching the other’s price is a best reply for both; there is thus a continuum of equilibria featuring positive payoffs for both firms. If capacities are too dissimilar, there are no pure strategies equilibria. Priori, the multiplicity of equilibria prevents the straightforward application of backward induction. Nevertheless, we are able to construct a SPE of G where firms play almost collusively by choosing top quality, the collusive capacity and the monopoly price; they are deterred from large capacity deviation by the credible threat of Bertrand cutthroat competition with arbitrary small payoff.

**Proposition 2.** There exists a symmetric SPE of G where firms select top quality (\( s = 1 \)) and share the monopoly profits.

**Proof.** On the equilibrium path, firms play \( s = 1 \), \( k = \frac{1}{4} \) and \( p = \frac{1}{7} \). Firms earn \( \frac{5}{2} \) in equilibrium. If a firm deviates to \( s \neq 1 \), she earns \( \Pi_1(s,1) \) as defined by Eq. (13). Straightforward computations indicate that \( \Pi_1(s,1) \leq \frac{1}{4} \); this is thus a dominated choice. We now tackle capacity deviations in G(1,1). If a firm deviates to \( k < \frac{1}{4} \), the best she can do is sell exactly her capacity at monopoly price, thus earn \( k < \frac{1}{2} \); this is a dominated choice. For a deviation to \( k < \frac{1}{2} \), we construct in Lemma 3 of Appendix A, an equilibrium of G(1,1,4,k) where the deviant firm earns an arbitrary small payoff. This particular continuation price equilibrium allows to prevent upwards capacity deviations. \( \square \)

Three comments are appropriate at this step. First, Proposition 2 establishes the existence of a SPE where firms choose the best available quality. Notice that other SPE displaying no-differentiation and a lower quality level exist as well in game G. However, lower qualities are associated with lower profits and since a firm can jump over her competitor to earn a high-quality differentiated payoff, there is a lower bound to the quality level that can be sustained in a SPE. In Boccard and Wauthy (2009), we characterize the domain of quality levels that can be sustained in a SPE of G.

Notice also that Proposition 2 extends to costly quality. The no-differentiation result is exactly preserved whenever the cost of quality is small, otherwise product differentiation prevails in a SPE of G but to a lesser degree than in the no-commitment situation. (cf. Boccard and Wauthy, 2009). Last, although the time sequence we assumed seems natural, the robustness of our result to the ordering of strategic choices can be questioned. Boccard and Wauthy (2009) study the alternative sequence where firms commit to capacities and then choose qualities; it is shown that if there is no cost for quality, firms choose identical qualities in a SPE. More generally, equilibrium product differentiation is systematically lower under Cournot competition than under Bertrand competition without capacity commitment.

5. Conclusion

In this article, we have shown that quality differentiation as a tool for relaxing price competition is not a robust principle once capacity commitment is allowed. More precisely, our analysis concludes that capacity commitment and Bertrand competition systematically induce less product differentiation relative to the game where capacity commitment is not possible. Furthermore, if the cost of quality is low enough, then the ability to commit to capacities before Bertrand competition leaves no room for quality differentiation as a strategic decision aimed at relaxing competition.

Considering a richer game where capacity precommitment is possible, we shed new light on quality choice as well as on price competition. In our setting, capacity commitment relaxes price competition so effectively that differentiation may become unprofitable. More generally, the residual incentive to differentiate by quality is the one that prevails under quantity competition. It is well-known in this respect that quantity competition induces less differentiation than price competition (cf. Motta, 1993). Our analysis therefore leads us to consider that quality differentiation may rely more heavily on quality costs considerations than on a desire to relax competition per se.

As we make apparent in the analysis of the capacity-setting subgame, Bertrand competition (as opposed to Bertrand–Edgeworth competition) is central in obtaining our minimum-differentiation

\(^{10}\) Care must be taken though that undercutting is not a properly defined optimal response.
result so easily. Allowing for rationing severely complicates the analysis because the non-existence of pure strategy equilibria is endemic in the pricing subgames where product differentiation prevails. Moreover, the computation of mixed strategy equilibria in such games is not straightforward. Preliminary results obtained in a more simple setting suggest that our present findings could generalize to Bertrand–Edgeworth games. At this step however, this remains an open conjecture.

