"Optimal operation of multiple reservoirs in hydropower-irrigation systems: a stochastic dual dynamic programming approach"

Goor, Quentin

Abstract
The increasing global water demand due to population growth and higher living standards exerts a significant stress to the limited global freshwater resources. Meeting the future global water demand therefore implies that the operational water allocation policies are efficient and effective. Computational tools relying on optimization algorithms can be used to derive efficient allocation policies. Yet, due to computational constraints, existing tools have only been applied to the analysis and design of small-scale water resources systems. As integrated water resources management requires that allocation policies be determined at the river basin scale, traditional optimization models have limited applicability due to the high dimensionality of basin-wide allocation problems. The overall objective of the thesis is to contribute to the improvement of the operational efficiency and effectiveness of existing and planned water resources systems and particularly the reservoirs operation. Th...

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Optimal Operation of Multiple Reservoirs in Hydropower-Irrigation Systems

A Stochastic Dual Dynamic Programming Approach

QUENTIN GOOR

December 2010

Thèse présentée en vue de l’obtention du grade de Docteur en Sciences Agronomiques et Ingénierie Biologique
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Integrated Water Resources Management; Stochastic optimization; Reservoir (re-)operation; Hydropower; Irrigation; Stochastic Dual Dynamic Programming (SDDP); Nile river basin; Euphrates river basin
La publication de cette thèse de doctorat est pour moi la finalisation d’un projet qui a débuté il y a maintenant cinq années. Nombreuses sont les personnes que je souhaite remercier pour leurs contributions, conseils, support, ...


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<tr>
<td>km³</td>
<td>billion cubic meters</td>
<td></td>
</tr>
<tr>
<td>hm³</td>
<td>million cubic meters</td>
<td></td>
</tr>
<tr>
<td>$c^{hp}$</td>
<td>MW/m³.s⁻¹</td>
<td>Vector (size Jx1) of production coefficient of hydropower plants.</td>
</tr>
<tr>
<td>$c(p)$</td>
<td>T/ha</td>
<td>Actual crop yield for crop $p$.</td>
</tr>
<tr>
<td>$\bar{c}(p)$</td>
<td>T/ha</td>
<td>Maximum crop yield for crop $p$.</td>
</tr>
<tr>
<td>$C^R$</td>
<td>-</td>
<td>Reservoir system connectivity matrix.</td>
</tr>
<tr>
<td>$C^I$</td>
<td>-</td>
<td>Irrigation system connectivity matrix.</td>
</tr>
<tr>
<td>$D$</td>
<td>-</td>
<td>Number of irrigation demand sites.</td>
</tr>
<tr>
<td>$e_t$</td>
<td>hm³</td>
<td>Vector (size Jx1) of evaporation losses from reservoirs, period $t$.</td>
</tr>
<tr>
<td>$E[\cdot]$</td>
<td></td>
<td>Expectation operator</td>
</tr>
<tr>
<td>$f_t$</td>
<td>US$</td>
<td>Immediate net benefits function from system operation, period $t$.</td>
</tr>
<tr>
<td>$g_t$</td>
<td>MW</td>
<td>Vector (size Jx1) of power generated by hydropower plants during period $t$.</td>
</tr>
<tr>
<td>$h$</td>
<td>m</td>
<td>Net head on the turbine.</td>
</tr>
<tr>
<td>$h_t$</td>
<td>-</td>
<td>Vector (size Jx1) of hydrological state variables, period $t$.</td>
</tr>
<tr>
<td>$H$</td>
<td>-</td>
<td>Number of hyperplanes that approximate the hydropower functions.</td>
</tr>
<tr>
<td>$HP_t$</td>
<td>US$</td>
<td>Net benefits from hydropower generation, period $t$.</td>
</tr>
<tr>
<td>$i_t$</td>
<td>hm³</td>
<td>Vector (size Jx1) of water withdrawals for off-stream uses, period $t$.</td>
</tr>
<tr>
<td>$IR_t$</td>
<td>US$</td>
<td>Net benefits from the agricultural sector, period $t$.</td>
</tr>
<tr>
<td>$J$</td>
<td>-</td>
<td>Number of nodes considered in the hydro-system.</td>
</tr>
<tr>
<td>$K$</td>
<td>-</td>
<td>Number of backward openings in the backward optimization phase.</td>
</tr>
<tr>
<td>$K_y(p)$</td>
<td>-</td>
<td>yield response factor, crop $p$.</td>
</tr>
<tr>
<td>$l_t$</td>
<td>hm³</td>
<td>Vector (size Jx1) of spills, period $t$.</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
<td></td>
</tr>
<tr>
<td>$L$</td>
<td>Number of cuts that approximate the benefit-to-go function.</td>
<td></td>
</tr>
<tr>
<td>$M$</td>
<td>Number of hydrological sequences used in the forward simulation phase.</td>
<td></td>
</tr>
<tr>
<td>$P_t$</td>
<td>MW</td>
<td>Vector (size $J \times 1$) of true power generated by hydropower plants during period $t$.</td>
</tr>
<tr>
<td>$\hat{P}_t$</td>
<td>MW</td>
<td>Vector (size $J \times 1$) of approximated power generated by hydropower plants during period $t$.</td>
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<tr>
<td>$P$</td>
<td>MW</td>
<td>Vector (size $J \times 1$) of installed capacity of hydropower plants.</td>
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<tr>
<td>$q_t$</td>
<td>hm$^3$</td>
<td>Vector (size $J \times 1$) of current inflows to the system, period $t$.</td>
</tr>
<tr>
<td>$q_{t-1}$</td>
<td>hm$^3$</td>
<td>Vector (size $J \times 1$) of sampled inflows to the system, period $t-1$.</td>
</tr>
<tr>
<td>$r_t$</td>
<td>hm$^3$</td>
<td>Vector (size $J \times 1$) of turbining, period $t$.</td>
</tr>
<tr>
<td>$r_{max}$</td>
<td>hm$^3$</td>
<td>Vector (size $J \times 1$) of maximum turbining capacity of hydropower plants.</td>
</tr>
<tr>
<td>$s_t$</td>
<td>hm$^3$</td>
<td>Vector (size $J \times 1$) of beginning-of-period storage, period $t$.</td>
</tr>
<tr>
<td>$s_{t+1}$</td>
<td>hm$^3$</td>
<td>Vector (size $J \times 1$) of end-of-period storage, period $t$.</td>
</tr>
<tr>
<td>$t_f$</td>
<td>-</td>
<td>End-of-agricultural season. Period at which agricultural benefits are perceived.</td>
</tr>
<tr>
<td>$T$</td>
<td>months</td>
<td>Length of the planning horizon.</td>
</tr>
<tr>
<td>$x_t$</td>
<td>Vector (size $J \times 1$) of decision variables, period $t$.</td>
<td></td>
</tr>
<tr>
<td>$y_{t+1}$</td>
<td>hm$^3$</td>
<td>End-of-period accumulated water into &quot;dummy&quot; reservoirs for irrigation.</td>
</tr>
<tr>
<td>$y_t$</td>
<td>hm$^3$</td>
<td>Vector (size $J \times 1$) of sampled beginning-of-period storage in dummy reservoirs for irrigation, period $t$.</td>
</tr>
<tr>
<td>$\hat{y}_{t}(p,d)$</td>
<td>hm$^3$</td>
<td>Volume of water that has been delivered to the crop $p$ at demand site $d$ during the irrigation season.</td>
</tr>
<tr>
<td>$\hat{y}_{t}(p)$</td>
<td>hm$^3$</td>
<td>Seasonal crop water requirement for crop $p$.</td>
</tr>
<tr>
<td>$z_t$</td>
<td>unit of deficit or surplus</td>
<td>Vector (size $J \times 1$) of slack variables.</td>
</tr>
<tr>
<td>$Z$</td>
<td>US$</td>
<td>Overall net benefits from system operation over the planning period considered.</td>
</tr>
<tr>
<td>$\alpha_t$</td>
<td>-</td>
<td>Discount factor, period $t$.</td>
</tr>
<tr>
<td>$\delta^h$</td>
<td>MW</td>
<td>Vector (size $J \times 1$) of independent terms of the $h^{th}$ convex hull approximation of the true hydropower function.</td>
</tr>
<tr>
<td>$\eta$</td>
<td>-</td>
<td>Turbines/generators efficiency.</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>N/hm$^3$</td>
<td>Specific weight of water.</td>
</tr>
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<td>$\lambda_{w,t+1}^h$</td>
<td>US$/hm^3$</td>
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<td>Description</td>
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<td>$\nu$</td>
<td>US$</td>
<td>Terminal value function.</td>
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<tr>
<td>$\theta^h$</td>
<td>US$/MWh</td>
<td>Vector (size $J\times 1$) of O&amp;M costs of hydropower plants.</td>
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<td>$\omega^h$</td>
<td>MW/hm$^3$</td>
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<td>$\theta^i(p,d)$</td>
<td>US$/ha</td>
<td>Variable costs of crop $p$, at the given site $d$.</td>
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<td>$\pi^i_t$</td>
<td>US$/MWh</td>
<td>Vector (size $J\times 1$) of short-run marginal costs (SRMC) of the hydrothermal electrical system to which power plants contribute.</td>
</tr>
<tr>
<td>$\pi^i(p,d)$</td>
<td>US$/T</td>
<td>Farm gate price of crop $p$, at the given site $d$.</td>
</tr>
<tr>
<td>$\psi^h$</td>
<td>MW/hm$^3$</td>
<td>Vector (size $1\times J$) of storage terms of the $h^{th}$ convex hull approximation of the true hydropower function.</td>
</tr>
<tr>
<td>$\rho$</td>
<td>-</td>
<td>Coefficient ($\rho \in [0,1]$) to eventually adjust the energy generated to the expected production.</td>
</tr>
<tr>
<td>$\tau_t$</td>
<td>-</td>
<td>Number of hours in period $t$.</td>
</tr>
<tr>
<td>$\xi_t$</td>
<td>US$/unit of deficit or surplus</td>
<td>Vector (size $J\times 1$) of penalties associated with slack variables $z_t$, period $t$.</td>
</tr>
<tr>
<td>$\varphi^l_{t+1}$</td>
<td>US$/hm^3$</td>
<td>Vector (size $1\times J$) of storage terms of the $l^{th}$ cut that approximate the benefit-to-go function.</td>
</tr>
<tr>
<td>$\gamma^l_{t+1}$</td>
<td>US$/hm^3$</td>
<td>Vector (size $1\times J$) of inflows terms of the $l^{th}$ cut that approximate the benefit-to-go function.</td>
</tr>
<tr>
<td>$\eta^l_{t+1}$</td>
<td>US$/hm^3$</td>
<td>Vector (size $1\times J$) of irrigation terms of the $l^{th}$ cut that approximate the benefit-to-go function.</td>
</tr>
<tr>
<td>$\beta^l_{t+1}$</td>
<td>US$</td>
<td>Vector (size $1\times J$) of independent terms of the $l^{th}$ cut that approximate the benefit-to-go function.</td>
</tr>
<tr>
<td>$\mu_{q,t}$</td>
<td>hm$^3$</td>
<td>Vector (size $1\times J$) of periodic mean of historical inflow to reservoirs, period $t$.</td>
</tr>
<tr>
<td>$\sigma_{q,t}$</td>
<td>hm$^6$</td>
<td>Vector (size $1\times J$) of periodic standard deviation of historical inflow to reservoirs, period $t$.</td>
</tr>
<tr>
<td>$\epsilon_t$</td>
<td>hm$^3$</td>
<td>Vector (size $1\times J$) of time independent stochastic noise, period $t$.</td>
</tr>
<tr>
<td>$\Phi^t$</td>
<td>-</td>
<td>Vector (size $1\times J$) of periodic autoregressive parameters (hydrological model), period $t$.</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction and objectives

1.1 Context of the thesis

Human population is increasing and, according to United Nations estimates, more than 9 billions people will be living on Earth in 2050, with most of the global population growth expected to occur in developing countries [United Nations, 2008]. This spectacular increase in population (about +30% compared to the current situation) associated with higher living standards will boost the demand for energy [International Energy Agency (IEA), 2008] and water resources [Vorosmarty et al., 2000]. Worldwide, irrigated agriculture is by far the largest freshwater user, providing about 40% of the world food production. In 1995, it was responsible for 66% of worldwide total water withdrawals\(^1\) and this volume is estimated to be 27% higher for 2025, in order to supply the demand for food production [Shiklomanov, 1999]. In the meantime, the global primary energy demand will grow by 45% between the period 2006-2030, according to the reference scenario of the International Energy Agency (IEA) [2008]).

The uses of water and energy resources are closely interconnected: water is needed to make use of energy and energy is required to make use of water [USAID Global Environment Center, 2001]. Water resources are involved in the various stages of the energy generation process: it is not only directly used through turbines to produce hydro-electricity, but is indispensable for fossil fuel extraction, refining, processing, transportation, evaporative cooling process in thermo-electric and nuclear power stations, and more recently for dedicated energy crops [U.S. Department of Energy, 2006]. On the other hand, to provide water to users, the transportation, distribution, treatment and pumping processes require a large amount of energy. As a consequence, inefficient management practices of either water or energy resources may increase shortage in the use of the other resource. This obvious link between water and energy re-

\(^1\)The term water withdrawal refers to the amount of water diverted from a surface source or pumped from a groundwater source for a particular use. The distinction must be made with water consumption, which is the water withdrawn from a source (river, lake, reservoir) and made unsuitable for further use.
sources is called the water-energy nexus [USAID Global Environment Center, 2001].

Water resources are by nature unevenly distributed around the world and its availability is characterized by a high seasonal and inter-annual variability. Based on this observation, the International Water Management Institute (IWMI) [2007] classified the concept of water scarcity into two categories: physical or economical. Physical water scarcity occurs when the demand is high compared to available resources. On the other hand, economically water scarce regions refer to as regions where the resource is abundant compared to the use, but where lack of water appears to be due to human and/or financial limitations. According to the International Water Management Institute (IWMI) [2007], about one third of the world population faces some form of water scarcity, as illustrated in figure 1.1. For example, it is shown that in Africa, except for the Northern coastal strip and the extreme Southern part, the problem is not so much physical but rather economical scarcity.

![Physical Water Scarcity. More than 75% of the river flows are allocated to agriculture, industries or domestic purposes (accounting for recycling of return flows). This definition of scarcity—relating water availability to water demand—implies that dry areas are not necessarily water-scarce.
Approaching physical water scarcity. More than 60% of river flows are allocated. These basins will experience physical water scarcity in the near future.
Economic Water Scarcity. Water resources are abundant relative to water use, with less than 25% of water from rivers withdrawn for human purposes, but malnutrition exists. These areas could benefit by development of additional blue and green water, but human and financial capacity are limiting.
Little or no water scarcity. Abundant water resources relative to use: less than 25% of water from rivers is withdrawn for human purposes.
Not estimated

Figure 1.1: Area of physical and economic water scarcity around the world. Source: International Water Management Institute (IWMI) [2007].
1.1. Context of the thesis

While the physical quantity of water resources is finite, its availability can be influenced, amongst other things, by the presence of hydraulic infrastructures, their operating policies and the existence of a suitable legal and institutional framework to ensure the management of these infrastructures [Biswas, 2004a]. Amongst these infrastructures, reservoirs and dams are often central components of large water resources systems. They provide hydraulic head and storage for hydropower generation but also serve as seasonal (or over-year) storage capacity for multiple purposes (irrigation, industrial and municipal water uses, flood control). Reservoir operation policies can control the pool elevation for recreation activities on the lake and the downstream discharge of the dam to facilitate navigation and fishing. Moreover, the power generated by their storage hydropower station is characterized by short-, mid- and long-term flexibility, low carbon dioxide emission compared to thermal power plants, etc. However, alongside these well accepted benefits, the societal and environmental costs of building large infrastructures are important [Biswas, 2004b]: displacement of people living in the impounded area, disruption of natural ecosystem, perturbation of the natural river flow regime, reduction of fish migration, accumulation of sediments in the reservoirs, irreversible investment with high capital costs, safety aspects, etc [ICOLD, 1997].

The numerous objectives of reservoirs and dams are often conflicting, especially during extreme hydrological conditions. For example, if both hydropower generation and irrigated agriculture are operating objectives, a trade-off must be found between (i) storing water during the wet season to make it available during the dry season, when the crop water requirements are the highest and (ii) turbining water during the summer season, when the demand for hydroelectricity generation usually peaks. The operating objectives of reservoirs have also evolved over time: environmental and ecological concerns are becoming increasingly important. For example, the reservoir must be operated so as to restore environmental flows, in terms of quantity and seasonal flow distribution (artificial floods), as illustrated by Acreman and Dunbar [2004] and more recently by Tilmant et al. [2010]. In addition, reservoirs that were primarily designed for a single objective must now be operated as multipurpose projects. As pointed out by the World Commission on Dams [2000], the above-mentioned factors explains, to some extent, why many systems worldwide are failing to achieve the level of performance that justified their implementation [Labadie, 2004].

Traditionally, although highly interrelated, irrigated agriculture, energy generation, industrial and domestic supply used to be managed independently. The consequences of this sectoral approach to water resources management have contributed to the increasing water scarcity and have lead to inefficient uses of the resources. In the future, given the challenges of meeting the growing energy and food demands, the competition for water resources is likely to increase, especially between the agricultural and energy sectors. The global problem of increasing water scarcity calls for a coordinated planning and management of both sectors in order to provide reliable energy and water supplies [U.S. Department of Energy, 2006; Hightower and Pierce, 2008].
Chapter 1. Introduction and objectives

Since the 1990s, Integrated Water Resources Management (IWRM) is the response given by the International Community to address those water resources management challenges. IWRM relies on a holistic approach which aims at taking into account all stakeholders (including the environment), the temporal and spatial dimension of water resources as well as the institutional and legal framework [Loucks and van Beek, 2005]. According to the Global Water Partnership [2000], “IWRM is a process which promotes the coordinated development and management of water, land and related resources, in order to maximize the resultant economic and social welfare in an equitable manner without compromising the sustainability of vital ecosystems”. To achieve this, IWRM relies on the four Dublin Principles presented at the United Nations International Conference on Water and the Environment, held in Dublin, Ireland, in 1992, and listed in table 1.1. Since then, water has been officially recognized as an economic good. Although the interpretation of the fourth principle is still debated [Savenije and van der Zaag, 2002], water should be considered as a scarce, natural resource and should therefore be managed and allocated efficiently on the basis of an integrated costs and benefits analysis of all the alternative options. However, the economically efficient allocation of water resources should only be achieved taking into account the the priority of uses, defined by the decision makers. Despite the apparent attractiveness of the concept, its vagueness and therefore its operationality is still challenged by some authors [Biswas, 2004a; van der Zaag, 2005; Gyawali et al., 2006].

Table 1.1: Guideline principles of the Dublin statement on water and sustainable development (adopted January 31, 1992 in Dublin, Ireland).

<table>
<thead>
<tr>
<th>Principle No.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Principle No. 1</td>
<td>Fresh water is a finite and vulnerable resource, essential to sustain life, development and the environment.</td>
</tr>
<tr>
<td>Principle No. 2</td>
<td>Water development and management should be based on a participatory approach, involving users, planners and policy-makers at all levels.</td>
</tr>
<tr>
<td>Principle No. 3</td>
<td>Women play a central part in the provision, management and safeguarding of water.</td>
</tr>
<tr>
<td>Principle No. 4</td>
<td>Water has an economic value in all its competing uses and should be recognized as an economic good.</td>
</tr>
</tbody>
</table>

A key concept underlying IWRM is that water resources are best managed at the river basin scale. The reason has to be found in that water flows from upstream to downstream generating positive and/or negative third-party impacts, called externalities. The concept is complicated by the fact that, worldwide, rivers cross political boundaries indiscriminately. With 261 rivers known as international ones, nearly half of the land surface of the earth is covered by countries sharing their watersheds with at least another one [Wolf et al., 1999]. About 40% of the world population is impacted. In the context of international river basins, the basin-wide management creates an asymmetrical relationship between upstream and downstream countries and is therefore potentially source of tensions: while taking an action in the upper part of a
river basin will generate physical and/or political implications downstream, the opposite is only true in terms of political impacts. As a consequence, the water-wars literature (both scientific literature and popular press) is extensive and argues that water has already been the motivator for wars. However, the analysis of the International Crisis Dataset (ICB) by Wolf [1998] reveals that no real armed war have ever been observed over water since it is not economically and strategically rational. According the author, actual history of water disputes is rather intense political tensions and small scale to intensive military acts than water-wars. On the contrary, the sharing of international watercourses can be considered as a catalyst for cooperation between the riparian countries. Sadoff and Grey [2002] analyzed the various types of cooperation that can emerged around international watercourses: improving the ecology of the river (benefits to the river) would enable better resources management (for example in terms of agricultural and energy production) (benefits from the river) while cooperation between riparian countries would avoid the costs associated with political tensions (benefits because of the river). Finally, riparians involved in a cooperative process would benefit from a better regional integration of the various economic sectors and therefore support the economic growth (benefits beyond the river).

1.2 Current challenges in optimal reservoir operation

As pointed out by the World Commission on Dams [2000], there is an urgent need to improve the operational efficiency and effectiveness of existing reservoirs, while in the meantime, to better assess the performance of planned infrastructures. To achieve this, decision makers can be assisted in their task by quantitative analysis tools, classified by Wurbs [1993] into three categories: (i) descriptive simulation models are the representation of the water resources system to predict its behaviour, under a predefined set of conditions (boundary conditions and operating rules). They can be very detailed but are not efficient to provide optimal management policies; (ii) Prescriptive optimization models rely on mathematical programming and, given some simplification of the water resources system, it can identify optimal operating policies; (iii) Hybrid models are primarily descriptive simulation models with piecewise optimization specific aspects. Despite this classification, the planning and management of water resources systems gives the best results when both simulation and optimization tools are implemented [Labadie, 1993; Rani and Moreira, 2010]. The use of simulation and optimization methods is complementary, particularly in the present context, where management alternatives are numerous: optimization models may be used as screening tools to effectively identify optimal operating policies and provide a few management alternatives to more detailed simulation models [Philbrick and Kitanidis, 1999; Loucks and van Beek, 2005].

The planning and management of water resources systems is challenging regarding the numerous interactions between the institutional, economic and natural systems [Loucks and van Beek, 2005]. Until the 90’s, many reservoirs
Chapter 1. Introduction and objectives

were managed independently, usually with a single objective (usually hydro-
power generation or seasonal storage for irrigation). Nowadays, IWRM calls for
new methodologies to holistically operate water resources systems: the entire
hydro-system (river basin) must be considered while ensuring a cross-sectoral
approach. Moreover, the potential number of alternatives management policies
explodes with the number of (often conflicting) objectives, associated with the
various sources of uncertainty (hydrology, economics, social, ...) and possible
impacts of climate change [Labadie, 2004]. As a consequence, the size and the
complexity of the reservoir operation problem has substantially increased and
traditional approaches to solve the reservoir operation problem, while considering
explicitly stochastic variables, usually fails in the context of multireservoir
systems because of computational issues: the computational effort required to
solve the problem increases exponentially with the number of reservoirs (curse
of dimensionality).

