"Unemployment Benefits and Basic Income. Economic Performance, Income Inequality and Social Welfare"

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ABSTRACT

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Unemployment Benefits and Basic Income. An Evaluation

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Abstract

Two social security schemes are compared in the long run, one based on a basic income and on the other on unemployment benefits. The analysis is carried out in an overlapping generations model in which agents have to decide whether to become skilled or remain unskilled, and how much time to devote to work. We study the effects of both social security systems on the economic incentives determining individual behavior. We compare their implications in terms of economic performance, income inequality, and social welfare at the steady state. The laissez-faire situation is taken as the benchmark.

Keywords: Basic income, Unemployment benefit, Welfare, Inequality, Leisure.

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Introduction

According to Van Parijs and Salinas (1998) Europe is confronted with a “new social question” consisting of a dilemma between high unemployment and worsening poverty. Actual mechanisms of income redistribution operate mainly through the taxation of labor income, the revenues of which are distributed in form of benefits to the unemployed. While reducing income poverty, this form of redistribution also produces pervert effects, like generating persistently high unemployment rates. This is due to the fact that “a growing number of households becomes unable [...] to durably achieve through their labour a net income that exceeds the level of social protection” (Van Parijs and Salinas (1998, page 3)). Since actual unemployment policies are not able to provide an efficient answer to this dilemma, alternative types of transfer policies have been put forward as possible devices to fight unemployment without worsening poverty: reductions of social security contributions, the negative income tax, or the unconditional basic income.

In this paper we analyze a basic income scheme as compared to the classical unemployment benefit. We study the implications of both schemes in terms of income inequality, economic performance, and social and individual welfare, taking the laissez-faire as the benchmark economy. We do so in a simple model that allows us to clearly identify the different mechanisms which are likely to intervene. In particular we focus on the individual behavior: the decisions on whether to become skilled or not, and the fraction of the time endowment devoted to work. We allow for two different sources of heterogeneity: the effort required to become skilled and the individual productivity. The analysis of the individual behavior in an economy with unemployment benefits, on one hand, and a basic income, on the other hand, provides us with insights concerning the differences between the two economies.

A basic income scheme consists of an income unconditionally paid to all citizens on an individual basis, without means test or work requirement. More specifically a basic income is characterized by the fact that (a) it is paid to individuals rather than to households, (b) irrespective of any other income, and (c) it does not require any present or past work performance nor is it conditional on the willingness to accept a job (Van Parijs (1992))\(^1\). The main objective of a basic income scheme is to address the unemployment trap by making more attractive and less costly the acceptance of low productivity and therefore low paid jobs. Indeed, contrary to an unemployment benefit, it does not destroy pecuniary incentives to carry out paid work at the bottom of the income scale. The introduction of a basic income would have considerable implications; not in the least, as it is supposed to be accompanied

\(^1\)A negative income tax scheme shares the third property with the basic income proposal, but not necessarily the first two: a negative income tax often operates at the household level and implies that the level of the transfer to which one is entitled depends on the income from other sources.
by several other measures like a fiscal and a social security reform, and eventually a deregulation of the labor market. According to Atkinson (1995) a basic income would, in its pure form, replace all social security benefits; it would be accompanied by a flat tax rate replacing the existing income tax and social security contributions. By this way a basic income would also be an instrument to attack the poverty trap.²

Our analysis concentrates on the implications of a basic income in terms of individual incentives, as compared to an unemployment benefit. The paper also addresses the consequent distributive effects, the implications for economic performance, and social and individual welfare. We consider a three period overlapping generations model, in which individuals have to make three decisions: (a) whether to become skilled or remain unskilled, (b) which fraction of their time endowment to devote to work, and (c) which fraction of their income to save. We then introduce an unemployment benefit and a basic income in the benchmark economy. Both social security systems are shown to negatively affect economic performance compared to the laissez-faire economy. In the case of a basic income because it decreases labor supply by inducing all employees to devote less time to work (increasing leisure) thereby decreasing production. Similarly, an unemployment benefit reduces employed labor by generating voluntary unemployment, and therefore decreases production.³

The negative impact on production seems to confirm the presumed existence of a practical trade-off between decreasing income inequality and economic performance. Several authors point at the large social security systems characteristic of European welfare states as an obstacle to economic performance. In their view, these systems focus on income safety and distribution disregarding economic incentives (Lindbeck et al (1994, page 17)). Policies decreasing income inequality may have negative effects on economic performance, as they may discourage individuals to carry out economic performance enhancing activity. In their analysis of what they call “the crisis of the Swedish model” these authors argue that “the social security (or social insurance) system should not overburden the economy through distorted incentives or large benefits.” They also claim that larger wage differentials, reflecting skill differences, are necessary to generate stronger incentives to invest in human capital (Lindbeck et al (1993)). This idea is corroborated in the present paper: both a basic income and an unemployment benefit reduce the incentives to work, although the latter to a larger extent. A basic income also discourages effort to become skilled as it decreases the relative advantages of being skilled in terms of future income as compared to being unskilled. The result is a drop in the national product.

The above paragraph exclusively refers to economic efficiency or economic per-

²Poverty traps occur when after tax income fall as gross income rises, due to progressive tax rates combined with the loss of benefits (Clark and Kavanagh (1996)).

³A similar result was obtained in Dehez and Fitoussi (1996): they show that an increase in the minimum real wage always reduce total employment and production.
formance. According to Van Parijs (1992) there is, however, a second category of efficiency to be taken into account in the debate on social policy, namely target efficiency. This criterion refers to the degree to which a social security system succeeds in helping those who need it given the budget constraints, and it is induced by a concern for social justice. In light of this second criterion, we study and compare the redistributing effects and the implications for social welfare of both a basic income and an unemployment benefit.

The presence of leisure in our model is responsible for the initial increasing income inequality effect always observed in the case of an unemployment benefit, and which may occur in the case of a basic income. The explanation of this counterintuitive result is to be found in that both systems induce individuals at the bottom of the wage scale to renounce to labor earnings: through the creation of unemployment and the reduction of time spent working respectively. They do so because of the disutility of labor (the valuation of leisure).