Finally, from an empirical point of view, our analysis suggests that in industries whose technology exhibits rigid production capacities, quality differentiation should basically reflect cost differentials; if upgrading quality is not too costly, less product differentiation should be observed.

Appendix A

Proof of Theorem 1. If \( s_h \succ s_l \), then \( G(s_h, s_l, k_h, k_l) \) has a unique pure strategy equilibrium.

When firm \( h \) names a price \( p_h \), the demand addressed to firm \( l \) is \( D_l(p_l, p_h) \) since rationing is not allowed (Eq. (1)). Firm \( l \)'s profits is therefore

\[
\Pi_l(p_l, p_h) = \begin{cases} 
  p_l D_l(p_l, p_h) & \text{if } D_l(p_l, p_h) \leq k_l \\
  (1-k_l) p_l & \text{if } D_l(p_l, p_h) > k_l.
\end{cases}
\]

Given \( p_h \), firm \( l \) always has the opportunity to set its price so as maintain the equality \( D_l(p_l, p_h) = k_l \); this particular price is

\[
p_l^k(p_h) = \begin{cases} 
  \frac{s_l}{s_h} (p_h - k_l(s_h - s_l)) & \text{if } p_h \leq s_h - k_l s_l \\
  (1-k_l) s_l & \text{if } p_h > s_h - k_l s_l.
\end{cases}
\]

Observe now that since \( \theta = 1 \), firm \( l \) is better off serving exactly its capacity by raising price if necessary than meeting excess demand. Thus, there are only two best response candidates against any \( p_h \); the “classical” best response \( \psi(p_h) \) or the “strategic” \( \sigma_l(p_h) \). It is immediate to see that the best response is \( \sigma_l(p_h) = \max(\psi(p_h), \sigma_l(p_h)) \) and since the maximum operator is applied to a pair of continuous and piecewise linear functions, \( \sigma^l \) is likewise continuous and piecewise linear. We now proceed to derive its exact formulation. Observe firstly that since \( \psi_l(0) < 0 = \psi(0) \), \( \sigma_l = \psi \) in a neighborhood of 0. More precisely,

- if \( k_l \leq \frac{s_h}{2s_h - s_l} \), then
  \[
  \sigma_l(p_h) = \psi_l(p_h) = \begin{cases} 
    \frac{s_l}{2s_h} & \text{if } p_h \leq \frac{2s_l(s_h - s_l)}{2s_h - s_l} \\
    p_l - s_h + s_l & \text{if } \frac{2s_l(s_h - s_l)}{2s_h - s_l} \leq p_h \leq s_h - \frac{s_l}{2} \\
    \frac{s_l}{2} & \text{if } p_h \geq s_h - \frac{s_l}{2}.
  \end{cases}
  \]

- if \( \frac{s_h}{2s_h - s_l} \leq k_l \leq \frac{s_h}{2} \), then
  \[
  \sigma_l(p_h) = \psi_l(p_h) = \begin{cases} 
    \frac{s_l}{2s_h} & \text{if } p_h \leq 2k_l(s_h - s_l) \\
    \frac{s_l}{s_h} (p_h - k_l(s_h - s_l)) & \text{if } 2k_l(s_h - s_l) \leq p_h \leq s_h - k_l s_l \\
    p_l - s_h + s_l & \text{if } s_h - k_l s_l \leq p_h \leq s_h - \frac{s_l}{2} \\
    \frac{s_l}{2} & \text{if } p_h \geq s_h - \frac{s_l}{2}.
  \end{cases}
  \]

\( \cdot \) if \( k_l \leq \frac{1}{2} \), then

\[
\sigma^l(p_h) = \begin{cases} 
  \frac{s_l}{2s_h} & \text{if } p_h \leq 2k_l(s_h - s_l) \\
  \frac{s_l}{s_h} (p_h - k_l(s_h - s_l)) & \text{if } 2k_l(s_h - s_l) \leq p_h \leq s_h - k_l s_l \\
  (1-k_l) s_l & \text{if } p_h \geq s_h - k_l s_l
\end{cases}
\]