Despite the attractiveness and efficiency of optimization methods, simula-
tion models has always been more widespread amongst the decision makers
[Yeh, 1985; Wurbs, 1993; Labadie, 2004; Rani and Moreira, 2010]. One of
the reason is that, until recently, simplifications and approximations were re-
quired to circumvent the dimensionality issues: reservoir aggregation, approx-
imation of the non-linearities of hydropower generation, use of deterministic
formulations, etc. An other explanation lies in that no generally applicable
optimization method or package for solving reservoir operation problems exist.
It depends on system characteristics and specific objectives and constraints to
be modelled. Finally, optimization models are often perceived as tools that
attempt to replace water managers’ judgement.

1.3 Objectives of the thesis

Since the United Nations International Conference on Water and the Environ-
ment in 1992, water resources have been officially recognized as an economic
good and should therefore be managed and allocated efficiently.

The overall objective of the thesis is to contribute to the improvement of the
operational efficiency and effectiveness of existing and planned water resources
systems and particularly the reservoirs operation. To achieve this, basin-wide
water resources allocation models must be developed to support the implemen-
tation of integrated water resources management policies. This thesis focuses
on the modelling tools that can identify optimal operation strategies of mul-
tireservoir in hydropower-irrigation systems.

The specific objective is to adapt, test and assess the usefulness and ap-
propriability of a new optimization algorithm that can handle multireservoir sys-
tems while explicitly considering the hydrologic uncertainty and its impacts on
allocation decisions, making it suitable to analyze basin-wide allocation prob-
lems. The algorithm relies on Stochastic Dual Dynamic Programming (SDDP),
a modelling technique that originates from the energy sector [Pereira, 1989].
The expected results from this thesis can be summarized as follow:
1.4 Outline of the thesis

- A comprehensive overview of the classical methods to address the mid-
to long-term reservoir operation problem in the context of large-scale
hydro-systems where hydropower and irrigation are the main economic
sectors.

- An implementation of an extension of the SDDP algorithm to directly
incorporate revenues from irrigation into objective function. This exten-
sion is relevant when the model is used for planning purpose to determine
the economically efficient development of irrigated agriculture.

- An implementation of an extension of the SDDP algorithm to deal with
the variable and non-linear productivity of hydropower plants.

- An illustration of the implementation of the new optimization algorithm
and its proposed extensions (and analyze the operating consequences)
with two case studies: the Eastern Nile and the Euphrates river basins.

1.4 Outline of the thesis

This thesis addresses the problem of optimal (re-)operation of multipurpose
multireservoir systems. It focuses on Stochastic Dual Dynamic Programming,
one of the few methods available to solve this class of problems while considering
stochastic variables, and its application to large hydropower-irrigation systems.

The thesis is based on articles published in international peer-reviewed jour-
nals\textsuperscript{2}. In order to avoid redundancy in the various chapters, the document is
organized around two parts: the model and the case studies are described first,
while the model implementation is presented in the second part.

Chapter 2 starts with a description of the reservoir operation problem and
reviews the available solution strategies to solve the problem. Then, an in-
depth description of the Stochastic Dual Dynamic Programming algorithm is
given. In this chapter, the proposed developments of the SDDP algorithm to
deal with the variable productivity of hydropower plants and net benefits from
irrigated agriculture are highlighted. In chapter 3, the main features of the two
case studies are presented: the Euphrates and the Eastern Nile river basins.

The second part of the thesis concentrates on the model applications. In
chapter 4, we investigate the consequences of one of the key assumptions on
which SDDP relies: the simplified linear hydropower production function. The
consequences are evaluated by comparing two SDDP formulations: the first
one considers a variable productivity of hydropower plants while the second
one assumes that the production of hydroelectricity is governed by the release
term. The results are obtained on a network of reservoirs and hydropower
stations in the Eastern Nile river basin.

Chapter 5 deals with the application of the SDDP algorithm to the Eastern
Nile river basin in order (i) to evaluate the impacts of upstream development
in the Blue Nile basin on the allocation decisions and reservoirs operating

\textsuperscript{2}see appendix B.3
strategies and (ii) to assess the economic value of regulation (reservoirs) in Ethiopia.

Chapter 6 analyzes the impacts of development of the Southeastern Anatolia Project in Turkey on the Euphrates downstream riparian countries, Syria and Iraq, and especially on the performance of the Tabqa dam in Syria. The analysis relies on a two-step modelling approach coupling single- and multireservoir optimization models.

Finally, general conclusions and perspectives for further research are drawn in chapter 7.
Part I

Materials and Methods
Chapter 2

The reservoir operation problem

2.1 Outline

The purpose of this chapter is to provide a description of the reservoir operation problem and to review available solution strategies. The chapter starts with a description of the reservoir operation problem. Then, various solutions strategies are reviewed with an emphasis given to Stochastic Dynamic Dynamic (SDP) and its limitations. Finally, a comprehensive description of Stochastic Dual Dynamic Programming (SDDP), one of the few solution methods available to solve multipurpose multireservoir operation problems in a stochastic environment is presented.

2.2 Problem characteristics and formulation

2.2.1 Problem characteristics

Optimal reservoir operation has been extensively reported in the literature. Yeh [1985] and more recently Labadie [2004] provide state-of-the art review of multireservoir systems operation. According to Karamouz et al. [2003], optimal hydropower reservoir operation requires a time decomposition approach: optimal decisions are the result of a process consisting in a sequence of models with different time frames. Long-term planning models (monthly time step and over-year planning horizon) provide strategic and tactical policies information. Results of interest (future marginal water value, storage levels) from this class of models then provide boundary conditions for mid- and short-term

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1 This chapter is adapted from:
optimization tools (from weekly to daily time horizon), to finally feed real-time operation models focusing on load dispatching, unit commitment and transmission constraints. This sequential modelling process is illustrated in figure 2.1.

Figure 2.1: Sequential modelling process for reservoir operation. Adapted from Karamouz et al. [2003]

This thesis deals with mid- to long-term reservoir operation planning problems where the availability of water must obviously be considered as stochastic. This is especially true for systems with significant hydro-capacities. However, in the abundant literature about mid- and long-term optimal reservoir operation, few solutions exist to deal with multipurpose multireservoir operation problems in a stochastic environment. This is mainly due to the computational difficulties associated with stochastic formulations. Philbrick and Kitanidis [1999] demonstrate that deterministic optimization applied to reservoir operation can lead to suboptimal operation policies when the system is no "certainty equivalent", i.e. when expected values cannot be used to identify optimal policies without loss of accuracy. Since most energy investments and reservoir operation involves irreversible decisions, Wallace and Fleten [2003] stresses the need to consider a stochastic programming approach.
Two strategies have been developed to capture the stochastic nature of reservoir inflows into optimal reservoir operation: implicit and explicit stochastic optimization, respectively denoted ISO and ESO [Karamouz et al., 2003; Labadie, 2004]. In the implicit method, deterministic optimization problems are solved for a large number of historical and/or synthetically generated hydrological scenarios. To some extent, this approach is similar to a Monte Carlo analysis. Although computationally efficient, the main disadvantage is that the resulting optimal operation policies are conditioned to the hydrological scenarios considered. Despite multiple regression analysis can be carried out on the results, traditional methods lead to relatively poor correlations [Labadie, 2004]. However, inference methods based on neural networks seem to give more satisfactory results [Raman and Chandramouli, 1996; Chandramouli and Raman, 2001; Cancelliere et al., 2002]. Karamouz et al. [2003] stress that deterministic formulations may overestimate benefits from system operation since they consider a perfect knowledge of future conditions and therefore ignore the costs of extreme events (flood or draught). On the other hand, ESO performs directly on the probability distribution of stochastic variables (e.g. reservoirs inflows, energy and water demands, spot prices for energy, ...) to compute optimal decision. As a consequence, considering explicitly the hydrological uncertainties in the optimization problem provides allocation policies that naturally hedge against the hydrological risk. However, stochastic formulations are computationally challenging in the context of multipurpose multireservoir system.

### 2.2.2 Mathematical formulation

Given the state of the system and specific goals, the objective of a mid- to long-term reservoir operation problem is to determine a sequence of optimal decisions $x_t$ (vector of decision variables) that maximize the expected sum of benefits from system operation $Z$, over a planning period $T$, while meeting operational and/or institutional constraints. Considering explicitly stochastic reservoir inflows and assuming that the system status is represented by a vector $S_t$ (vector of state variables), the problem can be formulated as:

$$\max_{x_t} \{ Z \} = \max_{x_t} \left\{ E \left[ T \sum_{t=1}^{T} \alpha_t f_t(S_t, x_t) + \alpha_{T+1} \nu(S_{T+1}) \right] \right\} \quad (2.1)$$

where $\nu$ is a terminal value function, $\alpha_t$ is the discount factor at stage $t$, $f_t(\cdot)$ is the immediate benefits from system operation and $E[\cdot]$ is the expectation operator.

The solution of the multistage decision making problem (2.1) is not straightforward since (i) the future hydrological conditions are uncertain and (ii) the water resources allocation problem is coupled in time: a decision today affects the availability of the resources for the future and therefore the future benefits of the system. At each time step of the decision process, the reservoir operator is faced to the following trade-off: use hydro today or save it for the future when it becomes more valuable. The relationship between a decision and its
operating consequences, given the future hydrological conditions, is illustrated in figure 2.2.

![Decision tree in reservoir operation](image)

Figure 2.2: Decision tree in reservoir operation. Adapted from Pereira et al. [1998]

When dealing with hydroelectric reservoir operation, a typical benefit function $f_t(\cdot)$ includes benefits from hydropower generation and penalties for not meeting operational and/or institutional constraints [Tilmant and Kelman, 2007]. Assuming that the system consists of $J$ hydropower plants and that the vector of state variables $S_t$ includes the beginning-of-period storage $s_t$ and the hydrological information (summarized by the current inflows to the system $q_t$), the benefit function, at time $t$, can be written as:

$$f_t(s_t, q_t, r_t) = \tau_t g'_t (\pi^h_t - \theta^h_t) - \xi_t z_t$$

(2.2)

where:

- $r_t$ is the vector (size $1 \times J$) of turbed outflows ;
- $\tau_t$ is the number of hours in period $t$ ;
- $g_t$ [MW] is the vector (size $1 \times J$) of power generated by the hydropower plants during period $t$ ;
- $\pi^h(j)$ is the vector (size $1 \times J$) of short-run marginal cost (SRMC) of the hydrothermal electrical system to which power plants contribute [$/\text{MWh}]$ ;
- $\theta^h(j)$ is the vector (size $1 \times J$) of O&M cost of hydropower plants [$/\text{MWh}]$ ;
- $z_t$ is a vector (size $1 \times J$) of slack variables with the violations of operational constraints (energy deficit, environmental flows, etc.) [unit of deficit or surplus] which are penalized in the objective function by the vector (size $1 \times J$) of penalties $\xi_t$ [$/\text{unit of deficit or surplus}]$. 
2.2. Problem characteristics and formulation

The reservoir operation problem (2.1) is subject to a set of constraints among which the mass conservation for all periods $t$:

$$s_{t+1} - C^R(r_t + i_t) - C^I(i_t) + e_t(s_t, s_{t+1}) = s_t + q_t$$  \hspace{1cm} (2.3)

where $s_{t+1}$ is the vector of end-of-period storage, $i_t$ is the vector of water withdrawals for off-stream uses (e.g. irrigation), $I_t$ and $e_t$ are the vectors of spillage and evaporation losses respectively, $q_t$ is the vector of current period reservoir inflows. $C^R$ is the reservoir system connectivity matrix ($C^R_{j,k} = 1$ (-1) when reservoir $j$ receives (releases) water from (to) reservoir $k$). Irrigation system have its own topology modelled by the connectivity matrix $C^I$ where irrigation withdrawals and returns flows are connected to the reservoir system: $C^I_{j,i} = \mu$ (percentage of irrigation withdrawals that will drain back to the river) when reservoir $j$ receives return flows from the irrigation site $i$ and/or $C^I_{j,i} = -1$ when water is withdrawn from reservoir $j$ to the irrigation site $i$. An example of a multireservoir system and the associated connectivity matrices is illustrated on figure 2.3.

![Figure 2.3: Example of a multireservoir system and the associated connectivity matrices for the surface reservoirs ($C^R$) and the irrigation system ($C^I$).](image)

Lower and upper bounds can be assigned to storage levels:

$$s_{t+1} \leq s_t \leq s_{t+1}$$  \hspace{1cm} (2.4)

Limits on reservoir releases are introduced to account for the maximum turbin- ing capacity of the hydropower station, to maintain a desired downstream minimum flow for water quality, navigation, etc.

$$r_t \leq r_t \leq R_t$$  \hspace{1cm} (2.5)
Irrigation water withdrawals can be limited by the pumping station or channel capacity:

\[ \underline{i} \leq i_t \leq \overline{i} \]  

(Equation 2.6)

Equations (2.1) to (2.6) form the core of the reservoir operation optimization problem that can be addressed using the methods presented in the following sections.

### 2.3 Stochastic Dynamic Programming

Dynamic programming (DP) is a particularly suitable method to solve the multistage decision making problem of optimal reservoir operation (2.1) to (2.6). Yakowitz [1982] presents a comprehensive review of DP applications in water resources, ranging from water quality and irrigation management to multipurpose reservoir operation. Dynamic programming has been introduced by Bellman [1957] as “the theory of multistage decision processes”. In this approach, the original problem (2.1) is decomposed into sub-problems, called stages, that are solved recursively. At each stage \( t \), a decision \( x_t \) has to be made to move to the next stage \( (t+1) \), linked by a transition-state equation, i.e. here the mass balance equation (2.3). Each decision depends on the current state of the system, defined by the vector of state variables \( S_t \). A sequence of optimal decisions constitute an optimal policy. This procedure is illustrated in figure 2.4 in the context of reservoir operation.

![Figure 2.4: Illustration of dynamic programming mechanism for reservoir operation. Adapted from Labadie [2004]](image)

Stochastic dynamic programming (SDP) and extensions have been successfully applied in optimal reservoir operation with uncertain inflows [Stedinger et al., 1984; Huang et al., 1991; Karamouz and Vasiliadis, 1992; Tejada-Guibert et al., 1995]. In SDP, the reservoir operation problem (2.1) to (2.6) is formulated so as to maximize the expected sum of immediate \( f_t(\cdot) \) and future benefits \( F_{t+1}(\cdot) \) from system operation (benefit-to-go function). As illustrated in figure 2.5, the immediate benefits from system operation decrease as the end-of-period
storage increases. This is obvious as less water is available for immediate uses. In the meantime, future benefits increase as more water is made available for future uses. At the optimal solution, the immediate and future marginal water values must be equal. These marginal water values are the derivatives of the immediate and future benefits functions respectively.

SDP recursively evaluates a value function, which estimates the benefits of system operation from the current stage $t$ until the end of the planning period $T$. The SDP recursive equation, also known as Bellman’s equation, can be written:

$$F_t(S_t) = E_{h_{t+1}|h_t} \left[ \max_{x_t} \{ f_t(S_t, x_t) + \alpha_{t+1} F_{t+1}(S_{t+1}) \} \right]$$

(2.7)

where $E[\cdot]$ is the expectation operator to observe hydrological condition $h_{t+1}$ given the hydrological state $h_t$. The temporal persistence of the hydrological conditions can be summarized by many options. For example, Karamouz and Vasiliadis [1992] investigate the use of current period inflows and future flows forecast as hydrologic state variables. Tejada-Guibert et al. [1995] discuss the value of different hydrologic information, among which the current or previous period inflows, in multireservoir systems. The value of seasonal flow forecast has been studied by Kim and Palmer [1997]. According to Loucks et al. [1981], the most common options are the previous and current period reservoir inflows.

Assuming that the reservoir inflows are correlated in time and considering that the state of the system (vector of state variables) is described by the beginning-of-period storage $s_t$ and the previous period inflows $q_{t-1}$, the
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equation (2.7) becomes:

\[
F_t(s_t, q_{t-1}) = \max_{x_t} \left\{ f_t(s_t, q_t, x_t) + \alpha_{t+1} E_{q_{t+1}} \left[ F_{t+1}(s_{t+1}, q_{t+1}) \right] \right\} (2.8)
\]
\[
= \max_{x_t} \sum_{q_{t-1}} p(q_t | q_{t-1}) \left( f_t(s_t, q_t, x_t) + \cdots + \alpha_{t+1} F_{t+1}(s_{t+1}, q_{t+1}) \right) (2.9)
\]

The expectation operator mentioned in equation (2.8) is obtained from the conditional probabilities \( p(q_t | q_{t-1}) \) to observe \( q_t \), during the current period \( t \) given an observed inflow \( q_{t-1} \) during the previous period \( t-1 \). The correlation between successive inflows is usually assumed to be governed by a Markov chain and the continuous random variables \( q_{t-1} \) is approximated by a discrete random variable \( q_{t-1}(i) \) and the temporal persistence between successive flows is encoded into flow transition probabilities.

Assuming that the hydrological state variable is the current period reservoir inflow \( q_t \), equation (2.7) becomes:

\[
F_t(s_t, q_t) = \max_{x_t} \left\{ f_t(s_t, q_t, x_t) + \alpha_{t+1} E_{q_{t+1}} \left[ F_{t+1}(s_{t+1}, q_{t+1}) \right] \right\} (2.10)
\]
\[
= \max_{x_t} \sum_{q_t} p(q_{t+1} | q_t) \left( f_t(s_t, q_t, x_t) + \cdots + \alpha_{t+1} F_{t+1}(s_{t+1}, q_{t+1}) \right) (2.11)
\]

Here, the conditional expectation is obtained from the conditional probabilities \( p(q_{t+1} | q_t) \). An advantage of this formulation is that the release decision can be adjusted as more information concerning \( q_t \) becomes available during period \( t \).

In traditional discrete SDP, the state-space domain (i.e. beginning-of-period storage \( s_t \) and hydrologic information \( h_t \)) must be discretized and the functional equation \( F_t(s_t) \) is evaluated at each grid point. Various interpolation techniques can then be used to determine \( F_{t+1}(s_{t+1}) \) between grid points [Tejada-Guibert et al., 1993; Johnson et al., 1993].

Considering a finite planning horizon, SDP assumes that the value of the benefit-to-go function \( F_{t+1}(s_{t+1}) \) is known and then evaluates \( F_t(s_t) \) for each point of the state-space domain. As a consequence, the algorithm moves backward in time i.e. starting from the end-of-planning period \( T \) to \( t_0 \). At stage \( T \), the functional equation \( F_{T+1}(s_{T+1}) \) is estimated by a terminal value function, as defined in (2.1).

Considering an infinite planning horizon and cyclostationary periodic problem, the optimal solution is obtained when release decisions generated by the SDP algorithm reach a steady state i.e. when the change in the SDP recursive function (2.7) becomes nearly constant from one cycle to the next and for each point of the discrete state space domain. The resulting steady-state release policy and benefit-to-go functions constitute the set of solutions that can be used by reservoir operators.
2.4 Limitation of SDP and potential solution strategies

As mentioned above, discrete SDP relies on the discretization of the state-space domain. Then, the functional equation (2.7) is evaluated at each grid point. Assuming $k$ state variables, both discretized in $N$ values, the grid where the benefit-to-go function must be evaluated contains $N^k$ points. Therefore, the computational effort $W$ required to solve the SDP problem increases exponentially with the number of reservoirs $J$ according to $W \propto (N^k)^J$. This is the well known "curse of dimensionality" of SDP which restricts its use to small scale problems involving 3-4 reservoirs [Bellman, 1957].

Various strategies have been developed to cope with the dimensionality problem associated with SDP. They can be classified in three different categories.

2.4.1 System simplification

In this approach, the solution relies on the simplification of the reservoir network system by reducing the number of reservoirs and therefore the dimension of the optimization problem. For example, Turgeon [1981] proposes a decomposition approach where the reservoir operation problem consisting in $k$ state variables is reformulated as $(k - 1)$ sub-problems of two state variables which are solved by SDP. The two state variables are the water content of a reservoir and the total energy content of the downstream reservoirs. On the same idea, Archibald et al. [1997] developed a methodology where the $J$ reservoirs problem is decomposed in subproblems consisting in 3 reservoirs: one reservoir of the original problem and two others consisting on the aggregated remaining downstream and upstream reservoirs. Another option is to aggregate all the network in a single reservoir, described by Terry et al. [1986] for the Brazilian system, but difficulties arise when disaggregating the results [Saad et al., 1994].

2.4.2 Value function interpolation

Computational savings are possible by calculating the value function for a subset of points of the state-space domain and then approximate the value function by interpolation. Various interpolation algorithm have been proposed, as linear polynomials and splines [Johnson et al., 1993; Tejada-Guibert et al., 1993; Philbrick and Kitanidis, 2001]. More recently, Castelletti et al. [2007] propose a solution based on heuristics and DP called Neuro-DP where the benefit-to-go function is approximated by artificial neural networks. In the work of Dias et al. [2010], the solution relies on a convex hull approximation of the benefit-to-go function in a discrete SDP scheme. The piecewise linear approximation is computed by the Convex Hull algorithm and is compatible with linear programming. However, current applications of the above mentioned algorithms are limited to small systems of 2 and 3 reservoirs. Another promising solution, proposed by Cervellera et al. [2006], relies on SDP with a neural approximation of the benefit-to-go function combined with an efficient discretization of
the state-space domain, rather than a classical uniform grid. Although these efforts reduce the computation time, they do not remove the curse of dimensionality.

2.4.3 Other strategies

A variety of other strategies have been developed. The most significant are briefly presented below.

Lee and Labadie [2007] developed the Q-Learning method in reinforcement learning (RL) to derive optimal reservoirs operating strategies. In RL, alternative feasible decisions are explored randomly through a forward moving procedure instead of exploring all feasible possible decision for each state of the system. The decisions that improve the benefits from system operation are stored in the optimal operation policy for that particular state of the system. The convergence of the model is ensured as all decisions are iteratively sampled in all states.

The optimal reservoir trajectory approach (ORT) developed by Turgeon [2007] relies on the observation that raising the level of a reservoir feeding a power plant is profitable as long as the gain due to the higher head is greater than the loss due to the additional expected spillage. The methodology is applied to a system consisting of seven reservoirs. Although ORT can be applied to reservoirs in series and parallel, its main drawback lies in that it cannot be applied to reservoirs located on different rivers.

Pereira [1989] and Pereira and Pinto [1991] proposed an extension of traditional stochastic dynamic programming (SDP) that is not affected by the curse of dimensionality, making it suitable to solve larger problems while considering stochastic state variables. The new algorithm, called stochastic dual dynamic programming (SDDP), does not attempt at constructing $F_{t+1}$ over the entire domain but rather builds a locally-accurate approximation. As such, SDDP belongs to the field of approximate dynamic programming, an increasingly popular algorithmic framework. In SDDP, the locally-accurate approximation of $F_{t+1}$ relies on sampling and Benders decomposition. The use of decomposition methods to optimize water resources systems can be found, for example, in Pereira and Pinto [1985]; Jacobs et al. [1995]; Morton [1996]; Watkins and McKinney [1998]; Archibald et al. [1999]; Cai et al. [2001]. SDDP is also used worldwide in hydropower-dominated regions such as Scandinavia [Rotting and Gjelsvik, 1992; Mo et al., 2001; Kristiansen, 2004], New-Zealand [Halliburton, 2004], Latin-America [Pereira et al., 1998] and Turkey [Tilmant and Kelman, 2007]. It has been recently implemented in the Nile basin [Goor et al., 2010b, a] and in the Zambezi basin [Tilmant et al., 2010].