According to Artus (1998), switching from a minimum income to a basic income is likely to reduce social welfare as it decreases the equilibrium wage of the unskilled. This reduction in welfare occurs despite the fact that it reduces unemployment among the unskilled employees. A decrease of the unemployment rate does not have any effect on social welfare (the equilibrium wage of the unskilled is such that unskilled employees are indifferent between working or not). It is the taxes levied in order to finance the basic income what negatively affects social welfare. In the present paper despite the taxes and the fall in production, there is room for a social welfare improving effect on behalf of a basic income because of its income redistributing effect.

The paper is organized as follows: the first section describes the benchmark model. Sections 2 and 3 describe the economy in the case of an unemployment benefit and a basic income respectively. In section 4 both systems are compared in terms of economic performance, income distribution, and social and individual welfare. Section 5 concludes.

1 The benchmark economy

The economy is represented by an overlapping generations model inhabited by a continuum of individuals differing in their effort required to become skilled (as in Cahuc and Michel (1996)). A second source of heterogeneity concerns the individual productivity characterizing each agent, represented by his endowment in terms of

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4The way we model the unemployment benefit (no tracking of beneficiaries and no time limit) makes it very similar to a minimum income (both would generate voluntary unemployment), except for the fact that a minimum income would be identical for skilled and unskilled, while in our model the level of unemployment benefit is different for skilled and unskilled.
efficiency units. The supply side of the economy consists of a firm producing one physical good that can be either consumed or invested. The available technology requires the use of capital and of skilled and/or unskilled labor.

1.1 The households

Each period $t$ a new generation of $N_t$ individuals indexed $i$ is born. An equal number of individuals leaves the economy implying a stationary population which is normalized to 1. The individuals live for three periods. In the first period they neither work nor consume but decide whether they become skilled or remain unskilled. Becoming skilled requires carrying out a certain amount of effort $e^i_{t-1} \geq 0$. This level of effort differs among individuals and is assumed to be distributed uniformly and independently across individuals over the interval $[0, 1]$. If we assume that individuals care about leisure, $e$ can be interpreted as the fraction of the time endowment an individual has to spend studying when young to become skilled. The higher this fraction, the more reluctant an individual will be to do the effort in order to become skilled.

Moreover, individuals within the (un)skilled group are endowed with a different innate productivity $z^i_t$ when carrying out their (un)skilled work in period $t$. Individual productivity is assumed to be distributed uniformly over $[a, b]$, $a > 0$, and independently of the $e^i_{t-1}$. This amounts to say that there is no correlation between innate productivity and effort required to become skilled.

In the second period the individual supplies his efficiency units $z$ in order to obtain labor earnings: the labor market pays a real wage per efficiency unit $w_{jt}$ where $j = 1, 2$ indexes skilled and unskilled work respectively. As the individual values leisure, he only works a fraction $\lambda^i_t$ of his time endowment (normalized to one). He consumes part of his revenue and saves the rest in order to consume in his third life’s period when he retires. Individuals have a utility function defined over leisure when young, consumption and leisure when adult, and consumption when old. Preferences take the following form:

$$
\log (1 - e^i_{t-1}) + (1 + \eta) \log(1 - \lambda^i_t) + \log(e^i_t) + \rho \log(d^i_{t+1})
$$

(1)

where $\rho \in (0, 1)$ is the discount factor. For the sake of saving notation we will hereafter skip individual indices.

1.2 The firms

Technology is represented by a Cobb-Douglas production function with constant returns to scale. Skilled and unskilled labor are perfectly substitutable. Output is given by

$$
Y_t = K_t^\alpha (L_{1t} \beta L_{2t})^{1-\alpha}
$$

(2)
where $\beta \in (0,1)$ so that effective labor productivity of the unskilled is lower. The demand for skilled and unskilled labor, $L_{1t}$ and $L_{2t}$ respectively, is measured in efficiency units. The representative firm behaves competitively and hires labor and capital. It will choose in every period capital stock and next period effective labor so as to verify the usual first order conditions equalizing the marginal productivity of each production factor to its price. For simplicity, we assume full depreciation of capital from one period to another. Hence,

$$1 + r_t = \alpha K_t^{\alpha-1} (L_{1t} + \beta L_{2t})^{1-\alpha}$$

and

$$w_{1t} = (1 - \alpha) K_t^\alpha (L_{1t} + \beta L_{2t})^{-\alpha} \quad \text{and} \quad w_{2t} = \beta (1 - \alpha) K_t^\alpha (L_{1t} + \beta L_{2t})^{-\alpha}$$

from where

$$w_{2t} = \beta w_{1t}.$$ 

The wage per efficiency unit of the unskilled is lower than that of the skilled workers. Note also that the relative wage, $w_{2t}/w_{1t} = \beta$, which will play an important role in the decision to become skilled or not, is constant. This is due to the assumed perfect substitutability between skilled and unskilled labor in the production function. In section 4.2 we will discuss the possible consequences of relaxing this assumption.

## 2 Unemployment benefits

In this section we assume that the government provides an unemployment benefit $\Omega_t \geq 0$. Agents whose labor earnings are below a certain level will choose to join the social protection program and become unemployed. Hence, there is some critical level of productivity $z$ inducing a certain amount of labor earnings, —the reservation wage—, for which the individual is indifferent between working or being unemployed. The activity agents will carry out in the second period depends on their productivity endowment $z_t$, on their valuation of leisure, and on the unemployment benefit. Unemployment benefits are assumed to be financed by a proportional tax levied on the revenues of both employed and unemployed individuals.