For firm \( h \), a similar analysis takes place; the price solving \( \Pi_h(p_h) = k_h \) is

\[
p_h^k(p_l) = \begin{cases} 
  p_l + (1-k_h)(s_h - s_l) & \text{if } p_l \leq s_l(1-k_h) \\
  \frac{s_h}{s_l} & \text{if } p_l \geq s_l(1-k_h)
\end{cases}
\]

and as above the best response \( \sigma^h(p_l) \) is the maximum of \( p_l^k(p_l) \) and

\[
\sigma^h(p_l) = \begin{cases} 
  \frac{s_h - s_l + p_l}{2} & \text{if } p_l \leq \frac{s_l(s_h - s_l)}{2s_h - s_l} \\
  \frac{s_l}{s_h} & \text{if } \frac{s_l(s_h - s_l)}{2s_h - s_l} \leq p_l \leq \frac{s_l}{2} \\
  \frac{s_l}{2} & \text{if } p_l \geq \frac{s_l}{2}
\end{cases}
\]

More precisely,

- if \( k_h \leq \frac{1}{2} \), then \( \sigma^h(p_l) = p_l^k(p_l) \)
- if \( k_h > \frac{1}{2} \), then

\[
\sigma^h(p_l) = \begin{cases} 
  \frac{s_h - s_l + p_l}{2} & \text{if } p_l \leq \frac{s_l(s_h - s_l)}{2s_h - s_l} \\
  \frac{s_l}{s_h} & \text{if } \frac{s_l(s_h - s_l)}{2s_h - s_l} \leq p_l \leq \frac{s_l}{2} \\
  \frac{s_l}{2} & \text{if } p_l \geq \frac{s_l}{2}
\end{cases}
\]

We have seen that best response functions are continuous and piecewise linear, hence a pure strategy equilibrium must be at their intersection. Notice that when a firm is in a monopoly situation (\( \sigma^l \) or \( \sigma^h \) is constant) the other firm faces a zero demand. This latter firm will therefore decrease her price to secure a positive demand.

Accordingly, only branches

\[
\begin{align*}
\text{for firm } l \text{ and } h \text{ can arise in an equilibrium. We have thus 9 possible but mutually exclusive configurations for candidate equilibria. We rule out 5 of them.}
\end{align*}
\]

- (I-1-h1): the solution is denoted [A] with \( p_h^A = \frac{s_l(s_h - s_l)}{2s_h - s_l}, p_l^A = \frac{s_h(s_h - s_l)}{2s_h - s_l} \)
- (I-1-h2): the solution is denoted [B] with \( p_h^B = \frac{(1-k_h)(s_h - s_l)}{2s_h - s_l}, p_l^B = \frac{k_h(s_h - s_l)}{2s_h - s_l} \)
- (I-1-h3): leads to \( p_h = \frac{s_l}{2s_h} = \frac{s_l}{2} \), a contradiction.
- (I-2-h1): leads to \( 2p_l = p_h, \) a contradiction.
- (I-2-h2): leads to \( p_h = p_l = s_h - s_l = (1-k_h)(s_h - s_l), \) a contradiction.
• (I2-h3): leads to \( p_1 = \frac{k_2 h_1}{2h_1} - s_h + s_i < p_2 \), a contradiction.

• (I3-h1): the solution is denoted \( C \) with \( p_i^r = \frac{1-2b(k_h-s_h)}{2s_h-s_i} \), \( p_h^r = \frac{1-k_h}{s_h-k_h/2}, s_i \leq \frac{k_h}{s_h-k_h/2} \).

• (I3-h2): the solution is denoted \( D \) with \( p_i^r = (1-k_h-k_i)s_i \), \( p_h^r = (1-k_h)sh_k \).

• (I3-h3): leads to \( \frac{1}{2} (p_h-k_h(s_h-s_i)) = \frac{2h_i-k_h}{2h_i} \) for area \( B \).