Read [1989] developed a Dual Dynamic Programming-based (DDP-based) approach using the dual space of the reservoir operation problem by maximizing the marginal value of water instead of minimizing the costs from system operation. This approach differs from the DDP approach of Pereira and Pinto [1991] in that it considers an approximation of the whole marginal water value surface for few reservoirs instead of using sampling to approximate much larger
2.5 Approximate Dynamic Programming

Traditional discrete dynamic programming algorithms solve the reservoir operation problem (2.7) backward in time, i.e. starting at the end of the planning horizon and compute the value function $F_{t+1}(S_{t+1})$ for each discrete value of the vector $S_t$. As mentioned earlier, DP becomes ineffective as the number of states grows exponentially with the number of dimensions (curse of dimensionality of SDP).

Approximate dynamic programming (ADP) is a powerful algorithmic framework for solving large, stochastic, multistage, optimization problems. The key feature of ADP is the replacement of the "true" value function $F_{t+1}(S_{t+1})$ by an approximation denoted $\hat{F}_{t+1}(S_{t+1})$ [Powell, 2007]. In ADP, the problem (2.7) becomes:

$$\hat{F}_t(S_t) = E_{h_{t+1}|h_t} \left[ \max_{x_t} \left\{ f_t(S_t, x_t) + \alpha_{t+1} \hat{F}_{t+1}(S_{t+1}) \right\} \right]$$  \hspace{1cm} (2.12)

While traditional DP approach solves the problem moving backward in time, ADP solves the problem by moving forward (except for some variations that uses a cyclic backward and forward process to reach the solution). Assuming that, at time $t_0$, the system is in a given state and that an approximation of the value function has been initialized for all states, the algorithm moves iteratively forward in time to refine the quality of the approximation. The successive states visited are either sampled or computed using the decision at the previous time step. In contrast to discrete SDP, the methodology does not require to enumerate all states of the state space domain.

The key step of ADP relies on the strategy adopted to approximate the value function. According to Powell [2007], these strategies can be classified into three categories. The approach referred as look-up tables with aggregation, assumes that the state-space domain can be aggregated into a coarser representation. The size of the state-space domain is reduced and the value function approximation is given for each aggregated states. This is particularly suitable for problem characterized by very large state-space. An other strategy is to use basis functions to approximate the value function. Here, the basis functions are relations that retrieve relevant information from the state variables in order to calculate the approximated value function. Finally, other statistical methods are available to approximate the value function, particularly in the field of machine learning, such as neural network.

Stochastic Dual Dynamic Programming, described thoroughly in the next section, relies on the concept of basis functions to approximate the value function in a SDP recursive scheme.
2.6 Stochastic Dual Dynamic Programming

2.6.1 General principle

As mentioned earlier, to save computation time (and memory space), Stochastic Dual Dynamic Programming (SDDP) builds a locally-accurate approximation of \( F_{t+1} \) using piecewise linear segments: the state-space domain is sampled and a linear approximation of \( F_{t+1} \) (cut) is calculated at each sampled point (Fig. 2.6). SDDP uses a cyclic optimization/simulation strategy to increase the accuracy of the solution by adding new cuts through a Benders decomposition scheme \([\text{Pereira and Pinto, 1985}]\). In SDDP, \( F_{t+1} \) is a scalar, stored as a set of constraints representing the linear segments.

To implement the efficient Benders decomposition scheme, the one-stage optimization problem must be formulated as a convex problem, such as a linear program (LP) so that the Kuhn-Tucker conditions for optimality are necessary and sufficient (see appendix A). Then, the parameters of the linear segments, which provide an "outer" approximation of \( F_{t+1} \), can be calculated from the primal and the dual information available at the optimal solution of the one-stage optimization problem \([\text{Tilmant and Kelman, 2007}]\).

![Figure 2.6: Piecewise linear approximation of benefit-to-go function \( F_{t+1} \).](image)

The following section describes the principle of Benders decomposition that supports the SDDP algorithm.

2.6.2 Benders decomposition scheme

Benders decomposition is a technique to solve large linear optimization problems characterized by the presence of complicating variables i.e. variables which, when temporarily fixed, render the remaining optimization problem considerably more tractable \([\text{Geoffrion, 1972}]\). The original problem is reformulated as a simple LP, as a function of the complicating variables. The basic idea is to start with an approximation of the solution and then to increase successively its accuracy through an iterative process. The technique is particularly
2.6. Stochastic Dual Dynamic Programming

suitable for multistage decision making problems. The theoretical basis, adapted from Pereira and Pinto [1985], is presented below for a simple deterministic problem.

Let's assume a two-stage deterministic operation problem:

$$\max_x c_1 x_1 + c_2 x_2$$  \hspace{1cm} (2.13)

subject to

$$A_1 x_1 \leq b_1$$  \hspace{1cm} (2.14)

$$E_1 x_1 + A_2 x_2 \leq b_2$$  \hspace{1cm} (2.15)

where $x_1$ and $x_2$ are the decision variables (tubining, spillage, ... ) at the first and second stage respectively, $c_1 x_1$ and $c_2 x_2$ are the benefits from system operation, given the decisions $x_1$ and $x_2$. Equation (2.14) and (2.15) represent the operating constraints, such as the water balance equations, lower and upper bounds on storage, for the first and second stage respectively. The problem (2.13) to (2.15) is sequential and can be decomposed into two subproblems which can be solved recursively.

Assuming that $\hat{x}_1$ is a feasible solution of the first stage subproblem, the second stage optimization subproblem can be formulated as:

$$\max_x c_2 x_2$$  \hspace{1cm} (2.16)

subject to

$$A_2 x_2 \leq b_2 - E_1 \hat{x}_1$$  \hspace{1cm} (2.17)

As the decision $x_1$ affects the second-stage optimization problem, the original problem (2.13) to (2.15) can be rewritten as:

$$\max_x c_1 x_1 + F(x_1)$$  \hspace{1cm} (2.18)

subject to

$$A_1 x_1 \leq b_1$$  \hspace{1cm} (2.19)

where $F(x_1)$ is a function of the complicating variables $x_1$ at the first stage, and is defined by the optimal solution of the following problem:

$$F(x_1) = \max_x c_2 x_2$$  \hspace{1cm} (2.20)

subject to

$$A_2 x_2 \leq b_2 - E_1 x_1$$  \hspace{1cm} (2.21)

Dynamic programming is a particularly suitable method to solve this problem: $x_1$ is discretized and $F(x_1)$ is then evaluated at each point. Unfortunately, DP becomes ineffective when $x_1$ is a high dimensional vector. The proposed approach here is rather to approximate $F(x_1)$ by analytical functions and successively increase the accuracy of the approximation through an iterative process.

The principle of Dual Dynamic Programming is to construct this approximation directly from the primal and dual information available at the optimal solution of the LP subproblem [Pereira and Pinto, 1991].

Problem (2.20) to (2.21) has the following dual equivalent:

$$F(\lambda) = \min_{\lambda} \lambda (b_2 - E_1 x_1)$$  \hspace{1cm} (2.22)

subject to

$$\lambda A_2 \geq c_2$$  \hspace{1cm} (2.23)
where $\lambda$ is the vector of dual variables (simplex multipliers, shadow prices) associated with constraints (2.21). The geometric interpretation of the feasible region defined by the constraint (2.23) is a convex hull, i.e. the smallest convex set containing the $p$ vertices $\lambda^1, \lambda^2, \ldots, \lambda^p$. From the theory of linear programming, it is demonstrated that the solution of the problem belongs to the feasible region delimited by the convex hull and if the optimal solution is unique, then it is a vertex. As a consequence, the problem (2.22) to (2.23) can be solved by finding the vertex that gives the best value of the objective function:

$$F(\lambda) = \min_{\lambda} \{ \lambda^i (b_2 - Ex_1) \} \quad \text{for all} \quad i = 1, 2, \ldots, p$$ (2.24)

Problem (2.24) is equivalent to:

$$F(x_1) = \max_x F$$ (2.25)
subject to

$$F \leq \lambda^1 (b_2 - Ex_1)$$ (2.26)
$$F \leq \lambda^2 (b_2 - Ex_1)$$ (2.27)
$$\vdots$$
$$F \leq \lambda^p (b_2 - Ex_1)$$ (2.28)

where $F$ is a scalar variable. The geometric interpretation of the constraints is illustrated in figure 2.7: because the objective of the problem is a maximization, the constraints will be met at the equality, as defined by (2.24). The problem (2.25) to (2.28) demonstrates that the benefit-to-go function $F(x_1)$ is a piecewise linear function of the complicating variable $x_1$ and can be characterized without the discretization of $x_1$. The piecewise functions, called Benders cuts, are the constraints $F \leq \lambda^p (b_2 - Ex_1)$.

Figure 2.7: Geometric interpretation of the piecewise linear approximation.
By substituting the latter expression of $F(x_1)$ into (2.18), the original two-stage reservoir operation problem (2.13) to (2.15) becomes:

$$\text{max } x \quad c_1 x_1 + F$$

subject to

$$A x_1 \leq b_1$$

$$\lambda^1 (b_2 - E x_1) - F \leq 0$$  \hspace{1cm} (2.30)

$$\lambda^2 (b_2 - E x_1) - F \leq 0$$  \hspace{1cm} (2.31)

$$\vdots$$

$$\lambda^p (b_2 - E x_1) - F \leq 0$$  \hspace{1cm} (2.33)

where $c_1 x_1$ can be interpreted as the immediate benefits function while the benefit-to-go function is represented by a scalar $F$ and a set of constraints (Benders cuts).

However, it would be computationally challenging to calculate all supporting hyperplanes. The approach here is to use a relaxation technique where the constraints $\lambda^p (b_2 - E x_1) - F \leq 0$ are successively added to the optimization problem through an iterative procedure. By successively adding new cuts to approximate the piecewise linear function $F(x_1)$, the algorithm increases the accuracy of the solution as it moves towards the solution.

The solution procedure can be easily extended to the stochastic case, considering that the revenues from the second stage decision of problem (2.13) to (2.15) is conditioned by the value of $b_2$, as described in Pereira and Pinto [1985].

### 2.6.3 Objective function

Remember that the one-stage SDDP subproblem must be linear, or at least convex, so that the efficient decomposition scheme can be implemented. This section presents the strategies that have been implemented to incorporate revenues from hydropower generation and irrigated agriculture in the objective function of the one-stage SDDP subproblem.

#### 2.6.3.1 Hydropower representation in SDDP

The expression of the power $P_t$ [MW] generated by a hydropower plant during a period $t$ depends on the product of the turbined outflow $r_t$ [m$^3$.s$^{-1}$] and the net head $h$ [m] on the turbine:

$$P_t = \gamma \eta (s_t, s_{t+1}, r_t) h (s_t, s_{t+1}) r_t$$  \hspace{1cm} (2.34)

where $\gamma$ is the specific weight of water [N/m$^3$], $h$ is the net head on turbines [m] depending on the average storage $(s_t, s_{t+1})$ during time period $t$ (and eventually also the turbining $r_t$ if tailwater effects are included), $\eta [-]$ is the turbines/generators efficiency as a function of the average head and the turbining during period $t$ [Wood and Wollenberg, 1996]. For example, figure 2.8 illustrates the true hydropower function for the High Aswan Dam hydropower station, as a function of the outflow and the storage level. In this case, it is
clear that a substantial amount of energy comes from the storage term: even-
though the slope of the hydropower function regarding the outflow is greater
than regarding the storage, the latter is not negligible.

However, incorporating the variable productivity of hydropower plants di-
rectly in linear programming is not an option because of the dependence of
power production with the hydraulic head on turbines and the turbining it-
self. One way to circumvent the non-linearity of the hydropower production
function is to assume that the production of hydropower is dominated by the
release term $r_t$ [m$^3$.s$^{-1}$] and not by the head (storage) term. This is a common
assumption in mid- and long-term hydro-scheduling. For example, Archibald
et al. [1999]; Wallace and Fleten [2003]; Tilmant and Kelman [2007] define
a production coefficient $c_{hp}(j)$ [MW/m$^3$.s$^{-1}$] to characterize each hydropower
station $j$. The production coefficient is defined as the ratio between the in-
stalled capacity $\bar{P}(j)$ of the hydropower plant $j$ and the maximum turbining
capacity $r_{max}(j)$ of that hydropower plant. Generally speaking, this ratio is
multiplied by a coefficient $v(j)$ ($v(j) \in [0,1]$) to eventually adjust the energy
generated to the expected production:

$$c_{hp}(j) = v(j) \frac{\bar{P}(j)}{r_{max}(j)}$$

(2.35)
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The hydropower generated [MW] is therefore the product of release decisions [m^3.s^{-1}] and the above defined production coefficient [MW/m^3.s^{-1}]. However, assuming that the effect of the hydraulic head on turbines is negligible with respect to the turbining, is only valid when the difference between downstream and upstream water levels is small compared to the maximum level.

To deal with the variable productivity, some authors use a production coefficient as a function of the volume of water available in the reservoir [Bortolossi et al., 2002]. Pereira and Pinto [1983] take advantage of the iterative procedure of SDDP to refine the production coefficient of hydropower plants until satisfactory energy generation are obtained. Cunha et al. [1997] build an approximation of the true hydropower function by a linear and concave function of storage and total outflow of the power plant. This methodology is implemented here and described below, in the context of head-dependent multipurpose multireservoir systems.

The basic idea is to build a locally-accurate approximation of the true hydropower function using piecewise linear segments: let \( \hat{P}_t \) be a vector of new decision variables representing the power outputs during period \( t \). Let \( H \) be the number of linear approximations of the true hydropower functions, that provide upper bounds to the true functions. The \( h \)th hyperplane is defined as:

\[
\hat{P}_t - \psi_h s_{t+1}/2 - \omega_h r_t \leq \delta_h + \psi_h s_t/2 \tag{2.36}
\]

where \( \hat{P}_t \) is the (\( J \times 1 \)) vector of approximated hydropower generated during period \( t \), \( \psi_h, \omega_h \) and \( \delta_h \) are (\( 1 \times J \)) vectors of parameters for the hyperplanes \( h \), \( s_t \) and \( s_{t+1} \) are the (\( 1 \times J \)) vector of the beginning-of-period and end-of-period storage respectively, \( r_t \) is the (\( J \times 1 \)) vector of releases during period \( t \). \( \hat{P}_t \) is therefore a vector scalars, stored as an additional set of constraints representing the hyperplanes.

The methodology to calculate the hyperplanes parameters \( \psi_h, \omega_h \) and \( \delta_h \) is described below. First, the feasible domain of the storage \( s \) and the release \( r \) of each hydropower station is discretized and the true hydropower function \( P(s, r) \) is calculated at each grid point. Then, \( \hat{P}(s, r) \), which is an upper bound of \( P(s, r) \), is estimated through a convex hull approximation by piecewise linear functions of the storage and turbining. The calculation of the convex hull \( \hat{P}(s, r) \) at the set of points \( P(s, r) \) corresponds to the smallest convex set that contains these points. It relies on the practical and very efficient Quick Hull algorithm for general dimension convex hulls developed by Barber et al. [1996] and freely available from \texttt{http://www.qhull.org}.

By definition, the above-mentioned approximation obviously overestimates the true hydropower production function. Consequently, assuming the storage and the turbining are discretized into \( U \) and \( V \) values respectively, an adjustment coefficient \( \rho (\rho < 1) \) which minimizes the following expression must be defined:

\[
\sqrt{\frac{1}{U \cdot V} \sum_{u=1}^{U} \sum_{v=1}^{V} \left( P_t(s_u, r_v) - \rho \cdot \hat{P}_t(s_u, r_v) \right)^2} \tag{2.37}
\]
Finally, the hydropower generated is estimated by the product $\rho \hat{P}_t(s, r)$ and the $H \times J$ piecewise linear approximations of the $J$ true power functions:

\[
\begin{align*}
\hat{P}_t - \psi^1 s_{t+1}/2 - \omega^1 r_t & \leq \delta^1 + \psi^1 s_t/2 \\
\vdots \\
\hat{P}_t - \psi^H s_{t+1}/2 - \omega^H r_t & \leq \delta^H + \psi^H s_t/2
\end{align*}
\] (2.38)

must be added to the set of constraints of the one-stage LP subproblem.

The hydropower term $HP_t$ of the objective function $f_t$ (2.2) therefore becomes:

\[
HP_t = \tau_t \sum_{j=1}^J (\pi_t^j (j) - \theta^j(j))\rho(j)\hat{P}_t(j)
\] (2.39)

### 2.6.3.2 Irrigation representation in SDDP

Directly incorporating benefits from irrigation into the objective function of a backward optimization algorithm like SDDP is not an option as these benefits are neither additive nor independent from previous allocation decisions, i.e. from $i_{t-1}, i_{t-2}, i_{t-3}$ etc [Tilmant et al., 2008]. As a matter of fact, irrigation benefits can only be observed at the end of the irrigation season when both the timing and volume of water delivered to the crops during the irrigation season are adequate.

The net benefit from the agricultural sector, denoted $IR_t$, is the sum of the benefits obtained at each irrigation demand site $d$ as a function of the volume of water that has been delivered to the crops at that site during the irrigation season $y_{tf}(d)$. Note that these site-specific net benefit functions $\hat{f}_{tf}(d)(y_{tf}(d))$ must be linear or at least approximated by piecewise linear functions in order to be compatible with the Benders decomposition scheme in SDDP:

\[
IR_t = \left\{ \begin{array}{ll}
\sum_d \hat{f}_{tf}(d)(y_{tf}(d)) & \text{if } t = tf \\
0 & \text{if } t \neq tf
\end{array} \right.
\] (2.40)

Benefits from the irrigated agricultural sector $IR_t$ are introduced in the objective function by considering "dummy" reservoirs of accumulated water devoted to irrigation, which are being refilled throughout the irrigation season and depleted at the end of that season (stage $tf$). Net benefits from agricultural water use is therefore accounted for only when crops are harvested. To achieve this, an additional state variable $y_t$, representing the beginning-of-period accumulated water into those "dummy" reservoirs must be added to the state vector $S_t$. Figure 2.9 illustrates a schematic representation of a surface reservoir and its irrigation demand site modelled by a "dummy" reservoir of accumulated water for irrigation. Moreover, at each demand site $d$, the model can handle various types of crops $p$, with its own benefit function $\hat{f}_t^{p,d}(\cdot)$, as illustrated on figure 2.10. To assess the impact of variation in supplies on crop yields, the
linear relationship between crop yield deficit and the actual evapotranspiration ($ET_a$) is used [Doorenbos and Kassam, 1979]:

$$c(p,d) = \bar{c}(p,d) \left[ 1 - K_y(p) \left(1 - \frac{ET_a(p,d)}{ET_{max}(p,d)} \right) \right]$$  \hspace{1cm} (2.41)

where $c(p,d)$ [T/ha] and $\bar{c}(p,d)$ [T/ha] are the actual and maximum yield of crop $p$ respectively, $ET_a$ [mm] and $ET_{max}$ [mm] are the actual and maximum evapotranspiration respectively, $K_y(p)$ [-] is the crop yield coefficient specific to crop $p$. Generally speaking, a higher $K_y$ means a higher yield reduction for the same water deficit. We will further assume that $K_y(p)$ remains the same in each of the stage $t$ within the irrigation season [Vedula and Majumdar, 1992; Vedula and Kumar, 1996]. Assuming that rationing, when it occurs, is evenly
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distributed over the irrigation season, equation (2.41) can be rewritten:

\[ c(p, d) = \bar{c}(p, d) \left[ 1 - K_y(p) \left( 1 - \frac{y_t(p, d)}{\bar{y}_t(p, d)} \right) \right] \]  

(2.42)

where \( \bar{c}(p, d) \) [T/ha] is the maximum crop yield achieved when the seasonal crop water requirement \( \bar{y}_t(p, d) \) [hm\(^3\)] is supplied at the demand site \( d \). The short-run net benefits \( f_i(p, d) \) of crop \( p \), at demand site \( d \) is therefore expressed by:

\[ \hat{f}_i(p, d) = \left[ \pi_i(p, d) c(p, d) - \theta_i(p, d) \right] A(p, d) \]  

(2.43)

where \( \pi_i(p, d) \) [US$/T] and \( \theta_i(p, d) \) [US$/ha] are the farm gate price and the variable costs of crop \( p \) respectively, at site \( d \). \( A(p, d) \) [ha] is the maximum cultivated area.

Throughout the growing season, the mass balance in the dummy reservoirs is respected given the following additional constraint:

\[ y_{t+1} - \epsilon_t y_t = y_t \]  

(2.44)

where \( \epsilon \) is a vector of irrigation efficiencies. These dummy reservoirs have their own lower and upper bounds:

\[ \underline{y}_{t+1} \leq y_{t+1} \leq \bar{y}_{t+1} \]  

(2.45)

2.6.4 One-stage SDDP subproblem

With these extensions, the multi-stage decision making problem of optimal reservoir operation (2.1) to (2.6) is solved using the following recursive equation:

\[ F_t(s_t, q_{t-1}, y_t) = \max \{ f_t(s_t, q_t, s_{t+1}, r_t, y_t) + \alpha_{t+1} F_{t+1} \} \]  

(2.46)

with the objective function defined by the sum of the benefits from hydropower generation (eq. 2.39) and irrigation (eq. 2.40):

\[ f_t(\cdot) = HP_t + IR_t - \xi_t z_t \]  

(2.47)

where \( z_t \) is a vector of slack variables with the violations of operational constraints (energy deficit, environmental flows, etc.) which are penalized in the objective function by the vector of penalties \( \xi_t \) [$/unit of deficit or surplus]. Note that the end-of-period storage \( s_{t+1} \) is a decision variable because hydropower generation and evaporation losses depends on the average head during period \( t \).

The one-stage optimization problem (2.46) is subject to the following set of constraints, among which the mass conservation for all periods \( t \):

\[ s_{t+1} - C^R(r_t + l_t) - C^I(i_t) + e_t(s_t, s_{t+1}) = s_t + q_t \]  

(2.48)

Lower and upper bounds on storage:

\[ s_{t+1} \leq s_t \leq \bar{s}_{t+1} \]  

(2.49)
2.6. Stochastic Dual Dynamic Programming

Limits on reservoir releases:
\[ r_t \leq r_t \leq \bar{r}_t \]  (2.50)

Limits on irrigation water withdrawals:
\[ i_t \leq i_t \leq \bar{i}_t \]  (2.51)

Mass balance in the dummy reservoirs devoted to irrigation:
\[ y_{t+1} = y_t - \epsilon \bar{i}_t \]  (2.52)

Lower and upper bounds on dummy reservoirs:
\[ y_{t+1} \leq y_{t+1} \leq \bar{y}_{t+1} \]  (2.53)

The approximations of hydropower functions are also stored in the constraints set:
\[
\begin{align*}
\hat{P}_t - \psi^1 s_{t+1}/2 - \omega^1 r_t &\leq \delta^1 + \psi^1 s_t/2 \\
\vdots \\
\hat{P}_t - \psi^H s_{t+1}/2 - \omega^H r_t &\leq \delta^H + \psi^H s_t/2 
\end{align*}
\]  (2.54)

where \( H \) is the number of planes that approximate the true hydropower functions. Finally, as mentioned above, the benefit-to-go function \( F_{t+1} \) is represented by a scalar and a set of constraints representing hyperplanes that approximate the true benefit-to-go function:
\[
\begin{align*}
F_{t+1} - \varphi^1 s_{t+1} - \eta^1 y_{t+1} &\leq \gamma^1 y_{t+1} + \beta^1 \\
\vdots \\
F_{t+1} - \varphi^L s_{t+1} - \eta^L y_{t+1} &\leq \gamma^L y_{t+1} + \beta^L 
\end{align*}
\]  (2.55)

where \( L \) is the number of cuts that approximate the benefit-to-go function and \( \varphi_{t+1}^1, \gamma_{t+1}, \eta_{t+1}^1 \) and \( \beta_{t+1}^1 \) are cuts parameters calculated during the backward optimization phase, from the primal and dual information available at stage \( t+1 \). The remaining of this chapter explains how these parameters are derived from the primal and dual information available at the optimal solution when the algorithm progresses backward. But first the build-in hydrological model must be described.