### 2.1 The household’s behavior

Individuals have several decisions to make: in the first period they decide whether to become skilled or not. When adult they decide whether they will work or be unemployed; if working they have to choose the fraction of their time unit they will work. Finally, they choose how much to consume and how much to save from their
income. We solve this problem backwards. Savings are determined by maximizing utility (1) subject to the budget constraint

\[ c_t + s_t = \begin{cases} \Omega_{jt}(1 - \tau_t) & \text{when } z \leq \bar{z}_j \\ 2\lambda_{jt}u_{jt}w_{jt}(1 - \tau_t) & \text{otherwise} \end{cases} \quad (6) \]

\[ d_{2t+1} = (1 + r_{t+1}) s_t. \]

where \( w_{jt} \) and \( \Omega_{jt} \) are the wage per efficiency unit and the unemployment benefit for the skilled \((j = 1)\) and unskilled \((j = 2)\) individuals respectively. Optimal savings are then given by

\[ s_t = \begin{cases} \frac{\rho}{1 + \rho} \Omega_{jt}(1 - \tau_t) & \text{when } z \leq \bar{z}_j \\ \frac{\rho}{1 + \rho} 2\lambda_{jt}w_{jt}(1 - \tau_t) & \text{otherwise} \end{cases} \quad (7) \]

Individuals with a productivity endowment \( z > \bar{z}_j \) will choose to work; they will devote a fraction \( \lambda_{jt} \) of their time to work. They maximize their indirect utility with respect to \( \lambda_{jt} \), taking \( \tau_t \) and \( w_{jt} \) as given. In particular they solve

\[ \max_{\lambda_{jt}} (1 + \eta) \log(1 - \lambda_{jt}) + (1 + \rho) \log \lambda_{jt} + A, \]

where \( A \) represents the constant terms of the indirect utility function. This yields a constant optimal choice \( \lambda_{jt} = \lambda \) with

\[ \lambda = \frac{1 + \rho}{2 + \rho + \eta}. \quad (8) \]

Observe that (a) \( d\lambda/d\eta < 0 \), the more the individual values leisure, the less time he will devote to work, and (b) \( d\lambda/d\rho > 0 \), the higher \( \rho \), the more the individual weights consumption and therefore labor income so the more he will work. Note also that the fraction of time devoted to work is the same for skilled and unskilled; it does not depend on wages, nor on individual productivity \( z \). This is due to the type of preferences considered, additively separable between leisure and consumption, and homothetic over consumption.

In order to determine the fraction of population willing to work, we have to compute the critical productivity factor \( \bar{z}_{jt} \), that equalizing indirect utility when working and when unemployed:

\[ (1 + \eta) \log(1 - \lambda) + (1 + \rho) \log(\bar{z}_j \lambda w_{jt}) = (1 + \rho) \log(\Omega_{jt}) \]

We further assume that unemployment benefits are defined as \( \Omega_{jt} = \phi m w_{jt} \) for \( j = 1,2 \), with \( m \equiv (a + b)/2 \). That is, proportional to the average individual productivity over the entire population times the wage per efficiency unit of the
skilled and unskilled. Since $w_{2l} = \beta w_{1l}$, the unemployment benefit for the unskilled is lower than the one for skilled individuals. The critical level of the productivity endowment is then given by

$$z = \frac{\phi m}{\lambda (1 - \lambda)^{1+\rho}},$$

where $\lambda$ is given by (8). Individuals whose productivity endowment is below this critical level will choose to be unemployed, generating an unemployment rate of $(z - a)/(b - a)$ in both sectors. Unemployment rates are identical for both skills because the amount of the unemployment benefit is skill-related. The larger the unemployment benefit, the higher the fraction of individuals deciding to become unemployed. It is further easy to check that $dz/d\eta > 0$ and $dz/d\rho < 0$, that is, a higher valuation of leisure will induce more individuals to become unemployed, while if consumption is more valued, less individuals will be willing to renounce to paid labor resulting in higher income.

When deciding whether to become skilled or not, individuals compare the indirect utility they would obtain in both cases. At that time, an individual already knows whether he will work or be unemployed in the next period as $z$ does not depend on the qualification decision. Both future workers and future unemployed face a trade-off between the effort they have to carry out in order to become skilled, and the resulting higher income they would get in terms of higher labor earnings or unemployment benefits.\(^5\) Comparing indirect utilities, the level of effort $\tilde{e}$ characterizing an individual indifferent between getting qualified or not is given by

$$\tilde{e} = 1 - \beta^{1+\rho},$$

with $\tilde{e} \in [0, 1]$. Individuals with $e^i < \tilde{e}$ will decide to get skilled. Since $e^i$ follows a uniform distribution over $[0,1]$, expression (10) also represents the fraction of the population that decides to become skilled. Again, due to additive separability between effort and consumption, and because of homothetic preferences over consumption, this fraction is independent of individual productivity $z$. The critical effort $\tilde{e}$ is identical for workers and unemployed because the unemployment benefit has been defined proportionally to the wage per efficiency unit of skilled and unskilled workers.

\(^5\)It may seem odd that agents who decide not to work, because of their too low productivity, notwithstanding may decide to become qualified as they may obtain a higher unemployment benefit if doing so. A more reasonable way of presenting things would be to introduce uncertainty concerning the future productivity endowment $z$. This would imply that an agent, when deciding whether to become skilled or not, does not know yet if he will be working or not the next period. Due to the loglinear preferences, however, all the results in the stochastic version of the model in the unemployment benefit case are identical to the ones in the deterministic version. Introducing uncertainty would only complicate considerably and unnecessarily the basic income case.
Before proceeding with the competitive equilibrium, we have to redefine the critical level of individual productivity as

$$\bar{z} = \min \left\{ b, \max \left\{ a, \frac{\phi m}{\lambda (1 - \lambda)} \right\} \right\},$$

(11)
in order to correctly account for excessively low unemployment benefits, not inducing any unemployment, or excessively high benefits generating a 100% unemployment rate (corner solutions).