It is easily verified that the four regions \( A, B, C \) and \( D \) form a partition of the capacity space (see Fig. 1). Thus, we have identified the unique pure strategy equilibrium for all configurations of parameters.

In region \( A \), where installed capacities are large, the solution is valid if \( p_i^r \leq 2(k_h-1)(s_i-s_h) \) \( \leftrightarrow k_h \geq \frac{2s_h-s_i}{s_h} \) and if \( p_i^r \leq 2k_h(s_i-s_h) \) \( \leftrightarrow k_h \geq \frac{4s_h-2s_i}{2s_h} \).

In region \( B \), the high capacity firm is capacity-constrained, the solution is valid if \( p_i^r > 2(k_h-1)(s_i-s_h) \) \( \leftrightarrow k_h \leq \frac{2s_h-s_i}{s_h} \) and if \( p_i^r \leq 2k_h(s_i-s_h) \) \( \leftrightarrow k_h \geq \frac{4s_h-2s_i}{2s_h} \).

In region \( C \), the low quality firm is capacity-constrained, the solution is valid only if \( p_i^r \leq 2(k_h-1)(s_i-s_h) \) \( \leftrightarrow k_h \geq \frac{4s_h-2s_i}{2s_h} \). To prove that this candidate is the equilibrium, we only need to check that \( k_i^r < k_i \) \( \forall i \in \{ h, j \} \). Algebraic manipulations show that \( k_i^r < k_i \) if \( i = h, j \). In order to establish the equivalence of this equilibrium with Cournot outcomes, observe that the demand system decharacterizes the price equilibrium of region \( D \) i.e. \( p_i^r \) and \( p_h^r \) as functions of quantity variables \( k_h \) and \( k_i \). Solving for a Nash equilibrium of this new quantity game we immediately obtain \( (k_i^r, k_h^r) \). We show in Boccard and Wauthy (2009) that equilibrium \( G(s, s_h) \) is pure-strategy.

**Theorem 3.** The pricing game \( G(s, s_h, k_h, k_i) \) has a multiplicity of equilibria.

- If capacities are similar, a continuum of equilibria exists, in which firms name identical prices.
- Otherwise, there exists no pure strategy equilibrium.

**Proof.**

Step 1: Pseudo-best replies

In order to characterize the best reply of firm \( i \), we denote \( p = p_i \) the competitor’s price and adopt the short hands \( p^- \), \( p^+ \), \( p^\ast \) in order to identify respectively undercutting, matching and pricing above strategies. Unless the domain of admissible prices is finite, undercutting is not properly defined so that we speak of pseudo-best responses.

We first identify the critical price levels which define the relevant payoff regimes. Equating (14.a) to zero, we use the relevant root to define

\[
\nu_i(b) \equiv \max \left\{ 0, 1 + s - \sqrt{(1-s)^2 + 4ks} \right\}. \tag{17}
\]

The threshold \( \nu_i(b) \) defines the critical price for which undercutting yields so much demand that the profit made over inframarginal units is exactly compensated by the losses made over the units beyond the current capacity. This cutoff plays a role comparable to that of the marginal cost in the standard Bertrand competition: no firm would ever undercut the other’s price below this threshold. Notice that \( p \leq \nu_i(b) \) \( \forall i \), \( (p^-, p) \leq 0 = (p^\ast, p) \).

Next, we define the critical price level for which a firm is indifferent between matching and pricing above (the other’s price), i.e. is indifferent between sharing the market and facing no demand. To this end, we equate (14.d) to (14.e) and use the relevant root to define

\[
\phi_i(b) \equiv \max \left\{ 0, 1 + s - \sqrt{(1-s)^2 + 8ks} \right\}. \tag{18}
\]
We may state that \( p \leq \rho_i(k_i) \Rightarrow \Pi_i(p, p) \leq 0 = \Pi_i(p^+, p) \).

Last, equating (14:a) and (14:c), we define \( \rho_i(k_i) \) the threshold at which a firm is indifferent between serving full demand beyond capacity and matching the other’s price while selling below capacity.

\[
\rho_i(k_i) \equiv \frac{1}{2} \max \left\{ 0; s + 2 - \sqrt{(2-s)^2 + 8k_i s} \right\}. \tag{19}
\]

Notice that \( p < \rho_i(k_i) < \Pi_i(p^-, p) \leq \Pi_i(p, p) \).