2.6.5 Hydrological model

In SDDP, to avoid the discretization of the hydrologic state variable, and therefore to further reduce the computational effort required to solve the recursive equation (2.46), the natural reservoirs inflows \( q_t \) are estimated at each node of the hydro-system by an analytical build-in multi-site periodic autoregressive model of order \( p \) (MPAR(\( p \))). Again, in order to implement the efficient decomposition scheme of SDDP, the analytical expression of the hydrological model must be linear and differentiable. Only the residuals of the MPAR model are therefore cross-correlated in space and time.
Chapter 2. The reservoir operation problem

The MPAR model is used in both phases of the SDDP algorithm: in the backward optimization (i) to generate the inflows scenarios at each node of the hydro-system prior to solve the one-stage optimization problem for each of these scenarios and (ii) to calculate analytically cuts parameters after each one-stage optimization. In the forward phase, the MPAR model generates synthetic reservoir inflows to simulate the system behaviour over the planning period. The set of MPAR model parameters are adjusted on historical records of natural reservoir inflows.

Assuming that the periodic process can be modelled by an autoregressive model of order 1, the model can be written as (for period $t$):

$$q_t = \mu_{q,t} + \phi_t \frac{\sigma_{q,t}}{\sigma_{q,t-1}} (q_{t-1} - \mu_{q,t-1}) + \sigma_{q,t} \epsilon_t$$  \hspace{1cm} (2.56)

where $\mu_{q,t}$ and $\sigma_{q,t}$ are vectors (size 1 x $J$) of the periodic mean and standard deviation of the current period reservoir inflows $q_t$, $\phi_t$ is the vector (size 1 x $J$) of periodic autoregressive parameter of order 1 and $\epsilon_t$ is a time independent (but spatially correlated) stochastic noise of zero mean and variance $\sigma_{\epsilon,t}^2$. A comprehensive description of the methodology to estimate the model parameters is presented in appendix B.

### 2.6.6 Backward optimization

As mentioned above, the SDDP algorithm is organized around in two phases: a backward optimization and a forward simulation [Pereira and Pinto, 1985]. During the backward optimization, an upper bound to the true benefit-to-go function $F_{t+1}$ is calculated at the sampled points.

In order to account for the stochasticity of reservoirs inflows $q_t$, the one-stage SDDP subproblem (2.46) to (2.55) is solved for $K$ reservoir inflow branches $q^k_t$ (backward openings). Given a set of sampled hydrologic state variable $q_{t-1}^o, ..., q_{t-p}^o$, these $K$ openings are estimated from the analytical build-in multi-site and multi-period autoregressive model of order $p$ (MPAR($p$)) described in the previous section. The expected benefit-to-go function $F_{t+1}$, stored in the form of cuts, is the expected value of the $K$ benefit-to-go function $F^k_{t+1}$ calculated for each inflow branch [Tilmant and Kelman, 2007]. This approach, where the construction of the approximating cuts relies on aggregation, differs from the multicut approach, where all the $K$ cuts are stored and passed to the previous stage [Birge and Louveaux, 1989]. While the multicut algorithm provides a more accurate approximation of the the benefit-to-go function, it increases the size of the optimization problem by multiplying the number of constraints. For example, the SOCRATES model [Jacobs et al., 1995] uses a multicut approach for hydroelectric scheduling in the United States. The aggregated approach is illustrated on figure 2.11 and the pseudo-code of the backward optimization phase is presented on figure 2.12.

A key step of the SDDP algorithm is to calculate the vectors of parameters $\varphi_{t+1}^l, \gamma_{t+1}^l, \eta_{t+1}^l$ and $\beta_{t+1}^l$ of the $l$th cut that approximate the true benefit-to-go function $F_{t+1}$:

$$F_{t+1} - \varphi_{t+1}^l s_{t+1}(j) - \eta_{t+1}^l y_{t+1} \leq \gamma_{t+1}^l q_t + \beta_{t+1}^l$$  \hspace{1cm} (2.57)
2.6. Stochastic Dual Dynamic Programming

Figure 2.11: Illustration of the backwards openings and the approximation of the benefit-to-go function. Adapted from Tshmant and Kelman [2007].

Define $L$ number of linear approximations of $F_{t+1}$
Initialize end-of-horizon cuts parameters $\varphi_{T+1}^l$, $\eta_{T+1}^l$, $\gamma_{T+1}^l$, $\beta_{T+1}^l$
FOR $t=T$, $T-1$, ..., $t_0$
  sample state variables $s_{t+1}^l$, $y_{t+1}^l$, $q_{t+1}^l$, ..., $q_{t-p}^l$
  Generate $K$ backwards openings (inflow branches) $q_k^l$
  FOR each sampled storage $l = 1, ..., L$
    Retrieve cuts parameters calculated at stage $t+1$: $\varphi_{t+1}^l$, $\eta_{t+1}^l$, $\gamma_{t+1}^l$, $\beta_{t+1}^l$
    FOR each inflow branches $q_k^l$
      solve the one-stage SDDP problem
      calculate cuts parameters: $\varphi_t^l$, $\eta_t^l$, $\gamma_t^l$, $\beta_t^l$
    END
  END
END
Aggregate cuts

Figure 2.12: Backward optimization phase: pseudo-code.

Their calculation relies on the primal and dual information available at the optimal solution: At stage $t+1$, the triplet $(s_{t+1}^l, q_{t+1}^l, y_{t+1}^l)$ is sampled. According to the Kuhn-Tucker conditions for optimality, dual information of the optimization problem at stage $t+1$ can be used to derive the vector of slopes $\varphi_{t+1}^l$ with respect to the storage state variable $s_{t+1}$ to approximate the future
benefit function $F_t+1$ at stage $t$:

$$\frac{\partial F_t^k}{\partial s_{t+1}} = \lambda_{w,t+1}^l(j) + \sum_{h=1}^{H} \lambda_{hp,t+1}^{l,h,k}(j)\psi^h(j)/2$$  \hspace{1cm} (2.58)

where $\lambda_{w,t}^l$ and $\lambda_{hp,t}^{l,h,k}$ are the Lagrange multipliers associated with the mass balance equations (2.48) and with the set of constraints (2.54) respectively.

The expected slope of the $l^{th}$ cut with respect to the storage $s_{t+1}(j)$ can be calculated by taking the expectation of the $K$ backwards openings:

$$\varphi_{t+1}^l(j) = \frac{1}{K} \sum_{k=1}^{K} \left( \lambda_{w,t+1}^l(j) + \sum_{h=1}^{H} \lambda_{hp,t+1}^{l,h,k}(j)\psi^h(j)/2 \right)$$  \hspace{1cm} (2.59)

Similarly, the $j^{th}$ element of the vector of slopes $\gamma_{t+1}^l$ with respect to the hydrologic variable $q_{t+1}$ can be obtained from the dual information associated with the constraints (2.55) and the water balance (2.48), i.e. from the vectors $\lambda_{w,t+1}^l$ and $\lambda_{c,t+1}^l$ respectively:

$$\frac{\partial F_t^k}{\partial q_t} = \frac{\partial F_t^k}{\partial q_{t+1}} \frac{\partial q_{t+1}}{\partial q_t}$$

$$= \left( \lambda_{w,t+1}^l(j) + \sum_{i=1}^{L} \lambda_{c,t+1}^l \gamma_{t+2}^l(j) \right) \times \left( \frac{\sigma_{q,t+1}(j)}{\sigma_{q,t}(j)} \rho_{q,t}(j) \right)$$

$$= \gamma_{t+1}^l(j)$$  \hspace{1cm} (2.60)

The expected slope with respect to the inflows is then estimated by:

$$\gamma_{t+1}^l(j) = \frac{1}{K} \sum_{k=1}^{K} \gamma_{t+1}^l(j)$$  \hspace{1cm} (2.61)

As for the other gradients, $\eta_{t+1}^l$ is also derived at stage $t+1$ from the Lagrange multipliers $\lambda_{y,t+1}$ associated with the constraints (2.52):

$$\frac{\partial F_t^k}{\partial y_t} = \lambda_{y,t+1}^l$$  \hspace{1cm} (2.62)

Then, the expected slope of the $l^{th}$ cut with respect to $y_{t+1}$ can be calculated by:

$$\eta_{t+1}^l(j) = \frac{1}{K} \sum_{k=1}^{K} \lambda_{y,t+1}^l(j)$$  \hspace{1cm} (2.63)

The $j^{th}$ element of the vector constant terms is finally given by:

$$\beta_{t+1}^l(j) = \frac{1}{K} \sum_{k=1}^{K} F_{t+1}^k - \varphi_{t+1}^l(j) s_{t+1}(j) - \gamma_{t+1}^l(j) q_{t+1}^l(j) - \eta_{t+1}^l(j) y_{t+1}^l(j)$$  \hspace{1cm} (2.64)
2.6.7 Forward simulation

As mentioned earlier, the SDDP algorithm is an iterative procedure organized around two phases. The backward optimization phase generates an outer approximation of the benefit-to-go function $F_{t+1}$ in the form of Benders cuts. The resulting benefit-to-go function is therefore an upper bound to the "true" function, as illustrated in figure 2.6. The accuracy of this approximation is evaluated at the end of the forward simulation. Because the set of Benders cuts (2.55) provides only an approximation to the multidimensional benefit-to-go functions, simulating the system forward will give a lower bound $\underline{Z}$ to the solution of the multistage decision making problem. Let $M$ be the number of hydrologic sequences used in simulation, the expected lower bound on the optimal solution is given by:

$$\mu_{\underline{Z}} = \frac{1}{M} \sum_{t=1}^{T} f_t^m(s_t, q_t^m, r_t, y_t) = \frac{Z^m}{M}$$

where $f_t^m(.)$ is the immediate benefit at stage $t$ for the hydrologic sequence $m \in [1, 2, \ldots, M]$. The standard deviation of the estimated lower bound can also be calculated:

$$\sigma_{\underline{Z}} = \sqrt{\frac{1}{M-1} \sum_{m=1}^{M} (Z^m - \mu_{\underline{Z}})^2}$$

The 95% confidence interval around the estimated value of $\underline{Z}$ is given by

$$\left[ \mu_{\underline{Z}} - 1.96 \frac{\sigma_{\underline{Z}}}{\sqrt{M}}, \mu_{\underline{Z}} + 1.96 \frac{\sigma_{\underline{Z}}}{\sqrt{M}} \right]$$

---

Figure 2.13: Forward simulation phase: pseudo-code.

2.6.8 Convergence check

While simulating the system operation forward provides a lower bound $\underline{Z}$ to the solution of the reservoir operation problem, at the end of the backward
optimization phase, i.e. at stage one, the function $F_1(s^0_1, q^0_0, y^0_1)$ overestimates the benefits of system operation over the planning period. When the sampled values for the state variables are $s^0_1$, $q^0_0$ and $y^0_1$, the upper bound $Z$ can be written:

$$Z = F_1(s^0_1, q^0_0, y^0_1)$$  \hspace{1cm} (2.68)

If $Z$ is inside the confidence interval (2.67), then the approximation is statistically acceptable and the problem (2.46) to (2.55) is solved. Otherwise, a new iteration is needed: a new backward recursion is implemented and the natural candidates for the sampled values of $s_t$ and $y_t$ are the storage volumes and accumulated volumes the previous simulations pass through. This backward phase is then followed by a new forward simulation, which will exploit the cuts that have been generated during the previous backward recursions. The process is repeated until convergence. The pseudo-code of the forward simulation and the convergence check is illustrated in figure 2.13. The convergence of the algorithm is discussed and demonstrated in Philpott and Guan [2008].
Chapter 3

Description of the case studies

3.1 The Eastern Nile river basin

3.1.1 General description

The Nile river basin covers an area of about 3 million km$^2$ and is shared by ten countries (Burundi, Democratic Republic of Congo, Egypt, Eritrea, Ethiopia, Kenya, Rwanda, Sudan, Tanzania, Uganda), as illustrated in figure 3.1 and table 3.1. The Blue Nile and the White Nile merge at Karthoum, in Sudan, to form the main Nile that discharges into the Mediterranean Sea. The Blue Nile originates from the Lake Tana, in the Highlands of Ethiopia. After flowing into abrupt canyons in Ethiopia, characterized by a temperate climate, it enters the plain in Sudan where the climate is much more arid. Given its equatorial position and associated high precipitations, the discharge of the Blue Nile in Ethiopia grows rapidly as it receives water from the numerous tributaries flowing from the highlands. The flow regime of the Blue Nile is currently largely unregulated and is characterized by a very high seasonal and inter-annual variability (Fig. 3.2). The White Nile drains an area from the Lake Victoria to Karthoum. After flowing through the Sudd (one of the world’s largest wetlands), it receives water from the Baro-Akobo-Sobat sub-basin, flowing from Ethiopia to join Bahr Eljabel at Malakal to form the White Nile. Compared to the Blue Nile, the flow regime of the White Nile exhibits less seasonality given the natural regulation achieved by the Lake Victoria and the Sudd (Fig. 3.2). Before entering Egypt, the Main Nile receives water from the Atbara sub-basin, which is its last significant affluent. The Eastern Nile river basin is composed of: the Blue Nile, the Atbara, the Baro-Akobo-Sobat, the White Nile downstream Malakal and the Main Nile sub-basins, as illustrated in figure 3.1.
Chapter 3. Description of the case studies

Figure 3.1: The Eastern Nile river basin (Own elaboration based on the Hydro1K database USGS (US Geological Survey) [1996]).
3.1. The Eastern Nile river basin

Table 3.1: Area of Nile river basin in riparian countries. Source: Own calculation based on the Hydro1K database USGS (US Geological Survey) [1996]).

<table>
<thead>
<tr>
<th>Country</th>
<th>Percentage of the country within the watershed [%]</th>
<th>Percentage of the watershed within the county [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Burundi</td>
<td>47.2</td>
<td>0.4</td>
</tr>
<tr>
<td>Democratic Republic of Congo</td>
<td>0.9</td>
<td>0.7</td>
</tr>
<tr>
<td>Egypt</td>
<td>31.8</td>
<td>10.1</td>
</tr>
<tr>
<td>Eritrea</td>
<td>21.2</td>
<td>0.8</td>
</tr>
<tr>
<td>Ethiopia</td>
<td>32.2</td>
<td>11.7</td>
</tr>
<tr>
<td>Kenya</td>
<td>8.75</td>
<td>1.6</td>
</tr>
<tr>
<td>Rwanda</td>
<td>82.1</td>
<td>0.7</td>
</tr>
<tr>
<td>Sudan</td>
<td>77.6</td>
<td>62.5</td>
</tr>
<tr>
<td>Tanzania</td>
<td>12.9</td>
<td>39.0</td>
</tr>
<tr>
<td>Uganda</td>
<td>98.3</td>
<td>7.6</td>
</tr>
</tbody>
</table>

Figure 3.2: Naturalized discharge of the Nile at key locations in the Nile Basin (period 1954-2000). Source: Norplan, Norconsult & Lahmeyer International [2006] and EDF and Scott Wilson [2007].

3.1.2 Geo-political background

Egypt, the most downstream country of the basin consumes about 80% of the Nile waters, while Ethiopia has a negligible consumption, even though 85% of the Nile waters comes from the Ethiopian Highlands [Wu and Whittington, 2006]. To secure its share of the Nile waters, Egypt has signed several agreements with its riparians: Great Britain on behalf of Sudan, Kenya, Tanzania
and Uganda and with Sudan in 1959 after its independence. Historically, those agreements were very controversial between upstream (source countries), and downstream countries (Egypt and Sudan). The most important treaty is the 1959 agreement between Egypt and Sudan about the sharing of the Nile waters. It assumes that, given an agreed upon annual average of 84 km$^3$ (1 km$^3 = 10^9$ m$^3$) of Nile yield, the allotment for Egypt is 55.5 km$^3$ year$^{-1}$ while Sudan receives 18.5 km$^3$ year$^{-1}$. Ten km$^3$ year$^{-1}$ are left for evaporation losses from Lake Nasser [Haynes and Whittington, 1981]. According to the agreement, the request of another riparian country(ies) has to be met equally from Egypt’s and Sudan’s share. All countries never recognized the treaty [Okidi, 1990; Nicol, 2003]. It is important to note that, according to the 1959 bilateral agreement, the evaporation losses from man-made reservoirs in Sudan must be deducted from its share of the Nile waters.

The first basin-wide dialogue between countries has been initiated in 1997, with the Nile Basin Initiative (NBI). This organization aims at providing a framework to develop the river in a cooperative way, sharing socio-economic benefits and promoting regional security and peace. The NBI has successfully developed a number of so called shared-vision, and subsidiary action projects. However, the dialogue of the Nile riparians on a unified legal framework has not yet been completed. In May 2010, five countries signed (Ethiopia, Rwanda, Tanzania, Uganda and Kenya). Two countries are waiting (Democratic Republic of Congo and Burundi), while the remaining two countries (Sudan and Egypt) strongly opposed [Nile Basin Initiative, May 2010a].

### 3.1.3 Reservoirs and hydropower plants

Currently, the Eastern Nile hydro-system consists of eleven major hydraulic infrastructures, listed in table 3.2. Their localization is illustrated in figure ?? while the topology of the system is depicted in figure 3.4.

**Ethiopia** - The first one is the Chara Chara weir that regulates the Lake Tana outflows to the Tis Abbay power complex, located some 32 kilometers downstream the Lake. Then, the Tana-Beles scheme (started operation in May 2010) consists of an artificial link between the Lake Tana and the Beles river to generate hydroelectricity and aims to irrigate around 150,000 ha in the future. In the upper Ethiopian part of the Atbara sub-basin (Tekeze river in Ethiopia), the TK-5 dam is the largest Ethiopian hydraulic infrastructure. With an over-year storage capacity and an installed capacity of 300 MW, the TK-5 dam started to generate hydropower in 2009 and aims to produce around 30% of current total national electric production.

**Sudan** - Downstream, in Sudan, the main objective of the Roseires and Sennar dams is to provide seasonal regulation of the Blue Nile waters to irrigate more than 1 million hectares of crops distributed over 3 major schemes. Their associated hydropower stations supply Sudan in electricity but their production is relatively small given the low head available. Due to its physiographic characteristics, Sudan has only a few interesting sites to store water and suffers from a lack of over-year storage capacity to supply irrigation and reduce flood damage. To tackle this problem, the heightening of the Roseires dam started
3.1. The Eastern Nile river basin

Figure 3.3: Infrastructure of the Eastern Nile river basin (Own elaboration based on the Hydro1K database USGS (US Geological Survey) [1996]).
Chapter 3. Description of the case studies

Figure 3.4: Topological view of the Eastern Nile river basin hydro-system.
3.1. The Eastern Nile river basin

recently to bring its storage capacity up to 6.9 km³. The Atbara river (called Tekeze in Ethiopia) is dammed at Kashm El Girba where the installed capacity is relatively small and the reservoir is encountering reduction of storage capacity because of siltation. Located on the White Nile, near its confluence with the Blue Nile, the Jebel Aulia dam is presently operated to reduce pumping costs for the irrigated areas located around the reservoir. The Merowe dam, built close to the 4th cataract of the Nile, is the last significant infrastructure in Sudan. Controversial infrastructure [Giles, 2006], the Merowe project will significantly increase the Sudanese production of hydroelectricity thanks to its installed capacity of 1250 MW.

Egypt - In Egypt, the High Aswan Dam (HAD) is the largest infrastructure of the basin and it fully regulates the Nile waters downstream of the dam. Its over-year storage capacity and associated hydropower plant were designed to supply reliable irrigation water, meet increasing energy demand (around 9% of current total national electric production), improve downstream navigation and to protect Egypt against flooding. Therefore, the HAD plays a crucial role in the Egyptian economy [Abu-Zeid and El-Shibini, 1997]. Downstream HAD, the Old Aswan dam is operated as a run-of-river plant; it slightly regulates the daily outflows from HAD and contributes to the production of electricity. The Esna run-of-river plant is the latest significant hydropower facility on the main stream of the Nile. Located in the Western Desert, the New Valley project aims at irrigating 250 000 ha of crops by pumping water from the left bank of the Lake Nasser. The project is currently under construction but the pumping station and the major canals were already completed.

Development perspectives: The major challenge for the Nile waters management is to control its seasonal and inter-annual variability. To date, as described above, only relatively small hydraulic infrastructures have been constructed in the Blue Nile catchment in Ethiopia, despite the huge hydropower potential offered by the topography of the country. Since the beginning of the 20th century, large-scale projects have been on the drawing board to develop the upper part of the basin which is currently a largely untapped resource [Whittington, 2004].

Under the umbrella of NBI, the Nile countries recently initiated a joint study to develop the Blue Nile in Ethiopia. The objective of the proposed joint multipurpose projects (JMP projects) are to minimize evaporation losses in the basin, increase flow reliability, generate cheap hydropower and enhance downstream energy production, alleviate downstream sedimentation and mitigate floods and draughts along the Nile and Blue Nile. The projects will boost the production of hydroelectricity and will probably have significant impacts on the management of the Nile waters. The existing and most likely project to be implemented are listed in table 3.2.

Most of the information about infrastructures was obtained during field visits to the Eastern Nile Technical Regional Office (ENTRO, Addis Abeba,
Ethiopia), which is the executive arm of NBI for projects related to the Eastern Nile river basin, and the Ethiopian Ministry of Water Resources.

Table 3.2: Major infrastructures: Hydropower plants and reservoirs (* = planned, ET = Ethiopia, SU = Sudan, EG = Egypt). Sources: ENTRO (2009), Block [2007]; Whittington et al. [2005]; Norplan, Norconsult & Lahmeyer International [2006]; EDF and Scott Wilson [2007].