### 2.2 Competitive equilibrium

The government’s budget constraint equals tax revenues to government spendings on unemployment benefits. The equilibrium tax rate is constant $\tau_t = \tau$ for all $t$, and is given by

$$\tau = \frac{\phi m \int_{a}^{\bar{z}} dz}{\phi m \int_{a}^{\bar{z}} dz + \lambda \int_{\bar{z}}^{b} z dz}$$

(12)

Equilibrium on the labor market implies that the demand for labor is equal to the supply of both types of labor. Only those agents whose productivity factor $z$ is above $\bar{z}$ will choose to supply their efficiency units during the time fraction $\lambda$. The remain prefer to be unemployed and receive the unemployment benefit. Hence,

$$L_{1t} = \bar{e} \lambda \int_{\bar{z}}^{b} z f(z) dz \quad \text{and} \quad \beta L_{2t} = (1 - \bar{e}) \beta \lambda \int_{\bar{z}}^{b} z f(z) dz$$

(13)

where $f(z) = (b - a)^{-1}$ is the density function of the distribution. Aggregate savings are given by

$$S_t = \frac{\rho}{1 + \rho} (1 - \tau) (\bar{e} + (1 - \bar{e}) \beta) \left( \phi m \int_{a}^{\bar{z}} w_{1t} f(z) dz + \lambda \int_{\bar{z}}^{b} zw_{1t} f(z) dz \right)$$

The demand for capital should equal aggregate savings: the credit market clears when

$$K_{t+1} = S_t.$$  

(14)

Equations (4), (12), and the labor and credit markets clearing conditions yield the law of motion of capital

$$K_{t+1} = \frac{\rho}{1 + \rho} (1 - \alpha) \left[ (\bar{e} + \beta (1 - \bar{e})) \lambda \int_{\bar{z}}^{b} z f(z) dz \right]^{1-\alpha} K_t^{\alpha}.$$  

(15)
From (15) we compute the unique positive interior globally stable steady state $K$ for capital. Using (10) and (8), and after solving the integral, $K$ is obtained as

$$K = \left[ \frac{\rho}{1 + \rho} (1 - \alpha) \right]^{\frac{1}{1-\alpha}} \left[ (\bar{c} + \beta(1 - \bar{c}) \lambda \frac{b^2 - \bar{c}^2}{2(b - a)} \right].$$

Given the definition of $\bar{z}$ it is straightforward to see that capital is negatively related to the unemployment benefit. Wages and the interest rate at the steady state can easily be computed and are given by

$$w_1 = (1 - \alpha) \left( \frac{\rho(1 - \alpha)}{1 + \rho} \right)^{\frac{1}{1-\alpha}},$$

$$w_2 = \beta(1 - \alpha) \left( \frac{\rho(1 - \alpha)}{1 + \rho} \right)^{\frac{1}{1-\alpha}},$$

$$1 + r = \frac{\alpha(1 + \rho)}{\rho(1 - \alpha)}.$$

### 3 Basic income

In this section the government will provide a basic income, represented by an unconditional transfer $B_t \geq 0$. By unconditional we mean that it is paid to all individuals, independently of whether they are skilled or unskilled, and irrespective their productivity. In the broad interpretation a basic income is commonly understood to be paid to all citizens from the moment in which they reach majority until they die.

In the present model, however, we assume that the individuals are only entitled a basic income during their second life’s period. The reason for doing so is that extending the basic income to the third life’s period, when the individual retires, would require a counterpart in the same period in the unemployment benefit based social security system. As a basic income is supposed to replace all social security benefits, we should then provide in the unemployment benefit case an equivalent in terms of a retirement income. Including this third life’s period transfer would complicate substantially the model without contributing to the understanding of the differences between both systems. The basic income is assumed to be financed by a proportional tax $\tau_t$ on total revenue, labor earnings, and basic income.

#### 3.1 The household’s behavior

Here again an individual has to decide (a) whether to qualify or not, (b) the fraction of his unit of time he will devote to work, and (c) how much to save. We solve the
problem backwards. Optimal savings are the outcome of the maximization of utility (1) subject to the respective budget constraints

\[ c_t + s_t = (z\lambda_{jt}w_{jt} + B_t)(1 - \tau_t) \]
\[ d_{2t+1} = (1 + r_{t+1})s_t, \quad (18) \]

and are given by

\[ s_t = \frac{\rho}{1 + \rho}(z\lambda_{jt}w_{jt} + B_t)(1 - \tau_t), \quad (19) \]

where \( w_{jt} \) and \( \lambda_{jt} \) are the wage per efficiency unit and the fraction of time devoted to work by the skilled \( (j = 1) \) and unskilled \( (j = 2) \) individuals respectively. In order to decide the fraction of time to be devoted to work, an individual solves

\[ \max_{\lambda_{jt}} \quad (1 + \rho) \log(z\lambda_{jt}w_{jt} + B_t) + (1 + \eta) \log(1 - \lambda_{jt}) + A, \]

where \( A \) represents the constant terms of the indirect utility function. If we further assume that the basic income is defined as \( B_t = qnw_{1t} \), that is, proportional to the wage per efficiency unit of the skilled, the optimal fraction \( \lambda_t \) for the skilled is given by

\[ \lambda_1(z) = \begin{cases} \frac{(1 + \rho)z - (1 + \eta)qm}{(2 + \rho + \eta)z} & \text{when } z > \frac{1 + \eta qm}{1 + \rho qm} \\ 0 & \text{otherwise} \end{cases} \]

and for the unskilled

\[ \lambda_2(z) = \begin{cases} \frac{(1 + \rho)z - (1 + \eta)qm}{(2 + \rho + \eta)z} & \text{when } z > \frac{1 + \eta qm}{1 + \rho \beta} \\ 0 & \text{otherwise} \end{cases} \]

From these first order conditions we immediately see that the unskilled work, on the average, a smaller fraction than the skilled. Since their wage per efficiency unit is smaller, the opportunity cost of leisure is also smaller. Observe also that individuals whose productivity endowment is below a critical level will prefer not to work at all: they will devote their entire time unit to leisure. The optimal fraction \( \lambda_j(z) \) is positively related to the individual productivity. Indeed, the higher the productivity, the higher the opportunity cost of leisure. A basic income lowers the fraction devoted to work, since it provides the individual with an unconditional income, allowing him to take more into account the leisure aspect of his utility. Increasing the relative weight of leisure \( 1 + \eta \) also decreases \( \lambda_j(z) \) while increasing \( \rho \) will generate an increase in \( \lambda_j(z) \) as consumption now becomes relatively more important.