Bringing together this information, we may characterize the pseudo-best reply:

- If \( p < \rho_i(k_i) \), both undercutting and matching yield negative profits so that the best response is pricing above \( (p^+ \) \) to guarantee zero losses.
- If \( \rho_i(k_i) \leq p \leq \max \left\{ 0; (1 - 2k_i)s \right\} \), the first inequality means that firm \( i \) can secure positive profits by matching so that matching dominates pricing above. The second inequality means that matching leads to a constrained capacity \( (14:d) \) and as one can see from Eq. (14:a), undercutting leads to an even more stringent constraint (see the unit weight instead of \( \frac{1}{2} \) over negative term).
- If \( \max \{ 0; (1 - 2k_i)s \} \leq p \leq \rho_i(k_i) \) then matching is optimal.
  Since \( p \geq \rho_i(k_i) \) remains true, matching keeps dominating pricing above and by construction of \( \rho_i(k_i) \), matching dominates undercutting.
  If \( p > \rho_i(k_i) \), the reversal in Eq. (19) makes undercutting the optimal strategy.

Step 2: Equilibrium characterization
Assume w.l.o.g. \( k_i \leq k_j \) and observe that

\[
\rho_i(k_i) \leq \rho_j(k_j) \equiv \left\{ k_j < \frac{1}{2} \text{ and } k_j > \rho_i(k_i) \equiv k_i + \frac{s - 1 + \sqrt{(1-s)^2 + 8k_i s}}{4s} \right\}
\]

The analysis of Step 1 can now be used fairly easily. When \( \rho_i(k_i) \leq \rho_i(k_j) \), any pair \((p, p)\) where \( p \leq [\rho_i(k_j), \rho_i(k_j)] \) is a symmetric equilibrium since both undercutting and pricing above are dominated by matching. Observe that there are no other pure strategy equilibria. In such equilibria, firms earn \( \frac{0}{2} \leq \frac{2}{2} \). When \( \rho_i(k_j) > \rho_i(k_i) \), the previous equilibria cease to exist and given the nature of pseudo-best replies, no pair of single prices may define a pure strategy equilibrium. Equilibria, if they exist, are fully mixed. \( \square \)

**Lemma 3.** For \( k_1 < k_2 \), there exists an equilibrium of \( G(1,1,k_1,k_2) \) where firm 2 earns an arbitrary small payoff.

**Proof.** The proof is by construction. Let firm 2 play the pure strategy \( p \) while firm 1 plays a distribution \( F_1 \) over the support \( [p; \tilde{p}] \). Since \( k_2 \leq 1, \forall \epsilon \in (0; k_2) \), it is true that \( p \equiv v_1(k_2 - \epsilon) = 1 - \sqrt{k_2 - \epsilon} < p \equiv 1 - k_2 + \epsilon \) (cf. Eq. (17)). On the equilibrium path, firm 1 has zero demand and zero payoff while firm 2 is a monopoly earning \( v_2(p, F_1) = \epsilon \) as Eq. (14:a) applies. If firms 2 chooses \( p^+ \), she becomes an even more constrained monopoly and thus make a lower profit. If she picks \( p^+ \), she has zero demand and zero payoff. The case \( p \equiv (p; \tilde{p}) \) forces us to Taylor \( F_1 \).

- For \( p \leq 1 - k_2 \), Eq. (14:a) applies, thus \( v_2(p, F_1) = (1 - F_1(p_2)) (k_2 - (1-p)) \). A sufficient condition to make \( p \) optimal is
  \[
  \forall p \equiv [p; 1 - k_2], \quad \frac{\partial v_2}{\partial p_2} \leq 0 \iff \frac{f_1(p)}{1 - F_1(p)} \geq \frac{2(p - 1)}{k_2 - (1-p)^2}. \tag{20}
  \]

\[
\begin{align*}
\text{\textbf{References}} \\
\end{align*}
\]
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