<table>
<thead>
<tr>
<th>Name (country)</th>
<th>River</th>
<th>Live storage [hm$^3$]</th>
<th>Capacity [MW]</th>
<th>Lateral irrig. yes/no</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tis Abbay I &amp; II (ET)</td>
<td>Blue Nile</td>
<td>run-of-river</td>
<td>86</td>
<td>no</td>
</tr>
<tr>
<td>Tana-Beles link (ET)</td>
<td>Blue Nile</td>
<td>run-of-river</td>
<td>270</td>
<td>no</td>
</tr>
<tr>
<td>Karadobi* (ET)</td>
<td>Blue Nile</td>
<td>17 000</td>
<td>1 600</td>
<td>no</td>
</tr>
<tr>
<td>Beko-Abo* (ET)</td>
<td>Blue Nile</td>
<td>20 000</td>
<td>2 100</td>
<td>no</td>
</tr>
<tr>
<td>Mandaya* (ET)</td>
<td>Blue Nile</td>
<td>24 600</td>
<td>1 620</td>
<td>no</td>
</tr>
<tr>
<td>Border* (ET)</td>
<td>Blue Nile</td>
<td>8 500</td>
<td>1 400</td>
<td>no</td>
</tr>
<tr>
<td>TK-5 (ET)</td>
<td>Atbara</td>
<td>9 200</td>
<td>300</td>
<td>no</td>
</tr>
<tr>
<td>Roseires (SU)</td>
<td>Blue Nile</td>
<td>6 900</td>
<td>275</td>
<td>no</td>
</tr>
<tr>
<td>Sennar (SU)</td>
<td>Blue Nile</td>
<td>480</td>
<td>15</td>
<td>yes</td>
</tr>
<tr>
<td>Khashm El Girba (SU)</td>
<td>Atbara</td>
<td>630</td>
<td>17</td>
<td>yes</td>
</tr>
<tr>
<td>Jebel Aulia (SU)</td>
<td>White Nile</td>
<td>2 800</td>
<td>30</td>
<td>yes</td>
</tr>
<tr>
<td>Merowe (SU)</td>
<td>Main Nile</td>
<td>8 300</td>
<td>1 250</td>
<td>no</td>
</tr>
<tr>
<td>High Aswan (EG)</td>
<td>Main Nile</td>
<td>105 900</td>
<td>2 100</td>
<td>no</td>
</tr>
<tr>
<td>Old Aswan (EG)</td>
<td>Main Nile</td>
<td>run-of-river</td>
<td>500</td>
<td>no</td>
</tr>
<tr>
<td>Esna (EG)</td>
<td>Main Nile</td>
<td>run-of-river</td>
<td>90</td>
<td>no</td>
</tr>
</tbody>
</table>

3.1.4 Irrigated areas

Ethiopia - There are some potential irrigation developments directly in the Ethiopian part of the Blue Nile basin, but they are relatively of limited size. Steep slopes and the deep incised valleys limit the possibilities for cheap irrigation in the Ethiopian Highlands. As a consequence, and except for very small areas, none of the proposed irrigation sites is taking water directly from the Blue Nile but rather around the Beles river where around 150 000 ha are planned [Norplan, Norconsult & Lahmeyer International, 2006].

Sudan - Irrigation in Sudan is organized around six major schemes identified by the location of their water abstraction point (Fig. 3.4). Along the Blue Nile, water is pumped for irrigation between Roseires and Sennar dams to irrigate around 300 500 ha of crops (cotton, groundnut, wheat, dura, kenaf and sugar cane). The Sennar reservoir feeds the Gezira-Managil scheme, located on the left bank of the Blue Nile. It is the largest in Sudan (870 000 ha of a mix of cotton, groundnut, wheat, dura) and the only gravity irrigation scheme along the Blue Nile. Water is also abstract by pumps from the Blue Nile bewteen Sennar and Karthoum to irrigate 74 500 ha of cotton, groundnut, dura and sugar cane. On the White Nile, about 180 000 ha are currently irrigated by pumped water from the Jebel Aulia reservoir. The New Halfa scheme (120 000 ha), located downstream the Kashm El Girba reservoir, was established to resettle Sudanese farmers for the filling of the Lake Nasser. The last significant
3.1. The Eastern Nile river basin

The Eastern Nile river basin

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3.1. The Eastern Nile river basin

The cropping pattern in Sudan consists mainly in cereals (47%), fodder (13%), industrial crops (31%) and horticulture (10%), as illustrated in figure 3.5. Figure 3.6 illustrates the monthly crop water requirement (CWR) for the different irrigation schemes in Sudan [Norplan, Norconsult & Lahmeyer International, 2006]. It is shown that the CWR exhibits a high seasonality with the water demand peaking from November to April.

Egypt - In Egypt, the annual water demand downstream the High Aswan Dam corresponds to the 1959 allotment of 55.5 km$^3$. This volume of water is primarily used for irrigation purposes. The monthly distribution of the demand has been taken from Oven-Thompson et al. [1982] with the peaking water demand observed from May to August, as illustrated in figure 3.7.
Chapter 3. Description of the case studies

Figure 3.6: Crop water requirements (CWR) by irrigation scheme in Sudan [Norplan, Norconsult & Lahmeyer International, 2006].

Figure 3.7: Gross water requirements (GWR) downstream the High Aswan Dam in Egypt [Owen-Thompson et al., 1982].
3.2 The Euphrates river basin

3.2.1 General description

With a joint area of 765,831 km$^2$, the Euphrates and Tigris river basins are shared by Turkey, Syria, Iran, Iraq and Saudi Arabia (figure 3.8). The two river basins are usually considered as a single unit since the two rivers are linked by their natural confluence in the Shatt-Al-Arab (in Iraq) which constitute a delta [Altinbilek, 2004]. The Euphrates and Tigris originate in the Highlands of Turkey, Iraq and Iran, and discharge into the Persian Gulf, as illustrated in figure 3.8. In the Euphrates, two third of the river flow comes from snow precipitation. The hydrograph exhibits therefore a high seasonal variability, with a flood peak in April and May corresponding to the snowmelt period (Fig. 3.9). Precipitation are unevenly distributed in the basin: annual rainfall are usually higher than 1000 mm in the Turkish Highlands while they are only around 400 mm at the border between Turkey and Syria, on the Euphrates. Downstream the border, rainfed agriculture becomes inopportune.

There is a major difference between the two rivers: despite the relatively small contribution of Turkey in terms of shared area of the total basin (table 3.3), at least 95% of the annual flow of the Euphrates (measured at mouth) is generated in Turkey while it is only 44% for the Tigris [Beaumont, 1998]. According to Altinbilek [2004], these figures are 98% and 53% respectively. Tigris receives water from a series of affluents during its course in Syria and Iraq while it is not the case for the Euphrates. This study will focus on the Euphrates river.

Table 3.3: Area of Tigris-Euphrates drainage basin in riparian countries. Source: UNEP. Partow, H. [2001].

<table>
<thead>
<tr>
<th>Country</th>
<th>Euphrates Area [km$^2$]</th>
<th>Euphrates [%]</th>
<th>Tigris Area [km$^2$]</th>
<th>Tigris [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Iran</td>
<td>0</td>
<td>0</td>
<td>175,386</td>
<td>47.2</td>
</tr>
<tr>
<td>Iraq</td>
<td>282,532</td>
<td>48.8</td>
<td>142,175</td>
<td>38.0</td>
</tr>
<tr>
<td>Saudi Arabia</td>
<td>77,090</td>
<td>13.3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Syria</td>
<td>95,405</td>
<td>16.5</td>
<td>948</td>
<td>0.3</td>
</tr>
<tr>
<td>Turkey</td>
<td>121,787</td>
<td>21.0</td>
<td>53,052</td>
<td>14.0</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>579,314</strong></td>
<td><strong>100.0</strong></td>
<td><strong>371,562</strong></td>
<td><strong>100.0</strong></td>
</tr>
</tbody>
</table>

3.2.2 The GAP in Turkey

The potential to develop the Euphrates and Tigris rivers for the production of hydroelectricity in the Southeastern Anatolia region has been recognized since the 1920s by Mustafa Kemal Atatürk, the founder of the modern Turkish Republic [Brismar, 2002]. Since the early 1960s, several surveys and development projects have been initiated by the Turkish government to finally lead to the Southeastern Anatolia Project (referred to as GAP from the Turkish acronym
Figure 3.8: The Euphrates and Tigris river basin (Own elaboration based on the Hydro1K database USGS (US Geological Survey) [1996]).
3.2. The Euphrates river basin

Figure 3.9: Naturalized discharge of the Euphrates at key locations in the Euphrates river basin (period 1937-1967). Source: Kolars and Mitchell [1994].

for Güneydoğu Anadolu Projesi), a multidimensional water resources development project which aims to improve the entire economy of the Southeastern Anatolia region in Turkey. This will be achieved by boosting the agricultural production by 300% and providing 22% of Turkey’s viable hydroelectric potential. This project is one of the largest water resources development projects in the world involving the construction of 22 dams, 19 hydroelectric power plants with an installed capacity of 7526 MW, and the irrigation of 1.7 million ha in the Euphrates and Tigris river basin [Tilmant et al., 2007]. Initiated in 1991, the project involves 9.5% of the Turkish territory [ILISU Engineering Group, 2001].

In this study, only the largest reservoirs and hydropower plants on the Euphrates are considered. The localization of these infrastructures is presented on figure 3.8 while their key characteristics are listed in table 3.4. The topology of the hydro-system is depicted on figure 3.10. The Keban dam is the first of the cascade: with the largest active storage capacity, it has been designed to regulate the seasonal fluctuations of the river and expect to produce annually around 6000 GWh of energy. Located 166 km downstream, Karakaya has the same objectives as Keban: stream flow regulation and hydropower generation (1800 MW installed and expected annual production around 7500 GWh). The Atatürk dam is the master piece of the GAP. Its storage capacity (48.7 km$^3$) can control almost two years of the Euphrates discharge at the dam location. The hydropower station is the largest of the river basin (2400 MW) and the reservoir aims to provide seasonal regulation to irrigate more than 1 million hectares (in majority cotton and wheat) in the plain upstream the Syrian border.
located hundreds of kilometers downstream the dam, via the Ufra tunnel. The diverted water will therefore not be available for the cascade of downstream power stations in Turkey (Ataturk, Birecik and Karkamis), representing 3250 MW. Birecik and Karkamis, located at the border between Turkey and Syria, are operated as run-of-river plants to produce annually 2500 GWh and 800 GWh respectively [Kolars and Mitchell, 1994].

Data on irrigation water usage (gross water requirement - GWR) in the Euphrates river basin are taken from Beaumont [1998], Kliot [1994] and Kolars and Mitchell [1994] who suggest 10000 to 12000 m$^3$.ha$^{-1}$.y$^{-1}$. These figures have been confirmed and disaggregated to monthly requirement using the irrigation model CROPWAT and the climatic database CLIMWAT [Allen et al., 1998] by Tilmant et al. [2007]. The cropping pattern consists in majority in wheat and cotton. Most of the irrigation water withdrawals take place from April to July and May to November for wheat and cotton respectively, as illustrated in figure 3.11 and 3.12.
3.2. The Euphrates river basin

Table 3.4: Major infrastructures: Hydropower plants and reservoirs (TK = Turkey, SY = Syria). Source: Kolars and Mitchell [1994] and Alia [2007].

<table>
<thead>
<tr>
<th>Name</th>
<th>Live storage [hm$^3$]</th>
<th>Capacity [MW]</th>
<th>Lateral irrigation yes/no</th>
</tr>
</thead>
<tbody>
<tr>
<td>Keban (TK)</td>
<td>13 926</td>
<td>1 240</td>
<td>no</td>
</tr>
<tr>
<td>Karakaya (TK)</td>
<td>4 691</td>
<td>1 800</td>
<td>no</td>
</tr>
<tr>
<td>Ataturk (TK)</td>
<td>19 300</td>
<td>2 400</td>
<td>yes</td>
</tr>
<tr>
<td>Birecik (TK)</td>
<td>583</td>
<td>670</td>
<td>yes</td>
</tr>
<tr>
<td>Karkamis (TK)</td>
<td>0 (run-of-river)</td>
<td>180</td>
<td>no</td>
</tr>
<tr>
<td>Tishreen (SY)</td>
<td>0 (run-of-river)</td>
<td>630</td>
<td>no</td>
</tr>
<tr>
<td>Tabqa (SY)</td>
<td>4 400</td>
<td>800</td>
<td>yes</td>
</tr>
</tbody>
</table>

Figure 3.11: Crop water requirements (CWR) in the Euphrates river basin [Tilmant et al., 2007; Alia, 2007].

Figure 3.12: Cumulated crop water requirements (CWR) in the Euphrates river basin [Tilmant et al., 2007; Alia, 2007].
3.2.3 The Syrian system

The first infrastructure downstream the border is the Tishreen run-of-river hydropower station (Fig. 3.10). Then, the Tabqa scheme constitutes the cornerstone the Syrian hydro-system. With the construction of the Tabqa dam in the 1970s, Syria planned to develop considerably irrigation along the Euphrates. The Tabqa reservoir has a storage capacity of 12 km$^3$ but due to its geographical location and the large area of the lake (Lake Assad, maximum area of 610 km$^2$), the annual evaporation losses are significant (around 1.5 km$^3$y$^{-1}$). The installed capacity of the hydropower plant is 800 MW and the annual energy generation target is 1600 GWh. Irrigation and hydropower generation are the main operating objectives but the priority is given to the irrigation while hydroelectricity is produced only during peak energy demands [Alia, 2007].

Whereas only 280 000 ha of irrigated lands were developed by the mid 1980s [Kolars and Mitchell, 1994], considerable developments were observed since the early 1990s. Beaumont [1996] and Beaumont [1998] estimates an irrigated area around 475 000 ha by the year 2000. The cropping pattern is mainly composed of a mix of cotton and wheat. The crop water requirement, estimated using the CROPWAT model [Allen et al., 1998], is illustrated in figure 3.11. The seasonal CWR pattern is the same as Turkey but the requirements are slightly higher given the arid climate that characterizes the region in Syria.
Part II

Case studies analysis
Chapter 4

Impacts of the variable productivity of hydropower plants on the allocation decisions in the Eastern Nile river basin

4.1 Outline

Stochastic Dual Dynamic Programming (SDDP) is one of the few solution methods available to solve multipurpose multireservoir operation problems in a stochastic environment. As explained in chapter 2, this algorithm requires that the one-stage optimization problem be a convex program so that the efficient Benders decomposition scheme can be implemented to handle the large state-space that characterizes multireservoir operation problems. When dealing with hydropower systems, one usually assumes that the production of hydro-electricity is dominated by the release term and not by the head (storage) term to circumvent the non-linearity of the hydropower production function. Although this approximation is satisfactory for high head power stations where the difference between the maximum and minimum head is small compared to the maximum head, it may no longer be acceptable when a significant portion of the energy originates from low and/or medium head power plants.

The theoretical developments to improve the representation of the non-linear hydropower function through a convex hull approximation have been presented in chapter 2. In this chapter, a network of hydropower plants and

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1 This chapter is adapted from: Goor, Q.; Kelman, R. & Tilmant, A., Optimal multipurpose-multireservoir operation model with variable productivity of hydropower plants in Journal of Water Resources Planning and Management, ASCE, (accepted) 2010.
irrigated areas in the Eastern Nile river basin is used to illustrate the impacts of the variable productivity of hydropower plants on the allocation decisions.

4.2 Methodology

4.2.1 SDDP models for the Eastern Nile river basin

Two SDDP formulations are compared: the first one considers a variable productivity of hydropower plants (convex hull approximation), while the second one assumes that the production of hydroelectricity is governed by the release term through a production coefficient. In this study, the production coefficient is defined by the ratio between the installed capacity of the hydropower plant and its maximum turbining capacity. To illustrate the difference between the two SDDP formulations (production coefficient versus convex hull), a system of 7 reservoirs in the Eastern Nile River basin is optimized. Both formulations provide, among other things, optimal release and storage decisions for each power plant in the system. In this chapter, the analysis will concentrate on the High Aswan Dam because (1) a substantial amount of energy comes from the storage term (Fig. 4.1), and (2) it is the largest existing infrastructure in the basin, both in terms of storage and installed capacity. The two SDDP formulations are thoroughly detailed in chapter 2.

Figure 4.1: Head versus storage relationship - High Aswan dam reservoir.
4.2. Methodology

The network of reservoirs, hydropower stations and irrigated areas considered in this chapter corresponds to the current (or close to current) situation in the Eastern Nile river basin. The topology of the system is depicted on figure 4.2 and detailed in table 4.1.

Table 4.1: Detailed description of the current (or close to current) situation of the Eastern Nile river basin hydro-system.

<table>
<thead>
<tr>
<th>Infrastructures</th>
<th>Ethiopia</th>
<th>Tis Abbay I&amp;II Tana-Beles link TK-5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Sudan</td>
<td>Roseires</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Sennar</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Jebel Aulia</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Kashm El Girba</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Merowe</td>
</tr>
<tr>
<td></td>
<td>Egypt</td>
<td>High Aswan</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Old Aswan</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Esna</td>
</tr>
<tr>
<td>Installed capacity [MW]</td>
<td>4933</td>
<td></td>
</tr>
<tr>
<td>Irrigated area [10^6 ha]</td>
<td>Ethiopia</td>
<td>~0</td>
</tr>
<tr>
<td></td>
<td>Sudan</td>
<td>1.62</td>
</tr>
<tr>
<td></td>
<td>Egypt</td>
<td>5.68</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>7.30</td>
</tr>
</tbody>
</table>

4.2.2 Model parameters and assumptions

**Model.** Given the multi-year storage capacity of the system, a 5 years ahead planning horizon is used (\(T=60\) month). Twenty backwards openings (\(K=20\)) are set-up and the forward simulation is carried out on 50 synthetic hydrological scenarios (\(M=50\)). For each reservoir, a 21 years long historical record (from 1980 to 2000) of lateral inflows was available to estimate the parameters of the build-in multi-site PAR\((p)\) hydrological model. As a consequence, we make the assumption that historical weather patterns are representative of possible future conditions.

**Economic costs and values.** For hydropower, we consider a seasonal short-run marginal cost (SRMC) averaging 80US$/MWh which remains identical throughout the countries [Whittington et al., 2005; Blackmore and Whittington, 2009]. Given the lack of accurate data about irrigated agriculture in the basin, we consider flat demand curves for irrigation water withdrawals with a net return of 0.05 US$/m^3, which is the same assumption as in Whittington et al. [2005]; Blackmore and Whittington [2009]. This value is consistent with international experience.
Figure 4.2: Topological view of the Eastern Nile hydro-system - situation close to current.
Hydropower functions. The discretization of the feasible domain of the storage and the release (i.e. $U$ and $V$ in equation (2.37)), will obviously impact the accuracy of the convex hull approximation of the true power function. Results for the High Aswan Dam (HAD) power station are illustrated on figure 4.3. This figure shows the relationship between (i) the discretization of the feasible domain of the storage and the release ($U,V$), (ii) the root mean square error (RMSE) of the convex hull approximation of the true hydropower function (in terms of percentage of the installed capacity), and (iii) the resulting number $H$ of hyperplanes of the convex hull approximation.

For the HAD hydropower station, the feasible domains of the storage and the release are both equally discretized in 11 values ($U = V = 11$), building a grid of 121 points where the true hydropower function is calculated. The approximation shows a RMSE of 53.77 MW, which represents less than 3% of the installed capacity. The number $H$ of hyperplanes that approximate the true hydropower function is 27; they represent the additional constraints (2.54) to the one-stage SDDP problem (2.46) to (2.55). The adjustment coefficient $\alpha$ [-] that multiplies the approximated hydropower is equal to 0.984.

In order to support and justify the adoption of the new SDDP formulation, the figure 4.4 represents, for the High Aswan Dam hydropower station, a plot between the storage and the difference in net power generation calculated for different release rates, with a variable and with a constant productivity. The reference case being the hydropower generation calculated with the true hydropower function. The figure clearly shows that considering a constant produc-
tivity of hydropower plant overestimates the true power generated, particularly for low pool elevation and high release rates. On the other hand, the convex hull approximation of the true hydropower function exhibits its higher error for medium storage level but it remains much smaller than the formulation with constant production coefficient.

Figure 4.4: Relationship between the storage and the difference in net power generation calculated with (1) the constant productivity coefficient (upper part of the figure) and (2) the convex hull approximation (lower part of the figure), for different release rates - High Aswan dam hydropower station.
4.3 Results Analysis

4.3.1 Convergence of the algorithm

The upper part of figure 4.5 illustrates the convergence of the SDDP algorithm, for both formulations, while the CPU time and the difference between the calculated lower bound $Z_{lo}$ (calculated during the forward simulation) and upper bound $Z_{up}$ (calculated during the backward optimization) on the expected benefits from system operation $Z$, are shown in the lower part of the figure. The SDDP algorithm with production coefficient (left part of the figure) converges after 4 iterations whereas 5 iterations are needed with the variable productivity of hydropower plants (right part of the figure). The CPU time required was 45 and 107 seconds respectively on a Intel® Centrino™ 1.86GHz machine with 1GB of RAM running GNU-Linux. Even though the new SDDP formulation is more than 2 times slower, the computational time remains fairly low, making it possible to imbed SDDP in decision support systems [Loucks and van Beek, 2005].

![Figure 4.5: Convergence of the SDDP algorithm with production coefficient (left) and with variable productivity of hydropower plants (right).](image)

4.3.2 Hydropower generation

Figure 4.6 compares the empirical cumulative density function (CDF) of annual energy generation at the High Aswan Dam hydropower station for the 2 different SDDP formulations. The empirical cumulative density function of a random variable $X$, defined as $F(X) = P(X \leq x)$, represents the probability that the random variable $X$ takes a value less than or equal to $x$. The annual production of hydroelectricity ranges from around 8 TWh for both formulations to 13 TWh and 15 TWh if a variable productivity or a production coefficient are respectively implemented. With a constant production coefficient, the power generation is only turbining dependant. It is therefore not surprising that, on
average, energy generation is higher since there is no reduction in power due to the head effect. In that case, the maximum power is reduced if the water level in the reservoir decreases and the simulated energy generation is around the observed historical average of 9,000 GWh/year. As a matter of fact, we can observe that the risk of not meeting the planned annual energy generation of 10,000 GWh/year increases from 21% to 68% if the geometry of the reservoir is considered.

Moreover, figure 4.6 shows the calculated energy that would be generated based on the storage and release terms estimated by both SDDP formulations. While the difference of energy generation estimated (1) by the formulation with variable productivity and (2) from the storage and release terms of the same SDDP model are very close, it is not the case for the model considering a constant production coefficient. In that case, hydropower is systematically overestimated and the difference ranges from 0 to 3.6 TWh per year.

![Empirical cumulative distribution function (CDF) of energy generation at High Aswan Dam power station.](image)

**4.3.3 Drawdown-refill cycles**

Figure 4.7 shows the boxplots of the simulated monthly pool elevation in the HAD reservoir. The behaviour of the water level in the reservoir is completely different according to the model implemented. The lower part of the figure is related to the SDDP formulation with the production coefficient while the upper part deals with SDDP formulation with the variable productivity. Water level is kept much higher in the reservoir when the production is head dependent as there are now incentives to keep high head on turbines. Assuming (1)
the HAD reservoir is at its lowest water level and (2) a constant production coefficient defined as the ratio between the installed capacity and the maximum turbining, the hydropower capacity is overestimated up to 30% compared to the reality (considering the hydraulic head).

Figure 4.7: Boxplots of simulated monthly storage of High Aswan Dam reservoir.

4.3.4 Evaporation losses

Empirical CDF of evaporation losses are depicted on figure 4.8 for both SDDP formulations. One can observe that when the production of hydroelectricity is only turbining dependent, the low water level implies lower evaporation losses. On average, evaporation losses are higher by 4.3 BCM.year\(^{-1}\) when head-dependant productivity is taken into account; they can be as low as 10,000 hm\(^3\)/yr during dry years, and as high as 14,000 hm\(^3\)/yr during wet years. The above-mentioned 30% increase in hydropower production seems to counterbalance the higher evaporation losses.