The critical level of effort \( \bar{e} \) for which an individual characterized by a productivity \( z \) is indifferent between qualifying or not is the one that equals both indirect utilities

\[ \log(1 - \bar{e}(z)) + (1 + \eta) \log(1 - \lambda_1(z)) + (1 + \rho) \log(z\lambda_1(z) + qm) \]
\[ = (1 + \eta) \log(1 - \lambda_2(z)) + (1 + \rho) \log(z\lambda_2(z)\beta + qm). \]
Let us first of all define some critical levels concerning the individual productivity endowment:

\[
z_1 = \min \left\{ b, \max \left\{ a, \frac{1 + \eta}{1 + \rho} qm \right\} \right\} \quad \text{and} \quad z_2 = \min \left\{ b, \max \left\{ a, \frac{1 + \eta}{1 + \rho} \beta \right\} \right\}.
\]

Hence \( z_1 \) and \( z_2 \) are the levels below which skilled and unskilled individuals respectively will devote no time at all to work. Based on their productivity endowment \( z \) we can distinguish three groups of individuals.

- **Case 1**, \( z < z_1 < z_2 \). Individuals are so little productive, qualified or not, that they choose not to work at all \( (\lambda_1 = \lambda_2 = 0) \). The disutility of work dominates its positive effect on income. Stated differently, the opportunity cost of leisure, which is foregone income, is very small. Hence, none of the individuals characterized by such a low productivity will decide to become qualified, no matter the (small) effort this may require:

\[
\bar{e}_1(z) = 0 \quad \text{for} \quad z < z_1. \tag{22}
\]

- **Case 2**, \( z > z_2 > z_1 \). These individuals are endowed with a sufficiently high productivity so as to devote time to work, whether qualified or not: \( u_1, u_2 > 0 \). The critical level of effort, for which an individual endowed with a productivity \( z \) is indifferent between qualifying or not (also the fraction of individuals belonging to this group who will become qualified), is given by

\[
\bar{e}_2(z) = 1 - \left( \frac{z \lambda_2(z) \beta + qm}{z \lambda_1(z) + qm} \right)^{1+\rho} \left( \frac{1 - \lambda_2(z)}{1 - \lambda_1(z)} \right)^{1+\eta} \quad \text{for} \quad z > z_2. \tag{23}
\]

After substituting for \( \lambda_1(z) \) and \( \lambda_2(z) \) for their respective expressions we get

\[
\bar{e}_2(z) = 1 - \left( \frac{z \beta + qm}{z + qm} \right)^{2+\rho+\eta} \left( \frac{1}{\beta} \right)^{1+\eta} \quad \text{for} \quad z > z_2. \tag{24}
\]

The proportion of individuals willing to get skilled is increasing in the individual productivity because \( z \) positively affects the relative increase in labor earnings when skilled, compared to when unskilled.

- **Case 3**, \( z_1 < z < z_2 \). In this case the productivity level of individuals is such that when qualified they work a fraction of their time \( (\lambda_1 > 0) \), but when not, they prefer not to work at all \( (\lambda_2 = 0) \) due to the low opportunity cost of leisure. Hence, when deciding whether to qualify or not, an individual has to take this into account. If qualifying he will earn a higher wage per efficiency unit than if not. Nevertheless, he has to cope with two sources of disutility—of qualifying and of working—, which do not operate if he decides not to
qualify. In other words, the earnings differential he obtains if qualified has to be sufficiently large so as to compensate the disutilities of both qualifying and working.

In order to decide whether to qualify or not, the individual compares the utilities he would obtain in both cases. The level of effort equalizing these two utilities is the critical level below which an individual will decide to become qualified. From

$$\log(1 - \bar{e}(z)) + (1 + \eta) \log(1 - \lambda_1(z)) + (1 + \rho) \log(z \lambda_1(z) + q m) = (1 + \rho) \log(q m),$$

we obtain

$$\bar{e}_3(z) = 1 - \left( \frac{q m}{z \lambda_1(z) + q m} \right)^{1+\rho} \frac{1}{(1 - \lambda_1(z))^{1+\eta}}. \quad (25)$$

After substituting for $\lambda_1(z)$ we get

$$\bar{e}_3(z) = 1 - \left( \frac{q m}{1 + \rho} \right)^{1+\rho} \left( \frac{z}{1 + \eta} \right)^{1+\eta} \left( \frac{2 + \rho + \eta}{z + q m} \right)^{2+\rho+\eta}$$

for $z_1 < z < z_2$. Here, again, the proportion of skilled will increase with the individual productivity of the individuals considered.

From (22), (24) and (26) we can compute the aggregate proportion of skilled individuals in the economy by summing these proportions over all individuals:

$$E^* = 1 - \int_{z_1}^{z_2} f(z) \, dz$$

Contrary to the case with an unemployment benefit, where the fraction of skilled is not affected by the presence of such a benefit, the basic income does affect the proportion of skilled. It can be easily verified that $dE^*/d(q m) < 0$. This results from two negative effects: first, the basic income, as it provides all individuals with a same unconditional transfer, decreases the income differential between skilled and unskilled, therefore decreasing the incentive to become skilled; second, the basic income also increases the fraction of people that decides not to work at all and therefore not to get skilled.

It is further straightforward to show that $\rho$ and $\eta$ have positive and negative effects, respectively, on the proportion of skilled. These effects again concern the relative weight of consumption and leisure in the decision making: the higher $\rho$, the more important is consumption relative to leisure, the larger will be the fraction of time unit devoted to work, and hence the more rewarding it is to do the effort of becoming skilled. For higher values of $\eta$ the reverse reasoning applies.
3.2 Equilibrium and dynamics

In order to get the budget balanced, the government should fix the tax rate $\tau_t$ such that tax revenues are equal to the government spendings on basic income. That is

$$\tau_t \left[ qm w_{jt} + \int_a^{z_1} 0 f(z) \, dz + \int_{z_1}^{z_2} \nu_2(z) z_1(z) w_{1t} f(z) \, dz + \int_{z_2}^b \nu_2(z) z_1(z) w_{1t} f(z) \, dz \right] = B_t$$

which implies $\tau_t = \tau$ for all $t$ with

$$\tau = \frac{qm}{qm + \int_{z_1}^{z_2} \nu_2(z) z_1(z) \, dz + \int_{z_2}^b \nu_2(z) z_1(z) + (1 - \nu_2(z)) z_2(z) \beta \, dz}$$

where we used (4) and $B_t = qm w_{1t}$.

At each period $t$ aggregate effective labor supplied by the skilled and the unskilled households should equal demand in the respective sectors in order to have the labor market clear. That is,

$$L_{1t} = \int_{z_1}^{z_2} \nu_2(z) z_1(z) f(z) \, dz + \int_{z_2}^b \nu_2(z) z_1(z) f(z) \, dz$$

and

$$\beta L_{2t} = \int_{z_2}^b (1 - \nu_2(z)) z_2(z) \beta f(z) \, dz.$$

The demand for credit consists of the investment $K_{t+1}$. The capital market clearing requires investment in physical capital to be equal to aggregate savings, the supply of credit. That is,

$$K_{t+1} = S_t$$

where

$$S_t = \frac{\rho}{1 + \rho} (1 - \tau) \left[ B_t + \int_{z_1}^{z_2} \nu_2(z) z_1(z) w_{1t} f(z) \, dz + \int_{z_2}^b \nu_2(z) z_1(z) w_{1t} f(z) \, dz \right. + \left. \int_{z_1}^{z_2} (1 - \nu_2(z)) z_2(z) w_{2t} f(z) \, dz \right].$$

Equations (4), (27), the labor and credit market clearing conditions, the definition of the basic income, and after substituting for $w_{1t}$, yield the law of motion of capital

$$K_{t+1} = \frac{\rho}{1 + \rho} (1 - \alpha) \left[ \int_{z_1}^{z_2} \nu_2(z) z_1(z) f(z) \, dz + \int_{z_2}^b \nu_2(z) z_1(z) f(z) \, dz \right.$$  
$$+ \left. \int_{z_2}^b (1 - \nu_2(z)) z_2(z) \beta f(z) \, dz \right]^{1-\alpha} K_t^{\alpha}.$$

$$\int_{z_1}^{z_2} \nu_2(z) z_1(z) f(z) \, dz + \int_{z_2}^b \nu_2(z) z_1(z) f(z) \, dz$$

with $\nu_2(z) z_1(z) + (1 - \nu_2(z)) z_2(z) \beta \, dz$ and $\beta L_{2t} = \int_{z_2}^b (1 - \nu_2(z)) z_2(z) \beta f(z) \, dz$.  

The demand for credit consists of the investment $K_{t+1}$. The capital market clearing requires investment in physical capital to be equal to aggregate savings, the supply of credit. That is,

$$K_{t+1} = S_t$$

where

$$S_t = \frac{\rho}{1 + \rho} (1 - \tau) \left[ B_t + \int_{z_1}^{z_2} \nu_2(z) z_1(z) w_{1t} f(z) \, dz + \int_{z_2}^b \nu_2(z) z_1(z) w_{1t} f(z) \, dz \right.$$  
$$+ \left. \int_{z_1}^{z_2} (1 - \nu_2(z)) z_2(z) w_{2t} f(z) \, dz \right].$$

Equations (4), (27), the labor and credit market clearing conditions, the definition of the basic income, and after substituting for $w_{1t}$, yield the law of motion of capital

$$K_{t+1} = \frac{\rho}{1 + \rho} (1 - \alpha) \left[ \int_{z_1}^{z_2} \nu_2(z) z_1(z) f(z) \, dz + \int_{z_2}^b \nu_2(z) z_1(z) f(z) \, dz \right.$$  
$$+ \left. \int_{z_2}^b (1 - \nu_2(z)) z_2(z) \beta f(z) \, dz \right]^{1-\alpha} K_t^{\alpha}.$$

$$\int_{z_1}^{z_2} \nu_2(z) z_1(z) f(z) \, dz + \int_{z_2}^b \nu_2(z) z_1(z) f(z) \, dz$$

with $\nu_2(z) z_1(z) + (1 - \nu_2(z)) z_2(z) \beta \, dz$ and $\beta L_{2t} = \int_{z_2}^b (1 - \nu_2(z)) z_2(z) \beta f(z) \, dz$.  

The demand for credit consists of the investment $K_{t+1}$. The capital market clearing requires investment in physical capital to be equal to aggregate savings, the supply of credit. That is,

$$K_{t+1} = S_t$$

where

$$S_t = \frac{\rho}{1 + \rho} (1 - \tau) \left[ B_t + \int_{z_1}^{z_2} \nu_2(z) z_1(z) w_{1t} f(z) \, dz + \int_{z_2}^b \nu_2(z) z_1(z) w_{1t} f(z) \, dz \right.$$  
$$+ \left. \int_{z_1}^{z_2} (1 - \nu_2(z)) z_2(z) w_{2t} f(z) \, dz \right].$$

Equations (4), (27), the labor and credit market clearing conditions, the definition of the basic income, and after substituting for $w_{1t}$, yield the law of motion of capital

$$K_{t+1} = \frac{\rho}{1 + \rho} (1 - \alpha) \left[ \int_{z_1}^{z_2} \nu_2(z) z_1(z) f(z) \, dz + \int_{z_2}^b \nu_2(z) z_1(z) f(z) \, dz \right.$$  
$$+ \left. \int_{z_2}^b (1 - \nu_2(z)) z_2(z) \beta f(z) \, dz \right]^{1-\alpha} K_t^{\alpha}.$$
From this expression we compute the unique positive interior globally stable steady state for capital. Using (10) and (8) and after solving the integral we get

\[
K = \left[ \frac{\rho(1 - \alpha)}{1 + \rho} \right]^{1-\alpha} \left[ \int_{z_1}^{z_2} \bar{e}_3(z) z \lambda_1(z) f(z) \, dz + \int_{z_2}^{b} \bar{e}_2(z) z \lambda_1(z) f(z) \, dz 
+ \int_{z_2}^{b} (1 - \bar{e}_2(z)) z \lambda_2(z) \beta f(z) \, dz \right].
\] (30)

Given that both the proportion of skilled individuals and the fraction of time devoted to work are negatively affected by the basic income, the latter will have a non ambiguous negative effect on the steady state value of capital. Wages and interest rate at the steady state can be computed and checked to be identical to the ones in the unemployment benefit case.