4.3.5 Consequences for multipurpose projects

In addition to the lower revenues for the hydropower sector, inaccurate representation of hydropower generation might lead to the inexact estimation of evaporation losses and water levels in the reservoir. This can have serious impacts on the performance of the system, especially for multipurpose reservoirs where optimal operation seeks to allocate water so as to maximize aggregated
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Chapter 4. Variable productivity of hydropower plants

Figure 4.8: Empirical cumulative distribution function (CDF) of annual evaporation losses from Lake Nasser.

Figure 4.9 represents the empirical CDF of percentage of supplied irrigation water demand in the Nile delta, downstream HAD. The left side of the graph corresponds to relatively dry conditions, where rationing can occur. If hydrologic conditions are rather dry, the risk of not supplying 90% of the irrigation water requirements in the Nile delta is 13% with the SDDP model with production coefficient, while it is reduced to 2% if variable productivity of hydropower plant is considered. With the first model, the water level is on average kept so low (upper part of Fig. 5.3) that if a dry year occurs, downstream irrigation demand cannot be fully met. However, as evaporation losses increase with water level in the reservoir, downstream supplied irrigation water demand is on average higher with SDDP model with production coefficient given the evaporation savings.

4.4 Conclusion

In this chapter, the consequences of one of the key assumptions on which SDDP relies (the simplified linear hydropower production function) has been investigated. One of the main findings is that the non-linearities inherent to the hydropower function can be approximated in SDDP by piecewise linear functions, identified by the Quick Hull algorithm. This more accurate representation of the hydropower functions results in larger one-stage SDDP optimization problems and slightly increases computation time. However, the computational
Figure 4.9: Empirical cumulative distribution function (CDF) of percentage of supplied irrigation water demand in the Nile delta.

time remains fairly low, making it possible to imbed SDDP in decision support systems. Taking the head effect into account changes the reservoir operating policies which in turn impact not only the production of energy but also the availability of water in the system. For example, it is shown that the risk for the High Aswan Dam of not meeting its planned annual energy generation increases from 21% to 68% if the geometry of the reservoir is considered. The results demonstrate that considering variable productivity of hydropower plants is of utmost importance in hydropower systems such as the Eastern Nile river basin.
Chapter 5

Optimal operation of a multipurpose multireservoir system in the Eastern Nile river basin

5.1 Outline

The upper Blue Nile river basin in Ethiopia is a largely untapped resource despite its huge potential for hydropower generation. Controversies exist as to whether the numerous infrastructural development projects that are on the drawing board in Ethiopia will generate positive or negative externalities downstream in Sudan and Egypt. This study attempts at (i) examining the (re-)operation of infrastructures, in particular the proposed reservoirs in Ethiopia and the High Aswan dam and (ii) assessing the economic benefits and costs associated with the storage infrastructures in Ethiopia and their spatial and temporal distribution. To achieve this, a basin-wide integrated hydro-economic model has been developed. The model integrates essential hydrologic, economic and institutional components of the river basin in order to explore both the hydrologic and economic consequences of various policy options and planned infrastructural projects. Unlike most of the deterministic economic-hydrologic models reported in the literature, a stochastic programming formulation has been adopted in order to: (i) understand the effect of the hydrologic uncertainty on management decisions, (ii) determine allocation policies that naturally hedge against the hydrological risk, and (iii) assess the relevant risk indicators. The study reveals that the development of four mega dams in the upper part of the Blue Nile basin would change the drawdown refill

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1 This chapter is adapted from: Goor, Q.; Halleux, C.; Mohamed, Y. & Tilmant, A. Optimal operation of a multipurpose multireservoir system in the Eastern Nile river basin in *Hydrology and Earth System Sciences.*, 14, 1-14, 2010 (doi:10.5194/hess-14-1-2010)
Chapter 5. Optimal operation of the ENB hydro-system

cycle of the High Aswan dam. Should the operation of the reservoirs be coordinated, they would enable an average annual saving of at least 2.5 billion cubic meters through reduced evaporation losses from the Lake Nasser. Moreover, the new reservoirs (Karadobi, Beko-Abo, Mandaya and Border) in Ethiopia would have significant positive impacts on hydropower generation and irrigation in Ethiopia and Sudan: at the basin scale, the annual energy generation is boosted by 38.5 TWh amongst which 14.2 TWh due to storage. Moreover, the regulation capacity of the above mentioned reservoirs would enable an increase of the Sudanese irrigated area by 5.5%.

5.2 Introduction

The Nile river basin covers an area representing one tenth of Africa (about 3 million km$^2$) and is shared by ten countries. The river is characterized by a considerable seasonal and inter-annual variability that challenges the management of the water resources. In the top of that, the water resources availability and uses are unevenly distributed amongst the countries: Egypt and Sudan are the largest water consumers while this is negligible for Ethiopia, even though 85% of the Nile waters comes from Ethiopian highlands. To meet the growing demand for food and energy, the Nile riparian countries will further develop their water resources. For example, in the Blue Nile River Basin, between lake Tana in Ethiopia and Karthoum in Sudan, large reservoirs, hydropower stations and irrigation areas are being planned with the ultimate goal of boosting the production of cheap hydroelectricity and increasing food security [Guariso and Whittington, 1987; Block, 2007; Block and Strzepek, 2010; Georgakakos, 2006; Nile Basin Initiative, May 2010a]. Due to the fugitive nature of water, those developments will generate both positive and negative externalities downstream and must therefore be carefully planned, ideally in a cooperative way with downstream riparians. This is precisely the raison d’etre of the Nile Basin Initiative (and the associated two Subsidiary Action programmes (SAPs)), an international institution expected to provide a framework for basin-wide cooperation including the identification and implementation of new infrastructural projects [Nile Basin Initiative, May 2010b].

Assessing the positive and negative externalities of infrastructural projects calls for integrated basin-wide modelling studies. Integrated basin-wide models are typically built around arcs and nodes: the former may represent natural inflows to the system, canals, the river network, whereas the nodes are used to represent confluences, reservoirs, abstraction points, demand sites, etc [Harou et al., 2009]. Ringler et al. [2004] analyze the optimal flow allocation in the Mekong river basin using an integrated economic-hydroplogic model. In a series of papers, Ward and Michelsen [2002]; Ward et al. [2006] investigate the hydrologic and economic impacts of various policy options in the Rio Grande basin using a hydro-economic model. In Whittington et al. [2005], a deterministic hydro-economic model was developed for the entire Nile River Basin and several development scenarios were analyzed. Their study revealed, amongst other things, that the annual benefits of cooperation between the Nile countries can be as high as 4.9 billion US$/y. In Georgakakos [2006], a DSS is developed
for the Nile river basin: the model can simulate the Nile response to different hydrological scenarios and provide reservoir operating strategies for real-time control through the Extended Linear Quadratic Gaussian (ELQG) optimization algorithm. Recently, Block and Strzepek [2010] analyzed the transient conditions associated with the period of filling some of the proposed dams in Ethiopia, under climate change scenarios.

In this study, unlike most of the deterministic economic-hydraulic models reported in the literature, a stochastic programming formulation has been adopted for mid- to long-term water resources planning and management of the Nile river basin. The objectives of the study presented in this chapter is to assesses (1) the (re-)operation of the largest hydraulic infrastructures, in particular the proposed reservoirs in Ethiopia and the High Aswan dam and (2) the economic benefits and costs associated with new storages in Ethiopia.

5.3 Methodology

5.3.1 SDDP model for the Eastern Nile river basin

The basin-wide water resources allocation model relies on SDDP, an algorithm that can solve large-scale stochastic optimization problems, as described in chapter 2. The model determines economically efficient allocation policies, including reservoir releases, and then simulates the system operation for various hydrological scenarios. The SDDP formulation adopted in this study considers a dynamic allocation of water resources between hydropower and irrigation [Tilmant et al., 2009]. In this approach, water resources are allocated to its most productive use throughout the entire river basin without any political or legal constraints. The model seeks to maximize the aggregated net benefits from both the irrigation and hydropower sectors by identifying optimal release and irrigation withdrawal decisions at each node of the hydro-system and for each stage of the planning period.

Since most of the infrastructures considered in the analysis are in the planning phase, a dynamic management approach allows us to determine optimal (economically efficient) irrigation developments/extensions within the river basin.

5.3.2 Scenarios description

Four scenarios were analyzed using the above mentioned stochastic hydro-economic model. Each scenario is characterized by an installed capacity and an irrigated area (table 5.1).
Table 5.1: Detailed description of the scenarios.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>S1</th>
<th>S2</th>
<th>S3</th>
<th>S4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ethiopia</strong></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Infrastructures</td>
<td></td>
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<tr>
<td>Tana-Beles link</td>
<td>Lake Tana</td>
<td>Lake Tana</td>
<td>Lake Tana</td>
<td>Lake Tana</td>
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<tr>
<td>Tis Abbay I&amp;II</td>
<td>Tis Abbay I&amp;II</td>
<td>Tis Abbay I&amp;II</td>
<td>Tis Abbay I&amp;II</td>
<td>Tis Abbay I&amp;II</td>
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<td>TK-5</td>
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<tr>
<td>Mandaya</td>
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<tr>
<td>Karadobi</td>
<td>Karadobi</td>
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<td>Beko-Abo</td>
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<td>Beko-Abo</td>
<td>Beko-Abo</td>
<td>Beko-Abo</td>
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<tr>
<td>Border</td>
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<tr>
<td><strong>Sudan</strong></td>
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<tr>
<td>Roseires</td>
<td>Roseires</td>
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<td>Sennar</td>
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<td>Jebel Aulia</td>
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<td>Merowe</td>
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<td><strong>Egypt</strong></td>
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<td>High Aswan</td>
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<td>Old Aswan</td>
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<tr>
<td>Installed capacity [MW]</td>
<td>4933</td>
<td>6933</td>
<td>11833</td>
<td>11833</td>
</tr>
<tr>
<td>Irrigated area [10^6 ha]</td>
<td><strong>Ethiopia</strong></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>~0</td>
<td>0.02</td>
<td>0.15</td>
<td>0.15</td>
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<tr>
<td></td>
<td><strong>Sudan</strong></td>
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<tr>
<td></td>
<td>1.62</td>
<td>1.74</td>
<td>2.12</td>
<td>2.12</td>
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<tr>
<td></td>
<td><strong>Egypt</strong></td>
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<tr>
<td></td>
<td>5.68</td>
<td>5.71</td>
<td>5.90</td>
<td>5.90</td>
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<tr>
<td></td>
<td><strong>Total</strong></td>
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<tr>
<td></td>
<td>7.30</td>
<td>7.74</td>
<td>8.10</td>
<td>8.10</td>
</tr>
</tbody>
</table>
5.3. Methodology

The first scenario (S1) corresponds to the current situation (base line scenario). In this scenario, no irrigation takes place in the Blue Nile catchment in Ethiopia while 1.62 million hectares are irrigated in Sudan (mainly downstream Roseires and around the Sennar dam). In Egypt, in addition to the cultivated area in the delta, we consider that 20000 hectares are operational in the New Valley project.

The second scenario (S2) corresponds to the situation around 2025, with the most likely infrastructure to be built on the Blue Nile (Mandaya reservoir and hydropower plant) and a 10 % increase of irrigation water demand in Sudan. In Egypt, it is assumed that the cultivated area remains unchanged in the delta while 50000 ha are irrigated in the New Valley project. It is also considered that, in Ethiopia, 20000 hectares in the Tana-Beles irrigation scheme are operational.

The third scenario (S3) is defined by the full development of the basin: the four mega dams in Ethiopia are implemented. We make the assumption that the full irrigation potential in the New Valley project (Egypt), around Sennar and Roseires (Sudan) and in the Tana-Beles irrigation scheme (Ethiopia) are operational.

The fourth scenario (S4) is imaginary: it is the same as the third one except that storage hydropower plants are replaced by run-of-river ones. In other words, S4 considers that there is no regulation capacity in Ethiopia (table 5.1).

The comparison of the three first scenarios will evaluate the impacts of upstream development on the allocation decisions and reservoirs operating strategies (first objective) while the comparison of the third and fourth scenarios will assess the economic value of regulation (storage) in Ethiopia (second objective). The scenarios are detailed in table 5.1 while the topology of the hydro-system is depicted on figure 3.4 in chapter 3.

5.3.3 Model parameters and assumptions

Since the model solves the water resources allocation problem with a monthly time step with a planning horizon over one year, we are dealing with mid-to long-term hydro-scheduling. In that context and given the over-year storage capacity of the hydro-system, a 10 years ahead planning horizon is used ($T=120$ month). Thirty backwards openings ($K=30$) are set-up and the forward simulation is carried out on 30 synthetic hydrological scenarios ($M=30$). For each reservoir, a 47 years long historical record (from 1953 to 2000) of lateral inflows was available to estimate the parameters of the build-in multi-site periodic autoregressive hydrological model. As a consequence, we make the assumption that historical weather patterns are representative of possible future conditions.

The model described in chapter 2 assumes the coordinated operation of all the infrastructures of the hydro-system. This implies, among other things, the existence of an institutional framework to ensure a basin-wide management of the system. In this study, we assume that the system is in steady state conditions and we do not consider the cost of building the infrastructures (considered as sunk cost). Block and Strzepek [2010] analyzed the transient conditions associated with the period of filling the reservoirs, under climate change scenarios.
Chapter 5. Optimal operation of the ENB hydro-system

Given the lack of accurate economic information about irrigated agriculture in the basin, we made the following assumptions. We consider flat demand curves for irrigation water withdrawals with a net return of 0.05 US$/m³, which is the same assumption as in Whittington et al. [2005]; Blackmore and Whittington [2009]. This value is consistent with international experience. For hydropower generation, we consider a seasonal SRMC averaging 80US$/MWh and identical throughout the countries of the region [Whittington et al., 2005; Blackmore and Whittington, 2009].

5.4 Results and analysis

5.4.1 Major reservoirs drawdown-refill cycles

The drawdown-refill cycles of the cascade of planned infrastructures on the Blue Nile in Ethiopia (S3) are illustrated in figure 5.1. Karadobi, the first reservoir of the cascade, fully exploits its storage capacity: the pool elevation decreases during the dry season while the reservoir fills-up during the wet season. With an active storage capacity of 17 km³ and an average annual reservoir inflows estimated around 24.1 km³ y⁻¹, Karadobi regulates around 70% of its natural inflows and therefore controls the water availability for the rest of the cascade. Downstream of Karadobi, the Beko-Abo reservoir controls the spills of Karadobi and the relatively small contribution of its sub-basin. As a consequence, the Beko-Abo reservoir exploits only the upper part of its storage capacity in order to maintain a high pool elevation that increases the productivity of its hydropower plant, which is the largest of the cascade (table 3.2). The operation of the Mandaya reservoir differs depending on S2 or S3 (Fig. 5.2). In S2, Mandaya is the only large infrastructure on the Blue Nile in Ethiopia. The reservoir is therefore operated so as to control the large inflows of its upstream sub-basins. On the other hand, the regulating role of the Mandaya reservoir decreases as other reservoirs are built upstream (S3), because the flow fluctuations are much smaller thanks to these reservoirs. Considering the full development scenario (S3), the increased upstream regulation allows Mandaya to be operated both at higher reservoir levels and with reduced spill. Downstream of the Mandaya reservoir and hydropower station, the discharge of the Blue Nile has already been regulated by the upstream reservoirs. The role of the Border dam, the latest before the border between Ethiopia and Sudan, is therefore to control the seasonal flow of its sub-basins. The reservoir exploits one third of its active storage capacity, which represents a trade-off between regulation capacity and reduced head on turbines. Building new storage facilities in Ethiopia would impact the management strategies of downstream infrastructures. Figure 5.3 illustrates the drawdown-refill cycles of the High Aswan Dam reservoir, for the different scenarios. With any regulation capacity in the upper part of the Blue Nile basin (S1 and S4), the water level in the reservoir decreases during the low flow season while the reservoir fills-up during the flood season. The drawdown-refill cycles vary seasonally according to the downstream water demand for irrigation purposes in the Nile delta. The lower pool elevation observed in S4 compared to S1 is explained by the higher water
Figure 5.1: Boxplots of the drawdown-refill cycles of the four multipurpose reservoirs in Ethiopia (Blue Nile) - scenario 3 (S3).
withdrawals for irrigation in Sudan and higher evaporation losses in Ethiopia. On the other hand, for the second and third scenario, the drawdown-refill cycles computed by the optimization model are reduced and the reservoir is operated at a much lower water level, especially in S3. The reason is that the flow has already been regulated by new Ethiopian infrastructures. Lower inflows and lower pool elevations will impact hydropower generation and evaporation losses in Egypt.

5.4.2 Evaporation losses

Box plots of annual evaporation losses from man-made reservoirs, obtained by the optimization model, are displayed in figure 5.4. Currently (S1), the Nile waters are regulated and stored in the High Aswan Dam reservoir, located at the border between Egypt and Sudan, and characterized by a very arid climate. It is therefore not surprising to observe evaporation losses ranging from 10.8 to 13.6 km$^3\cdot y^{-1}$. We can observe in figure 5.4 that the evaporation losses are reduced as more water is stored and regulated upstream in the basin (moving from S1 to S3). On average, the basin-wide water savings reach 2.5 km$^3\cdot y^{-1}$. This is due to the equatorial location of the Ethiopian reservoirs with lower temperatures and higher precipitations. Moreover, at full supply level (FSL), the cumulated impounded area of the Ethiopian reservoirs would represent 38% of the Lake Nasser’s. Collectively, Ethiopian reservoirs represent 66% of Lake Nasser’s potential in terms of active storage capacity and present therefore a great potential in terms of storage and flow regulation.
5.4. Results and analysis

![Boxplots of the drawdown-refill cycles of the High Aswan Dam reservoir, for each scenario.](image)

Figure 5.3: Boxplots of the drawdown-refill cycles of the High Aswan Dam reservoir, for each scenario.
Figure 5.4: Box-plots of annual evaporation losses, for the major countries of the basin and for each scenario.
5.4. Results and analysis

5.4.3 Hydrological risk

Fig. 5.5 displays the statistical distributions of the annual flows at key locations in the river basin: (1) at the Sudanese and Ethiopian border, (2) at the Sudanese and Egyptian border, which represents the Lake Nasser inflows and (3) the High Aswan Dam releases. The three empirical cumulative distribution functions (CDF) available at each site give the non-exceedance probability of any given annual flow for the four scenarios.

As we will see later, the large storage capacities in Ethiopia would further increase irrigation withdrawals primarily in Sudan, where the productivities of irrigation districts can compete with that of downstream power stations (Merowe and HAD). Expanding crop irrigation in Ethiopia does not appear to be economically attractive as farmers are facing a coalition of downstream productive uses (a cascade of hydropower plants and irrigated agriculture in the delta) that prevent the expansion of consumptive uses upstream by attracting as much water as possible downstream in Sudan and Egypt.

The limited increase in irrigation withdrawals in Ethiopia and the fairly low evaporation losses from the proposed reservoirs are shown on Fig. 5.5-c where we can see that the CDF of annual flows at the Ethiopian/Sudanese border for the second and third scenario is not significantly different from those of scenario 1. Fifty percent of the time, the annual discharge at the border will be greater than 49.5 km$^3$.y$^{-1}$, whatever the scenario is.

According to the 1959 bilateral agreement between Egypt and Sudan, the annual discharge into the Lake Nasser must be 65.5 km$^3$.y$^{-1}$ (55.5 km$^3$.y$^{-1}$ for Egypt and 10 km$^3$.y$^{-1}$ for evaporation losses at the Lake Nasser). We can see that the risk of not meeting the annual Egyptian allocation of 65 km$^3$ (hydrological risk) will decrease from 23 to about 20% when the major storage and irrigation infrastructures will be operational (S2 and S3 - Fig. 5.5-b). Finally, Fig. 5.5-a illustrates that one should not downplay the role of HAD when the Ethiopian and Sudanese infrastructures will be operational; with its over-year storage capacity, HAD nullifies the cross-border hydrological risk by transferring water from wet to dry years, therefore preserving the reliability of supply to Egypt. In other words, Egypt still receives its annual allotment of 55.5 km$^3$.

Building the proposed infrastructures in Ethiopia would have significant impacts on the flow regime of the Nile. Figure 5.6 illustrates, for the thee first scenarios, the average monthly discharge of the Blue Nile at the border between Ethiopia and Sudan. The flow is decomposed into spillage and turbining from the immediately upstream power station and the natural inflow from the sub-basins. The first scenario is characterized by no regulation of the Blue Nile in Ethiopia. As more infrastructures are being implemented (S2 and S3, Fig. 5.6-b and Fig. 5.6-c respectively), reservoirs have the ability to move water from the wet to the dry season. The flood peak observed in figure 5.6-a (S1) from July to October is reduced by about one third and the discharge is much higher during the low flow season. Consequently, less frequent and reduced floodings, particularly in Sudan but also downstream would occur.
Figure 5.5: Empirical cumulative distribution function (CDF) of the annual discharge at key locations. (a) High Aswan Dam outflows, (b) High Aswan dam inflows and (c) Sudanese - Ethiopian border.
5.4. Results and analysis

![Graphs showing average monthly discharge downstream the cascade of Ethiopian reservoirs.](image)

Figure 5.6: Average monthly discharge downstream the cascade of Ethiopian reservoirs (Sudanese / Ethiopian border).
5.4.4 Hydropower generation

Boxplots of the annual hydropower generation, for each scenario and for each country are depicted in figure 5.8. As mentioned earlier, with both lower inflows and pool elevations, the production of hydroelectricity from HAD in Egypt would be reduced by 9% in S3 compared to S1. On the other hand, partial development of the basin (S2) would have no significant impact on Egyptian hydropower generation. Obviously, Ethiopia will be net beneficiary with an average increase of 469% and 1423% of annual energy generated from S2 and S3 respectively, and become therefore the largest hydroelectric producer of the Eastern Nile Basin. Sudan would also benefit from the upstream infrastructures: reduced spillage makes more water available for hydropower generation.

![Figure 5.7: Stacked monthly average energy generated by the major power plants throughout the basin - scenario 3.](image-url)
5.4. Results and analysis

Figure 5.8: Box-plots of annual hydropower generation, for the major countries of the basin and for each scenario.
Figure 5.7 illustrates, for the full development scenario (S3), the monthly average energy generated by the major hydropower plants throughout the basin. The temporal distribution is coherent with the hydrology and the seasonality of reservoirs releases. On the other hand, the wet season is characterized by lower energy values and the reservoirs operators are consequently keen to store water to release it during the next dry season, when it becomes more valuable.

By analyzing the difference between S3 and S4, we can assess the added value of the regulation capacity of the proposed reservoirs located in Ethiopia. We can see on figure 5.8 that, moving from S1 to S3 would increase the hydropower generation in Ethiopia by + 40 TWh (+1666%), amongst which 14.3 TWh due to storage (table 5.2). At the basin-scale, the annual production of hydroelectricity is boosted by + 38.5 TWh (+163%) amongst which 14.2 TWh due to the regulation capacity of Ethiopian reservoirs. Positive impacts are also observed for Sudan where less spillage occurs. On the other hand, less power is generated in Egypt. Figure 5.9 analyzes the temporal distribution of the benefits from upstream regulation on hydropower generation, by illustrating the monthly difference between S3 and S4. The figure reveals that, during the low flow season, Ethiopia and Sudan take advantage from the regulation. The main beneficiary is obviously Ethiopia. We can also observe that the production of hydroelectricity is differed from the wet season to the dry season, when the SRMC of the hydrothermal electrical system to which hydropower plants contribute are increasing.

5.4.5 Irrigation

We saw on Fig. 5.5-b that the development of the upstream part of the basin (S2 and S3) induces a reduction of the flow to Egypt: the annual volume of water crossing the border between Sudan and Egypt would be lower than 65.5 km$^3$ 23% or 20% of the time for S2 and S3 respectively. That reduction essentially comes from increased irrigation withdrawals in Sudan but, thanks to the over-year storage capacity of Aswan, that reduction is not accompanied by a reduction in the reliability of supply; Egypt still receives its annual allotment of 55.5 km$^3 \cdot y^{-1}$ with no risk of failure. The role of the High Aswan dam should therefore not be downplayed, especially when dry or wet years occur. In the third scenario, nearly all potential irrigated areas in Sudan and in Egypt are effectively irrigated. Irrigated agriculture in Sudan therefore benefits from upstream storage in Ethiopia since the Sudanese annual withdrawals are lower in scenarios 1, 2 and 4. Those allocation decisions illustrate that once water has passed through the Ethiopian hydropower plants, irrigated agriculture starts competing with hydropower generation and irrigation withdrawals become more economically sound.

Table 5.2 illustrates the benefits of storage in Ethiopia, on the irrigated agriculture sector. The regulation capacity of the reservoirs located in Ethiopia would increase irrigation water withdrawals by 5.5% in Sudan. No significant impacts is observed for Egypt and Ethiopia.
Table 5.2: Benefits from upstream regulation (Difference between S3 and S4).

<table>
<thead>
<tr>
<th>Country</th>
<th>Hydropower [GWh.y⁻¹]</th>
<th>Irrigation withdrawals [km³.y⁻¹]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ethiopia</td>
<td>+ 14 348 (+50.9%)</td>
<td>+0 (+0%)</td>
</tr>
<tr>
<td>Sudan</td>
<td>+ 956 (+14.3%)</td>
<td>+1.1 (+5.5%)</td>
</tr>
<tr>
<td>Egypt</td>
<td>- 386 (-3.1%)</td>
<td>+0 (+0%)</td>
</tr>
</tbody>
</table>

Figure 5.9: Temporal repartition of benefits (hydropower) from upstream regulation.

5.4.6 Impacts of sediments

Soil erosion of the intensively farmed highlands in the Ethiopian Plateau is a major source of sedimentation in downstream reservoirs. The annual sediment load of the basin is estimated around 140 Mt/y at Roseires [Norplan, Norconsult & Lahmeyer International, 2006]. However, construction of the four mega dams on the Blue Nile within Ethiopia will significantly trap sediment which currently discharges down the Blue Nile in Sudan particularly in the flood season months of July to September. Secondly, the regulated flow will substantially reduce the flood plain area along the rivers in Sudan. Although the impacts of sediments is an important issue, it had not been included in the model of the present study.

5.4.7 Short-run net benefits

Figure 5.10 reveals a huge increase in basin-wide benefits due to the new hydro-power stations in Ethiopia and irrigated areas in Sudan. On average, these infrastructures would increase annual basin-wide benefits by 3.48 billion US$ (+63.8%) or 1.29 billion US$ (+23.7%) for S3 or S2 respectively. Note that the development of run-of-river in lieu of storage hydropower plants in Ethiopia
Figure 5.10: Average annual short-run net benefits from hydropower generation and irrigation.
5.4. Results and analysis

(S4) would also yield significant basin-wide benefits (7.8 billion US$/y). The comparison of basin-wide benefits in S3 and S4 gives how much economic benefits can be obtained from storing water in Ethiopia. The storage services in Ethiopia would, on average, yield an annual extra value of 1.12 billion US$/y, which corresponds to 14.4% of the basin-wide benefits.

5.4.8 Analysis of the dynamic allocation

Remember that the present analysis relies on the assumption that the water resources are allocated using a dynamic allocation approach i.e. water resources are allocated to its most productive use, without any political or legal constraints: at a given site in the hydropower-irrigation system, the decision to release water downstream or withdraw it for irrigation is based on the comparison of the aggregated productivity of the farmers at that site and the sum of productivities of all downstream users at the margin [Tilmant et al., 2009].

To achieve this economically efficient allocation, a distinction must be made between the at-site and at-source water value: the at-source water value is the value of water calculated at the site where it is withdrawn (lake, river, reservoir or aquifer). On the other hand, the at-site water value is the value of water calculated at the site of use (farm, industry). The at-site water value will exceed the at-source value by whatever costs are required to capture, store, transport, and treat the water.

As previously mentioned, the at-site water value for irrigation is assumed around 0.05 US$/m³ [Whittington et al., 2005; Blackmore and Whittington, 2009]. The at-source water value is therefore the at-site water value multiplied by the irrigation system efficiency. For hydropower generation, the at-site water value [US$/m³] is calculated with the characteristics of the hydropower plants and the short-run marginal costs of the hydrothermal system to which the hydropower plant contributes $\pi_h$ [US$/MWh]:

$$\pi_{site} = \frac{\bar{P}}{R_{max} \cdot 3600} \pi_h$$

where $\bar{P}$ [MW] and $R_{max}$ [m³.s⁻¹] are respectively the installed capacity and the maximum turbining capacity of the hydropower plant.

Figure 5.11 and 5.12 compare the average annual marginal at-site water value for the cascade of hydropower plants and irrigated areas along the Blue Nile and the Main Nile. Based on the above mentioned assumptions on the marginal water values for irrigation and hydropower, the main findings of this analysis is that withdraw water from the Blue Nile for irrigation purposes in Ethiopia would not be economically efficient. Indeed, due to the cumulative effect of the downstream productivities for hydropower only (non-consumptive use), the at-site marginal water value for hydropower is always higher that the value for irrigation, for a particular site in the upper reach of the Blue Nile catchment. The reason lies in that each cubic meter of water turbined in Ethiopia is still available downstream in Sudan or Egypt to either generate hydroelectricity or irrigate. On the other contrary, water withdrawn for irrigation purposes in Ethiopia is no more available for downstream uses. More-
over, water withdrawals for the Tana-Beles irrigation scheme would by-pass the Karadobi, Beko-Abo and Mendaya hydropower station, representing 4320 MW. In Egypt, given the expected low efficiency of the irrigation system in the New Valley project, partly due to the high evaporation losses rates, the diversion from the Lake Nasser for irrigation in the Toshka Valley would not appear to be economically relevant. On the other hand, irrigation upstream Jebel Aulia reservoir on the White Nile, seems economically attractive.

However, since several downstream users may benefit from the reallocation of upstream water, we need a benefits sharing mechanism to assess the individual contribution of each beneficiary to the financial compensation of the affected upstream user, as proposed by Tilmant et al. [2009].

It is obvious that the optimal water allocation obtained using the SDDP model is highly dependent of the productivities of water for each specific uses i.e. the marginal water values of water. However, the above-mentioned analysis regarding irrigation in Ethiopia would likely to remain unchanged if the water value for irrigation and/or hydropower would be modified. In Egypt, any modification of the water productivities for irrigation or hydropower would not affect the water allocation in the delta since it is the very last water user in the river basin. On the other hand, the water allocation in the Toshka Valley and in Sudan is much more sensitive to the assumptions on the marginal water values.
5.4. Results and analysis

Figure 5.12: Average annual marginal water value for hydropower and irrigation in the White Nile and Main Nile catchment.

5.4.9 SDDP model allocation versus countries historical allotments

Figure 5.13 compares the average annual water allocation obtained using the SDDP model versus the historical allotments defined by the 1959 bilateral agreement. Remember that, according to the 1959 agreement, Egypt’s share of the Nile waters is 55.5 km$^3$.y$^{-1}$ while Sudan receives 18.5 km$^3$.y$^{-1}$.

Figure 5.13 highlights that, according to the hydrological data available, the average natural discharge of the Nile at Aswan, for the period considered in this study (1953-2000) has been estimated around 93 km$^3$.y$^{-1}$. The analysis of the figure reveals that the allocation optimized by the SDDP model are very close to the historical allotment of 1959: according to the optimal allocation obtained using the SDDP model and given the assumptions mentioned above, Sudan should, for the base-line scenario, receive an average of 14.40 km$^3$ annually for irrigation purposes while 5.67 km$^3$ would be lost by evaporation from Sudanese man-made reservoirs. In the same scenario, Egypt would receive 54.90 km$^3$.y$^{-1}$.

In the full development scenario (S3), Sudan would use 24.90 km$^3$.y$^{-1}$ (19.82 km$^3$.y$^{-1}$ for irrigation purposes and 5.08 km$^3$.y$^{-1}$ for evaporation losses from man-made reservoirs) which represents an increase of 6.4 km$^3$.y$^{-1}$ regarding its 1959 allotment of 18.5 km$^3$.y$^{-1}$. Considering the same development scenario (S3), Egypt would receive 57.6 km$^3$.y$^{-1}$ which represents an increase of 2.1 km$^3$.y$^{-1}$ used to irrigate in the New Valley Project.
Figure 5.13: Average annual water resources allocation obtained using the SDDP model versus the allotments defined by the 1959 bilateral agreement.
5.5 Conclusion

Four development scenarios for the Eastern Nile river basin were analyzed using a stochastic hydro-economic model. The objective was (1) to evaluate the impacts of upstream development in the Blue Nile basin on the allocation decisions and reservoirs operating strategies and (2) to assess the economic value of regulation (reservoirs) in Ethiopia. The analysis focused on two economic sectors: irrigation and hydropower generation.

The analysis reveals that building new large infrastructures in the upper part of the basin would have significant impacts on the operating strategies of the reservoirs: should the operation of the reservoirs be coordinated, the flood peak observed in the Blue Nile is reduced while the low flows are augmented. The main beneficiaries are hydropower in Ethiopia and irrigation in Sudan. Moreover, upstream storage in Ethiopia (and their regulation capacity) will generate positive externalities in Ethiopia and Sudan. In Ethiopia, the production of hydroelectricity is boosted by 40 TWh (+1666%), amongst which 14.3 TWh due to the regulation capacity of Karadobi, Beko-Abo, Mandaya and Border. In Sudan, the regulation capacity would increase irrigation water withdrawals by 5.5 %.

Coordinated operation of the reservoirs would also enable an average annual saving of at least 2.5 billion cubic meters through reduced evaporation losses from the High Aswan Dam. The High Aswan Dam inflows would be reduced and the reservoir would be operated a lower pool elevation but it will still reduce the hydrological risk exposure of Egypt: the reliability of supply to Egypt (according to the 1959 bilateral agreement) would not be affected. Such hydro-economic analysis helps Eastern Nile riparians on their endeavors for coordinated management of the basin.
Chapter 6

Impacts of the GAP in Turkey on the performance of the Tabqa dam and hydropower plant in Syria

6.1 Outline

Water resources systems development in the upper parts of a river basin can have major impacts on downstream users. This chapter analyzes the impacts of development of the Southeastern Anatolia Project (Turkey), commonly called GAP, on the Euphrates downstream riparian countries, Syria and Iraq, and especially on the performance of the Tabqa dam in Syria. A two stage modelling approach has been adopted. First, the operating rules of the largest GAP reservoirs are optimized and then simulated using a stochastic dual dynamic programming (SDDP) model to get, among other results, time series of simulated discharges at the borders with Syria and Iraq. This process is repeated for three development scenarios of the GAP. In the second stage, the reservoir operating policies of the Tabqa hydropower plant in Syria are derived from a stochastic dynamic programming (SDP) model and then simulated over a planning period of fifty years using the time series of inflows produced by SDDP for each development scenario. The analysis of results reveals, amongst other things, that if GAP is completed as planned, the risk of not meeting the annual Syrian energy target increases substantially (up to 60%).

1 This chapter is adapted from: 
6.2 Introduction

The southeastern Anatolia Project (GAP) is a multidimensional water resources development project in the Turkish part of the Euphrates-Tigris River basin. As described in chapter 3, the project involves the construction of 22 dams, 19 hydroelectric power plants with an installed capacity of 7526 MW, and the irrigation of 1.7 million ha. With the completion of the Atatürk reservoir, Turkey has now enough storage capacity to control the headwaters of the Euphrates and to potentially divert huge volumes of water for irrigation. The downstream riparian countries, Syria and Iraq, are concerned with the modification of the hydrological regime of the Euphrates River and its impact on the production of hydroelectricity from their hydropower plants and on the availability of water for irrigation purposes. This is especially important for Syria as Tabqa, its largest reservoir and hydropower plant (14.1 km³, 800 MW), is located immediately downstream of the Turkish border. In the Tigris River basin, the problem is less acute as the Turkish contribution to Tigris flows is much smaller than to the Euphrates (cf. chapter 3).

In response to the complaints formulated by its downstream neighbours, Turkey emphasizes the positive effects its storage capacity can have for the downstream countries by augmenting low flows during severe droughts and by absorbing flood waters. This study assesses the performance of the Tabqa reservoir and hydropower plant and quantifies the positive and negative effects of the altered hydrological regime on hydropower generation and on the reliability in meeting downstream water demands. The performance is evaluated for several development scenarios of GAP in Turkey, where each scenario is characterized by an irrigation area and by the largest GAP reservoirs are optimized and then simulated using a stochastic dual dynamic programming (SDDP) model, as described in chapter 2. One of the important results is a time series of simulated discharges at the Syrian border. The analysis is carried out for three development scenarios, characterized by the installed capacity and the irrigated area. Each scenario corresponds to a situation between the year 2000 (scenario low) and the full development, as

6.3 Methodology

To perform the analysis, a two-stage modelling approach has been adopted.

Step 1: Tilmant and Kelman [2007] analyzed the mid-term dispatch of the Turkish hydropower plants so as to minimize the operating costs of the hydrothermal system over a planning period of five years. These costs includes fuels costs of the thermal power plants and the rationing costs, when it occurs i.e. when the electricity demand cannot be met. The above mentioned system considers the largest infrastructures and consists in 40 hydropower plants, located in 13 river basins, and 185 thermal power plants. The operating rules of largest GAP reservoirs are optimized and then simulated using a stochastic dual dynamic programming (SDDP) model, as described in chapter 2. One of the important results is a time series of simulated discharges at the Syrian border. The analysis is carried out for three development scenarios, characterized by the installed capacity and the irrigated area. Each scenario corresponds to a situation between the year 2000 (scenario low) and the full development, as
6.3. Methodology

described in the master plan (scenario high). The scenarios are detailed in table 6.1 and illustrated in figure 6.2. Figure 6.1 illustrates the average monthly discharge simulated by the SDDP model of Tilmant and Kelman [2007] at the Syrian border. We can notice that, as claimed by the Turkish authorities, the natural flood peak in April-May is reduced while the low flow season is higher. Moreover, the total volume of water flowing through the border is reduced as the irrigated areas increase (moving from scenario "low" to "high").

**Step 2:** In the second stage, Tabqa reservoir operating policies are derived from a stochastic dynamic programming (SDP) model and then simulated over a planning period of 50 years using the time series of inflows produce by SDDP for each GAP development scenario. A comprehensive description of the SDP model is given in chapter 2. Those simulations are analyzed to evaluate the influence of the level of development of GAP on the performance of the Tabqa dam in Syria.

Table 6.1: GAP scenarios considered in Tilmant and Kelman [2007].

<table>
<thead>
<tr>
<th>Scenario</th>
<th>High [10^6 ha]</th>
<th>Medium</th>
<th>Low</th>
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</thead>
<tbody>
<tr>
<td>Irrigated area</td>
<td>1.70</td>
<td>0.833</td>
<td>0.20</td>
</tr>
<tr>
<td>Irrigation efficiency [%]</td>
<td>60</td>
<td>60</td>
<td>60</td>
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<table>
<thead>
<tr>
<th>Infrastructures</th>
<th>Euphrates</th>
<th>Keban</th>
<th>Karakaya</th>
<th>Karakaya</th>
<th>Karakaya</th>
<th>Karkamis</th>
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<td>Tigris</td>
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<td>Cizre</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Installed capacity [MW]</td>
<td>8 300</td>
<td>7 910</td>
<td>5 838</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Figure 6.1: Discharge at the Turkish/Syrian border, for each GAP development scenario.
6.3. Methodology

Figure 6.2: GAP development scenarios analyzed in Tilmant and Kelman [2007]. (a) = "Low" development, (b) = "Medium" development, (c) = "High" development.
6.4 Results analysis

The SDP algorithm has been used considering water withdrawals for irrigation in Tabqa reservoir as a priority so that the analysis will focus on energy production and consequently on downstream water availability. To maintain minimum flow for downstream users, essentially Iraq, an additional constraint has been activated: if enough water is available, monthly release cannot be less than 300 m$^3$.s$^{-1}$, which corresponds to the minimum flow that should cross the Iraqi border according to an outdated agreement between the three riparian countries. In the absence of a new agreement on the sharing of the Euphrates, we will use this minimum flow as a target value.

Fifty years of operation of the Tabqa reservoir are simulated (600 months) by a forward moving reoptimization method of the optimal policy calculated by the SDP algorithm [Tejada-Guibert et al., 1993], making possible the statistical analysis of performance indicators such as the risk faced by Syria not meeting his energy generation objective (1600 GWh.y$^{-1}$). For illustrative purposes, the reoptimized SDP release policy of the Tabqa reservoir (March), considering a medium GAP development scenario is illustrated in figure 6.3.

![Figure 6.3: Reoptimized SDP release policy for the Tabqa reservoir - "Medium GAP development scenario", May.](image-url)
6.4. Results analysis

6.4.1 Energy generation

Figure 6.4 displays empirical cumulative density functions (CDF) of annual energy generation for each GAP scenario. Let’s remind that the CDF gives the non-exceedance probability of any given annual energy generation, for each scenario. Examination of Figure 6.4 reveals that if the GAP project is fully implemented (high GAP scenario development), the risk of not meeting the annual Syrian energy target increases up to 60%. In the case of the medium GAP development this risk becomes 34% and to 16% for the current situation (low GAP scenario development).

![Empirical cumulative density function of annual energy production at Tabqa for the three GAP development scenarios.](image)

Table 6.2 shows the basic statistics associated with these results. It is shown that, on average, the annual energy generation would be reduced by 415 (17.9%) and 849 GWh (36.5 %) by moving from the current situation to the medium and high scenario respectively. Moreover, the variation coefficient increases as the number of infrastructures increases in the river basin.

Another statistical indicator used is the resilience. This can be expressed as the probability that if a system is in an unsatisfactory state, the next state will be satisfactory [Loucks and van Beek, 2005]. Resilience increases from 29% (high GAP scenario development) to 75% (low GAP scenario development) showing again that the current situation is more flexible and sustainable than the full GAP project implementation.
Chapter 6. Impacts of the GAP on the Syrian hydro-system

Table 6.2: Simulated annual energy generation - Tabqa.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>High</th>
<th>Medium</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average [GWh]</td>
<td>1474</td>
<td>1908</td>
<td>2323</td>
</tr>
<tr>
<td>Variation coefficient [-]</td>
<td>0.34</td>
<td>0.28</td>
<td>0.24</td>
</tr>
<tr>
<td>Minimum [GWh]</td>
<td>599</td>
<td>875</td>
<td>1198</td>
</tr>
<tr>
<td>Maximum [GWh]</td>
<td>2929</td>
<td>3330</td>
<td>3636</td>
</tr>
<tr>
<td>Reliability [-]</td>
<td>0.38</td>
<td>0.66</td>
<td>0.84</td>
</tr>
<tr>
<td>Resilience [-]</td>
<td>0.29</td>
<td>0.53</td>
<td>0.75</td>
</tr>
</tbody>
</table>

6.4.2 Downstream water flow

The empirical cumulative density functions of monthly release of the Tabqa dam is presented in Figure 6.5 for each GAP development scenario. The particular shape of the functions is due to the hard constraint put on the release: this cannot be less than 300 m$^3$s$^{-1}$ if enough water is available. This constraint is always satisfied except for the high GAP development scenario and under very dry years corresponding to a return period of 50 years. During those particularly dry years even the irrigation (also a hard constraint in the SDP model) cannot be fully met. The maximum flow observed is composed

![Figure 6.5: Empirical cumulative density function of monthly outflow (turbining and spillage) at Tabqa dam for the three GAP development scenarios.](image_url)
of the technical maximum release flowing through the eight turbines (8 x 285 m$^3$.s$^{-1}$) and an unforeseeable inflow of water in the reservoir forcing spillage. Those floods are less important in Syria with the increase of GAP project development. This observation corroborates Turkey’s argument that GAP’s development storage capacity has positive effects on the downstream riparian countries by augmenting low flows during severe droughts and by absorbing flood waters. The statistical properties of the simulated monthly release are detailed in table 6.3.

Table 6.3: Simulated monthly release - Tabqa.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>High</th>
<th>Medium</th>
<th>Low</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average [m$^3$.s$^{-1}$]</td>
<td>442</td>
<td>576</td>
<td>707</td>
</tr>
<tr>
<td>Variation coefficient [-]</td>
<td>0.87</td>
<td>0.94</td>
<td>0.94</td>
</tr>
<tr>
<td>Minimum [m$^3$.s$^{-1}$]</td>
<td>0</td>
<td>300</td>
<td>300</td>
</tr>
<tr>
<td>Maximum [m$^3$.s$^{-1}$]</td>
<td>2654</td>
<td>2977</td>
<td>3883</td>
</tr>
</tbody>
</table>

6.5 Conclusion

Until recently, huge simplifications and approximations were required to get optimization results for hydro-systems involving more than 3 to 4 reservoirs, while considering stochastic variables. However recent advances in mathematical programming have reduced this computational burden. The new algorithms can also better handle the hydrological uncertainty, which is inherent to the operation of a multireservoir system. One of these algorithm is Stochastic Dual Dynamic Programming (SDDP). Coupled with a classical SDP scheme, the analysis demonstrates that the development of the GAP project on the Euphrates will impact significantly on Syria. This is particularly true for energy generation as the GAP development scenario increases from the current situation to full project implementation. The risk of Syria not being able to meet its annual energy target may increase to 60%. Iraq, being downstream of Syria on the Euphrates, may also incur significant impacts.
Chapter 7

General conclusions, limitations and perspectives

7.1 General conclusions

Increasing water demands due to population growth and higher living standards implies that more attention should be paid to improve the operational effectiveness of water allocation policies. Computational tools relying on optimization algorithms can be used to derive efficient allocation policies but these have been traditionally limited to small scale systems due to computational difficulties. As integrated water resources management requires that allocation policies be determined at the river basin scale, traditional optimization models have limited applicability due to the high dimensionality of basin-wide allocation problems.

The overall objective of the thesis was to contribute to the improvement of the operational efficiency and effectiveness of existing and planned water resources systems and particularly the reservoirs operation. The specific objective was to adapt, test and assess the usefulness and applicability of a model that aims at developing optimal operating strategies for multiple reservoirs in hydropower-irrigation systems characterized by a stochastic environment where the reservoir inflows are spatially and temporally correlated. The methodology to solve the reservoir operation problem, described in-depth in chapter 2 (first expected result), relies on an advanced optimization method called Stochastic Dual Dynamic Programming.