4 Comparison in the long run

In this section we examine the effects at the steady state of the respective social security systems on some macroeconomic indicators: economic performance, income inequality, and social welfare.

4.1 Economic performance and distributional issues

As we will see, there is at best a trade-off between economic performance and less income inequality. In the case of an unemployment benefit, however, both measures are likely to worsen. Even in the case of a basic income, both economic performance and income inequality can worsen.

We represent economic performance by aggregate steady state consumption over all individuals (see appendix A for the explicit definition of aggregate consumption).

**Proposition 1** We have

\[
\frac{dC(\phi)}{d\phi} < 0 \quad \text{and} \quad \frac{dC(q)}{dq} < 0.
\]

That is, both an unemployment benefit and a basic income negatively affect economic performance as they decrease steady state aggregate consumption.

See appendix A for the proof. The basic income decreases the fraction of the qualified, those relatively more productive in the economy. It further negatively affects the aggregate labor supply, and therefore production, in two ways. On one hand, it decreases the fraction of time an individual is ready to devote to work. On the
other hand, it induces some people not to work at all. Indeed, the basic income, by providing an unconditional income, allows people to quit work and still receive an income. This effect is likely to be stronger among the unskilled and among individuals endowed with a small productivity because of their smaller opportunity cost of leisure. In the case of an unemployment benefit, the proportion of skilled is not affected. Nevertheless, it induces some agents to become unemployed and get the unemployment benefit, thereby reducing labor supply, hence production and ultimately consumption.

The distributional effects are assessed by the Atkinson index, a measure of income inequality. It is defined as $I = 1 - \exp\left[\sum_{i=1}^{n} f_i \log\left(\frac{Y_i}{\bar{Y}}\right)\right]$ (see Atkinson (1983)). $Y_i$ denotes the post-tax income of those in the $i$th income range, $f_i$ denotes the proportion of the population with incomes in the $i$th range, and $\bar{Y}$ denotes the post-tax mean income (see appendix A for an explicit expression of $I$). The larger $I$, the more important is income inequality. It should be pointed out, however, that our concern for the degree of income inequality corresponds to a positive, rather than a normative approach, in the sense that equality is not an objective per se. As already emphasized, from the economist’s point of view there is no doubt that, unless one is willing to accept important efficiency losses, equality cannot be achieved because individuals influence their own outcomes.

We evaluate the effect of an increase in the unemployment benefit when the latter is close to the level at which no effect is generated on the economy. This critical level is the one which makes the least productive individual (either skilled or unskilled) indifferent between working or being unemployed.

**Proposition 2** We have

$$\lim_{\phi \to \frac{2^{1-n} + 1}{n}} \frac{dI(\phi)}{d\phi} > 0.$$

That is, the introduction of an unemployment benefit will, at low levels, always increase income inequality at the steady state.

See appendix B for the proof. The valuation of leisure explains this rather counterintuitive result: leisure induces some individuals, who would have been able to earn a wage superior to the benefit, to quit their jobs in exchange of an unemployment benefit and a higher level of leisure time. The result is an increase in inequality: a fraction of the population (the less productive) renounces to their wage and receives a benefit lower than the wage, while the rest of the population keeps on working and earning as much as before (at least in terms of gross income).

As far as the basic income is concerned, matters are quite more complicated. Contrary to the unemployment benefit, it affects the decisions whether to become

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6 This is an appropriate way of measuring income dispersion as it abstracts of scale effects, for example, due to the height of the wage per efficiency unit.
skilled, and the fraction of time devoted to work. The economy’s equilibrium has been simulated numerically for a range of parameter’s specifications. The results show how inequality might be affected in either way by the introduction of a basic income. In particular, inequality can be raised or decreased depending on the relative productivity of skilled and unskilled workers. Indeed, for \( q \) close to zero, an increase of \( q \) will increase \( I \) for low \( \beta \) and decrease it for higher values of \( \beta \) (see appendix C).

The interpretation of this result is the following: a basic income is by definition the same amount given to everybody, so one would expect it to be inequality reducing. But it also generates other effects going in the opposite direction and which may eventually offset this inequality decreasing effect. The introduction of a basic income will induce all individuals to devote less time to work. Unskilled individuals will, however, reduce their working time to a larger extent than the skilled, thereby augmenting the inequality in terms of labor earnings. This is due to the lower opportunity cost of leisure for the unskilled (they only lose \( \beta w_1 \) as compared to \( w_1 \) per efficiency unit). The asymmetry in the response between skilled and unskilled will be stronger the lower \( \beta \). Indeed, the lower \( \beta \), the more different will be the unskilled compared to the skilled in their respective behavior. On the contrary, as \( \beta \) approaches 1, the more they are alike, and the more they will react in a similar way to changes in the economy, leading to less income inequality.

Numerical exercises are needed if the quantitative differences are to be assessed. We adopt the following settings for the parameters: while \( \alpha = 0.3 \) the technology parameter \( \beta \) is set to 0.75, implying that 31% of the population get skilled. (In the United States, the percentages of adult population having achieved non university superior education and university are 30% (see OECD (1993, page 86))). The subjective discount rate \( \rho \) is set to 0.3, guaranteeing a positive annual interest factor of 2.5% (each generation is assumed to last for 25 years). The weight of leisure in utility \( 1 + \eta \) is set to 0.5. The reason for considering a negative \( \eta \) is that we want leisure to have less weight \((<1)\) in utility than consumption in the first period (leisure only concerns the first period when the individual works and not the second one when he is retired). Underlying is the assumption that consumption is needed for the joy of leisure. The lower and upper bound \( a \) and \( b \) of the productivity endowment \( z \) are set to 2 and 4 respectively. Taking into account the technology parameter, this gives us a wage dispersion characterized by the ratio highest to lowest wage equal to 2.66, which can be considered as reasonable: in Germany and the UK, the ratios between deciles of the earnings distribution are given by \( D9/D1 = 2 \) and \( D9/D1 = 2.37 \) respectively (see Nickell and Bell (1996, page 310)).