Directly incorporating the demand curves for irrigation water implies that the model, when used in a planning mode, can be used to determine optimal (economically efficient) irrigation developments/extensions. The second expected result of the thesis was therefore an implementation of an extension of the SDDP algorithm to directly incorporate net benefits from irrigated agriculture into the global objective function (maximize the net benefits from system operation). The methodology, detailed in chapter 2, is applied on the Eastern Nile river basin (chapter 5) to demonstrate that, considering an economically efficient allocation without any political and/or legal constraints, it
would not be economically efficient to withdraw water from the Blue Nile for irrigation purposes in Ethiopia. Indeed, Ethiopian farmers face a coalition of non-consumptive hydropower stations and other irrigated areas downstream in Sudan and Egypt (productive uses) that prevent the expansion of consumptive uses in the upper part of the basin. However, this conclusion must be moderated: economic efficiency should only be achieved if suitable benefits sharing mechanisms are established (equity principle), as proposed by Tilmant et al. [2009].

The third expected result was an implementation of a methodology, compatible with the efficient decomposition scheme of the SDDP algorithm, to deal with the variable productivity of hydropower plants. The theoretical development of the representation of the non-linear hydropower function through a convex hull approximation of the true hydropower function is given in chapter 2. In chapter 4, a network of storage hydropower plants in the Eastern Nile river basin illustrates the impacts of the variable productivity of hydropower plants on the allocation decisions. It appears that the non-linear hydropower function can be accurately approximated by a set of hyperplanes (convex hull). The proposed extension of the model provides therefore a more realistic representation of the hydropower system. The analysis also revealed that considering the effect of the variable hydraulic head on the hydropower generation has major impacts on reservoirs operating strategies. Assuming a constant productivity of hydropower plants may lead to underestimated pool elevation, hydropower generation, evaporation losses and may therefore overestimate the water availability for other uses, especially irrigated agriculture. Although this extension of the model generates higher complexity of the optimization problem to solve, the increase in CPU time can be considered as fairly reasonable.

Finally the fourth expected result was an illustration of the model’s capabilities in analyzing the water resources allocation problem in large-scale hydropower-irrigation systems. Hence, the model has been applied to two case studies where the interpretation of the principles that govern the sharing of international watercourses (The United Nations Convention on the Law of the Non-Navigational Uses of International Watercourses) diverges depending of the position of the countries within the basins: The Eastern Nile and Euphrates river basins which have been described in chapter 3.

In the Euphrates river basin, with the implementation of its colossal water resources development project (called GAP), Turkey, the upstream country, plans to divert huge volumes of water for irrigation purposes and control the regime of the rivers thanks to huge reservoirs. The downstream riparian coun-

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1The United Nations Convention on the Law of the Non-Navigational Uses of International Watercourses (available at http://untreaty.un.org/ilc/texts/instruments/english/conventions/8_3_1997.pdf) has been adopted in 1997 (adopted by a vote of 103 for and 3 against, with 27 abstentions), aims at providing a coherent framework for negotiations at the international level. The main underlying principles of the convention about water use are the "right to equitable and reasonable use" of the shared resource (article 5) and the "obligation not to cause significant harm" (article 7) by using it. The Convention is therefore an effort to balance two opposite interests: (1) the right for countries to develop their water resources system and (2) the protection of existing users. However, there is no consensus on the interpretation of these principles, especially the meaning of "equitable", "reasonable" and "significant", and their hierarchy [McCaffrey, 2001].
tries, Syria and Iraq, who have been using the Euphrates and Tigris waters since immemorial times, are concerned with the modification of the hydrological regime of the Euphrates river and its impact on the production of hydroelectricity from their hydropower plants and on the availability of water for irrigation purposes. In chapter 6 the reoperation of the hydro-system in Syria is analyzed, given the unilateral development of the upper part of the Euphrates river basin in Turkey. Assuming the coordinated operation of the Turkish hydro-thermal energy system but that the Turkey and Syria operate their own system independently, the development of the GAP in Turkey would have significant impacts downstream in the basin. For example, the risk of Syria not being able to meet its annual energy target may increase to 60% because of the reduction of the Euphrates flow given the development of large irrigation schemes in Turkey.

In the Nile river basin, Egypt, the most downstream country is completely dependent of the Nile waters for its economy and food security. Despite of the approaching closure\(^2\) of the Nile river basin, Egypt aims at developing its irrigated areas, requiring therefore more water. In the mean time, upstream countries (source of water) are claiming the right to develop their irrigation and hydropower potentials, which is obviously incompatible with Egyptian ambitions. In chapter 5, four development scenarios of the Eastern Nile river basin were analyzed in order to (1) evaluate the impacts of upstream development in the Blue Nile basin on the allocation decisions and reservoirs operating strategies and (2) assess the economic value of regulation (reservoirs) in Ethiopia. Assuming an economically efficient allocation of water, without any political and legal constraints, the main findings of the analysis are that hydropower, mainly in Ethiopia, is the major beneficiary of the proposed development. It is shown that the coordinated development of the infrastructures could reduce evaporation losses from man-made reservoir and increase water withdrawals for irrigation purposes in Sudan without affecting the reliability of supply to Egypt (according to the 1959 bilateral agreement). Moreover, the analysis reveals a huge increase in basin-wide benefits due to the new hydropower stations in Ethiopia and irrigated areas in Sudan. It is important to stress that the storage services in Ethiopia would, on average, yield an annual extra value 14.4\% of the basin-wide short-run net benefits.

In the light of the above-mentioned model developments (better representation of the hydropower and irrigation systems) and applications, it appears clearly that optimization models and particularly Stochastic Dual Dynamic Programming are very promising tools to support the implementation of integrated water resources management: SDDP can identify mid-term optimal operating strategies in multireservoir systems that hedge against hydrological risk. It provides, for each node of the water resources system, key valuable information for shorter-term planning models, such as statistical distribution of allocation decisions (end-of-period storage, reservoir release, spillage, withdrawals for off-stream uses), economic information (marginal water value at-source and at-site, short-run net benefits for each country/demand site/hydropower plant),

\(^2\)A river basin is said to be closing when the water demand cannot be met, in terms of quantity and/or quality, within the basin, for part or all of the year [Molle et al., 2010].
evaporation losses from man-made reservoirs, etc. In SDDP, the computation burden found in traditional optimization approaches are removed and the computational time required to solve large-scale reservoir operation problems remains fairly low, making it possible to imbed SDDP in decision support systems [Loucks and van Beek, 2005]. In the context of international river basins, the hydro-economic analysis based on SDDP results could help riparian countries on their endeavours for coordinated management of the river basin.

7.2 Shortcomings and perspectives

In spite of the achievements presented above, the present study has several limitations detailed below.

**Data quantity and quality issues.** Clearly, basin-wide water resources systems analysis requires many data from various sources (public institutions, private companies, scientific community, etc) and from various backgrounds: hydrological (natural inflows that enters the system at each node), economical (short-run marginal costs of the hydrothermal system to which the hydropower plants contributes, water demand curves for agriculture and other off-stream uses, ...), agronomical (irrigated area and their cropping pattern, irrigation network efficiencies, ...), etc. As a consequence, we are left with no choice but to reduce the level of detail of particular components of the hydro-system to be modelled. For example, heterogeneous cropping patterns in irrigated areas are aggregated into homogeneous demand sites. Hydroelectric turbines are aggregated into hydropower stations.

In this thesis, most of the data come from official sources (Ministries and International organization) and are complemented with many readings in order to provide the most consistent and up-to-date database.

**Water quality and groundwater issues.** This thesis focus on surface water quantity issues. This does not mean that water quality issues are negligible and should be ignored. For example, sediments transport is a major concern in the Eastern Nile river basin, but in such a large river basin, we could not explicitly consider every aspects associated with infrastructural development in the basin and the growing water demands.

In SDDP, one way to deal with water quality issues could be to consider imaginary reservoirs of solutes, connected to the river system by physical relations that govern the solute transport processes. The same idea could be implemented to deal with groundwater: groundwater reservoirs could be additional nodes of the system (reservoirs) that are naturally and/or artificially recharged and where water is abstracted. If necessary, water quality constraints and objectives, as well as groundwater representation, could be implemented in SDDP, as far as the mathematical formulation of the various processes remain compatible with the Benders decomposition scheme.
7.2. Shortcomings and perspectives

Model approximations. It has been shown that SDDP is one of the few solutions that exist to deal with optimal operation of large-scale multipurpose multireservoir systems while considering a stochastic environment. Obviously, the computationally efficiency requires to resort to some approximations. For instance, SDDP relies on linear programming, therefore requiring that the problem be linear or at least convex. Fortunately, most relationships considered in this thesis are close to linear, making their approximation by piecewise linear functions rather accurate. One should also remember that SDDP provides locally-accurate solutions of the benefit-to-go functions (at the sampled points) instead of covering the entire state-space domain. However, the accuracy of the solution is increased by adding new cuts through the cyclic optimization/simulation phases. Finally, SDDP uses a finite time horizon, making the algorithm unsuitable to directly obtain operating rules for each reservoir of the system. However, these could be obtained by post-processing of the optimization/simulation results, by for example, multiple regression analysis of use them in an on-line mode.

Coordinated reservoir operation. The present analysis assumes the coordinated operation of all the infrastructures. This implies, among other things, the existence of an institutional framework and a central system operator to ensure a basin-wide management of the system. However, this assumption is relevant to support the implementation of integrated water resources management that requires a basin-wide, coordinated approach. In addition, at least coordinated development of infrastructures is usually part of the river basin agencies mandates.

Climate change. For each node of the hydro-system considered, an historical record of lateral inflows need to be available to estimate the parameters of the build-in multi-site periodic autoregressive hydrological model. As a consequence, we make the assumption that historical weather patterns are representative of possible future conditions. However, it is now recognized by the international community that climate change will affect the hydrology and therefore the freshwater resources availability [IPCC, 2007], making the use of past hydrology questionable. To address this issue, a preliminary study would be required to generate a "new" hydrology corresponding to "new" climate and land-use changes scenarios. The "new" time series of river discharges could then be processed by SDDP in the same way as the historical ones.

The consequence of a possible climate change has become a major concern for decision makers and reservoir operators who use to have a static approach for hydrological risk management. Information about inter-annual climate variability, via relevant available climatic indicators, could be used to develop adaptive reservoir management and operation strategies to evolve towards a dynamic risk management strategy.
Appendices
Appendix A

Kuhn-Tucker conditions for optimality

The Kuhn-Tucker conditions for optimality are necessary and sufficient for the characterization of an optimal solution for a LP problem. Let \( x \) be the vector of decision variables, \( F \) be the linear objective function to be maximized and \( g_i \) be the \( i \)th linear constraint. The optimization problem can be written as:

\[
\max_x F(x)
\]

subject to

\[
x \in X
\]

where \( X = \{ x | g_i(x) \leq 0, i = 1, 2, \ldots, m \} \)

If \( x^* \) is an optimal solution, then there exist \( \lambda \) with \( \lambda_i \geq 0 \) such that

\[
x^* \in X
\]

\[
\lambda_i g_i(x^*) = 0 \quad i = 1, 2, \ldots, m
\]

\[
\nabla F(x^*) = \sum_i \lambda_i \nabla g_i(x^*)
\]

Hence, at the optimal solution, the derivative of the objective function with respect to the variables \( x_i \) is given by:

\[
\frac{\partial F}{\partial x_i} = \sum_i \lambda_i \frac{\partial g_i}{\partial x_i} \quad i = 1, 2, \ldots, m
\]
Appendix A. Kuhn-Tucker conditions for optimality
Appendix B

Hydrological model MPAR

B.1 Introduction

It is largely advocated that hydrological time series have periodic and stochastic characteristics. Moreover, the discharges at various geographical locations in a river basin usually exhibit temporal and spatial correlation. For example, lag-one serial correlation in time and lag-zero and lag-one cross-correlation in space are illustrated in figure B.1.

Figure B.1: Schematic representation of the correlation structure of a general multivariate lag-one autoregressive model.

Let \( q_{v,t}(j) \) be \( N \) years of periodic stationary historical flow data \((v = 1, \ldots, N)\) available at the \( J \) nodes of the hydro-system \((j = 1, \ldots, J)\). The period (usually one month) is denoted \( t \) and the year \( v \). Assuming the periodic process can be modelled by an autoregressive model of order 1, the model can be written as (for period \( t \) and site \( j \)):

\[
\frac{q_t(j) - \mu_{q,t}(j)}{\sigma_{q,t}(j)} = \phi_{t}(j) \frac{q_{t-1}(j) - \mu_{q,t-1}(j)}{\sigma_{q,t-1}(j)} + \epsilon_t(j) \quad (B.1)
\]
where \( \mu_{q,t}(j) \) and \( \sigma_{q,t}(j) \) are the periodic mean and standard deviation of \( q_t(j) \), \( \phi_t(j) \) is the periodic autoregressive parameter of order 1 and \( \epsilon_t(j) \) is a time independent (but spatially correlated) stochastic noise of zero mean and variance \( \sigma_{\epsilon,t}^2(j) \).

### B.2 Modelling methodology

The natural reservoir inflows modelling procedure is decomposed into five steps:

(i) Estimation of the autoregressive model parameters set:
\[ \{ \mu_{q,t}(j), \sigma_{q,t}(j), \phi_t(j), \sigma_{\epsilon,t}^2(j) \} \] for each site \( j \);

(ii) Identification of statistical properties of the stochastic noise \( \epsilon_t(j) \) for each site \( j \);

(iii) Standardization of the stochastic noise \( \epsilon_t(j) \) into \( V_t(j) \);

(iv) Synthetic generation of \( \hat{V}_t(j) \) (spatially cross-correlated);

(v) Back transformation from \( \hat{V}_t(j) \) to synthetic natural reservoir inflows \( \hat{q}_t(j) \).

#### B.2.1 Autoregressive model parameters

Given \( N \) years of historical incremental reservoirs inflows data \( q_{v,t}(j) \) available at site \( j \), the parameters \( \mu_{q,t}(j) \) and \( \sigma_{q,t}(j) \) can be estimated by the first and second order moment of \( q_{v,t}(j) \) respectively:

\[
\hat{\mu}_{q,t}(j) = E[q_t(j)] = \frac{1}{N} \sum_{v=1}^{N} q_{v,t}(j) \quad \text{(B.2)}
\]

and

\[
\hat{\sigma}_{q,t}(j) = E[q^2_t(j)] = \frac{1}{N} \sum_{v=1}^{N} (q_{v,t}(j) - \hat{\mu}_{q,t}(j)) \quad \text{(B.3)}
\]

where \( E[\cdot] \) is the expectation operator.

Considering an autoregressive model of order 1 defined by equation (B.1), the autoregressive parameter \( \phi_t(j) \) is estimated by [Salas et al., 1980]:

\[
\phi_t(j) = E \left[ \left( \frac{q_t(j) - \mu_{q,t}(j)}{\sigma_{q,t}(j)} \right) \left( \frac{q_{t-1}(j) - \mu_{q,t-1}(j)}{\sigma_{q,t-1}(j)} \right) \right] = \rho_t(j) \quad \text{(B.4)}
\]

where \( \rho_t(j) \) is the temporal lag-one autocorrelation between \( q_t(j) \) and \( q_{t-1}(j) \).

According to Salas et al. [1980], the variance of the stochastic noise \( \sigma_{\epsilon,t}^2(j) \) can be written as a function of the periodic autoregression parameter \( \phi_t(j) \):

\[
\sigma_{\epsilon,t}^2(j) = 1 - \phi_t(j) \cdot \rho_t(j) = 1 - \rho_t(j)^2 \quad \text{(B.5)}
\]
B.2. Modelling methodology

B.2.2 Statistical properties of the stochastic noise and standardization

Let assume that the periodic stochastic noise \( \epsilon_t(j) \) follows a 3-parameter log-normal distribution of mean \( \mu_{y,t}(j) \) (equal to zero), standard deviation \( \sigma_{y,t}(j) \) and lower bound \( a_t(j) \):

\[
f_{\epsilon_t(j)} = \frac{1}{(\epsilon_t(j) - a_t(j))\sqrt{2\pi\sigma_{y,t}(j)}} e^{-0.5\left(\frac{\log{(\epsilon_t(j) - a_t(j))} - \mu_{y,t}(j))}{\sigma_{y,t}(j)}\right)^2} \tag{B.6}
\]

The lower bound \( a_t(j) \) is required to ensure non-negative inflows:

\[
\epsilon_t(j) > \frac{\mu_{y,t}(j)}{\sigma_{y,t}(j)} - \phi_t \left( \frac{\eta_t(j) - \mu_{y,t}(j)}{\sigma_{y,t}(j)} \right) = a_t(j) \tag{B.7}
\]

Equation (B.6) defines a new variable \( y_t(j) = \log{(\epsilon_t(j) - a_t(j))} \), where logarithm is to the base \( e \), normally distributed with \( \mu_{y,t}(j) \) and \( \sigma_{y,t}(j) \) being the mean and variance respectively:

\[
\begin{align*}
\mu_{y,t}(j) &= E[\log{(\epsilon_t(j) - a_t(j))}]) \tag{B.8} \\
\sigma_{y,t}^2(j) &= E[\log{(\epsilon_t(j) - a_t(j))}^2] \tag{B.9}
\end{align*}
\]

Statistical parameters of \( y_t(j) \) and \( \epsilon_t(j) \) are related by the following relations:

\[
\begin{align*}
\mu_{\epsilon_t}(j) &= a_t(j) + e^{\mu_{y,t}(j)} + \sigma_y^2 j \tag{B.10} \\
\sigma_{\epsilon_t}^2(j) &= e^{2(\mu_{y,t}(j) + \sigma_y^2 j)} - e^{2\mu_{y,t}(j) + \sigma_y^2 j} \tag{B.11}
\end{align*}
\]

Using equations (B.5), (B.7), (B.10) and (B.11), the two lower order moments of the variable \( y \), \( \mu_{y,t}(j) \) and \( \sigma_{y,t}^2(j) \) respectively, can now be estimated by :

\[
\begin{align*}
\sigma_{y,t}^2(j) &= \left(1 + \frac{\sigma_y^2(j)}{a_t^2} \right) \tag{B.12} \\
\mu_{y,t}^2(j) &= 0.5 \log \left( \frac{\sigma_y^2(j)}{e^{2\sigma_y^2(j)} (\sigma_{y,t}^2(j) - 1)} \right) \tag{B.13}
\end{align*}
\]

Finally, the standardized stochastic noise \( V_t(j) \), is estimated by:

\[
V_t(j) = \frac{\log{(\epsilon_t(j) - a_t(j)) - \mu_{y,t}(j)}}{\sigma_{y,t}(j)} \tag{B.14}
\]

B.2.3 Spatial cross-correlation modelling

Assuming the hydro-system consists in \( j \) nodes where an historical records of natural incremental reservoir inflows \( q_{\text{v},t}(j) \) is available at each node \( j \), the calculation of standardized stochastic noise \( V_t(j) \) for each node \( j \) is straightforward according to the above mentioned procedure. We can therefore define a column vector \( V_t \) (size \( J \times 1 \)) as follow:

\[
V_t = [V_t(1), \ldots, V_t(j), \ldots, V_t(J)]' \tag{B.15}
\]
Appendix B. Hydrological model MPAR

where $\cdot'$ is the matrix transpose operator.

The spatial statistical dependence of reservoir inflows at the various nodes is introduced by the observed lag-0 covariances and cross-covariances of the residuals (the model does not preserve the lag-1 cross covariance):

$$V_t = B_t W_t$$ (B.16)

where $W_t$ is a column vector of $J$ independent elements consisting of white noises, i.e. $W_t(j)$ is normally distributed with zero mean and variance equal to 1. The matrix $\hat{B}_t$ is obtained by Choleski factorization of the covariance matrix of standardized noise at each node $j$:

$$\hat{B}_t \hat{B}_t' = \text{Cov}(V_t) = E[V_t V_t']$$ (B.17)

where $\text{Cov}(V_t)$ is the covariance matrix of $V_t$ where diagonal elements are the variances of the natural reservoir inflows residuals at each nodes $j$ of the hydro-system.

B.2.4 Synthetic generation and back transformation

Given $J$ independent stochastic white noises $\hat{W}_t(j)$, the standardized spatially correlated noise $\hat{V}_t(j)$ are generated according to:

$$\hat{V}_t = \hat{B}_t \hat{W}_t$$ (B.18)

For each site $j$, the natural reservoir inflows is obtained by back-transformation from the standardized stochastic noises $\hat{V}_t(j)$ to $\hat{\epsilon}_t(j)$:

$$\hat{\epsilon}_t(j) = e^{(\hat{V}_t(j) - \mu_y + \mu_y)} + a_t(j)$$ (B.19)

and finally from equation (B.1) representing the autoregressive process.

B.3 Outliers identification

An outlier is defined as a an observation that lies outside the overall pattern of a distribution (Moore and McCabe 1999). The outliers are removed from the historical data using the Grubbs procedure [Grubbs, 1969], design to detect statistically outliers values in a univariate normally distributed dataset $x_1, x_2, \ldots, x_N$ where $N$ is the size of the sample. The test is an iterative procedure that detects and removes from sample one outlier value at time until no more outlier value is detected. In the two-sided version of the test, Grubbs statistic $G$ is defined as:

$$G = \frac{\max|x_i - \mu_x|}{\sigma_x}$$ (B.20)

where $x_i$ is the tested value and $\mu_x$ and $\sigma_x$ are the mean and standard deviation of the sample respectively.
The hypothesis of no outlier value is rejected (at significance level $\alpha$) if:

$$G > \frac{N - 1}{\sqrt{N}} \sqrt{\frac{t^2_{\left(\frac{\alpha}{2N}, N-2\right)}}{N - 2 + t^2_{\left(\frac{\alpha}{2N}, N-2\right)}}}$$

(B.21)

where $t^2_{\left(\frac{\alpha}{2N}, N-2\right)}$ is the critical value of the $t$-distribution with $(N-2)$ degrees of freedom and a significance level of $\alpha/(2N)$. 

B.3. Outliers identification

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About the author

Born in Charleroi (Belgium) on the 1st of February 1982, Quentin Goor has been graduated as an agricultural engineer from the Université catholique de Louvain (2005). He was then teaching assistant in integrated water resources management, hydrology, hydraulics and irrigation systems design. He was also involved in consultancy activities for the Belgian Cooperation Agency and for various private companies.
Summary

The increasing global water demand due to population growth and higher living standards exerts a significant stress to the limited global freshwater resources. Meeting the future global water demand therefore implies that the operational water allocation policies are efficient and effective. Computational tools relying on optimization algorithms can be used to derive efficient allocation policies. Yet, due to computational constraints, existing tools have only been applied to the analysis and design of small-scale water resources systems. As integrated water resources management requires that allocation policies be determined at the river basin scale, traditional optimization models have limited applicability due to the high dimensionality of basin-wide allocation problems.

The overall objective of the thesis is to contribute to the improvement of the operational efficiency and effectiveness of existing and planned water resources systems and particularly the reservoirs operation. The specific objective is to adapt, to test and to assess the usefulness and applicability of the Stochastic Dual Dynamic Programming (SDDP) algorithm for determining economically efficient water allocation policies in stochastic multipurpose multireservoir systems. SDDP has been designed to circumvent the curse dimensionality problem associated with traditional optimization methods. More specifically, SDDP is used to analyze the hydrologic and economic consequences associated with various development and management scenarios in large-scale river basin systems in which hydropower generation and irrigated agriculture are the main uses.

The algorithm has been successfully implemented and evaluated for analyzing water allocation in two case studies: the Eastern Nile and Euphrates river basins.