We compute aggregate consumption and the inequality index for different levels of the unemployment benefit and basic income, and plot this range of pairs of values out on a same graph. In particular we let \( q \) vary between its two extreme values, \( (a\lambda (1-\lambda)^{(1+\eta)/(1+\rho)})/m \) and \( (b\lambda (1-\lambda)^{(1+\eta)/(1+\rho)})/m \). These are the levels at which nobody and everybody decide to become unemployed respectively. The basic income
parameter $\phi$ varies from 0 to 2.5.

The dashed line on figure 1 consists of the pairs of aggregate consumption and income inequality achieved for the range of the different levels of basic income under consideration; the plain line represents the same but for the different levels of unemployment benefit. The point from which both lines start, coincides with the laissez-faire economy, where $q \leq \frac{a\lambda(1 - \lambda)^{(1+\eta)/(1+\rho)}}{m}$ and $\phi = 0$. If we worry about income distribution and total consumption we observe that we are unambiguously better off with a basic income. This is due to the fact that the introduction of an unemployment benefit, besides decreasing consumption, initially always increases income inequality, while the basic income decreases it. This inequality enhancing effect on behalf of the unemployment benefit is obviously only observed for small unemployment benefits. As the benefit further increases, income inequality will decline as an ever increasing part of the population will decide to get unemployed, receiving an identical benefit (at least within the group of the skilled and unskilled individuals). However, from figure 1 we observe that the unemployment benefit never succeeds to catch up with the basic income in terms of income equality. Moreover, the increase in equality is at the expense of aggregate consumption.

### 4.2 Complementarity between skilled and unskilled labor

In this section we introduce a certain degree of complementarity between the two types of labor in order to examine its impact on the results obtained this far. Let
us consider a production function of the type

\[ \frac{Y_t}{K_t} = \frac{L_{1t}^\gamma + (\beta L_{2t})^{-\gamma}}{1 + \gamma}, \]

where \( \gamma \in [-1, \infty) \) represents the degree of complementarity: the higher \( \gamma \), the more skilled and unskilled labor are complementary production factors (\( \gamma = -1 \) corresponds to the case of perfect substitution). Deriving the first order condition for this firm we obtain the following relationship between the wage of skilled and unskilled labor:

\[ \frac{w_{1t}}{w_{2t}} = \beta^\gamma \left( \frac{L_{2t}}{L_{1t}} \right)^{1+\gamma}. \]

When imposing a certain degree of complementarity, the relative wage is no longer a constant: it depends on the ratio of the unskilled to the skilled labor force.

In order to assess the effect of the degree of complementarity \( \gamma \) on the individual’s behavior, we first of all notice that

\[ \frac{d(w_1/w_2)}{d\gamma} \geq 0 \quad \text{when} \quad L_1 \leq \beta L_2, \]

\[ \frac{d(w_1/w_2)}{d\gamma} < 0 \quad \text{otherwise}. \quad (31) \]

An increasing degree of complementarity between the two types of labor will further increase the wage differential \( w_1/w_2 \) when the amount of skilled labor in terms of efficiency units is smaller than the amount of unskilled labor hired by the firm, and vice versa. That is, the more skilled and unskilled labor are complements, the more it becomes optimal to hire an equal amount of both types of labor. This is indicated by an increase of the relative wage of the more scarce labor type.

Since the fraction of population that decides to become skilled increases with the wage differential \( w_1/w_2 \), the effect of \( \gamma \) on \( e \) goes in the same direction as the effect of \( \gamma \) on \( w_1/w_2 \). This is the case for our three economies (the benchmark economy, the one with unemployment benefits, and the one with a basic income). In the benchmark and unemployment benefit cases, this is the only effect observed: the fraction of the time unit \( \lambda \) spent on working, and the unemployment rate are not affected by \( \gamma \). Stated differently, \( \gamma > -1 \) will not influence the way an unemployment benefit affects the economy.

In the case of the basic income economy, \( \gamma \) does affect the fraction of time devoted to work by the unskilled: as \( \lambda_2 \) negatively depends on the wage differential \( w_1/w_2 \), a change in the degree of complementarity will generate a change in \( \lambda_2 \) through its effect on \( w_1/w_2 \). Moreover, the extent to which a basic income will (negatively) affect \( \lambda_2 \) and the proportion of skilled, depends on the degree of complementarity: depending on the ratio skilled to unskilled labor, a certain degree of complementarity will either reinforce or attenuate these negative effects through the prices of labor. More precisely, when \( L_1 > \beta L_2 \), the negative effect of a basic income on the fraction of skilled is likely to be reinforced through a decrease in the wage differential; its
negative effect on $\lambda_2$, on the contrary, is likely to weaken. The reverse holds when $L_1 < \beta L_2$. Our conjecture is therefore that allowing for complementarity will not qualitatively change our previous results.

4.3 Social Welfare

Social welfare is represented by aggregate utility: the sum over all individuals of the utility of individual net consumption, leisure, and disutility of educational effort. Although a weak notion of welfare,\(^7\) it helps illustrating some of the effects of these social security systems. We will analyze the effect of the introduction of an unemployment benefit and a basic income on aggregate utility, and study the different components of this effect.

**Proposition 3** The introduction of an unemployment benefit decreases aggregate utility at the steady state for low levels of $\phi$.

The effect on aggregate indirect utility of an unemployment benefit, valued in its limit is given by

$$
\lim_{\phi \to m \lambda (1 - \lambda)^{1+\eta}} \frac{dU}{d\phi} = \frac{m}{b - a} \left[ \frac{1 + \rho}{\lambda (1 - \lambda)^{1+\eta}} \log(1 - \lambda) \right]^{1+\eta} \\
- \frac{1 + \eta}{\lambda (1 - \lambda)^{1+\eta}} \log(1 - \lambda) - \frac{m (1 + \rho)}{m \lambda}.
$$

The unemployment benefit induces a fraction of people to get unemployed, but does not affect the fraction of time devoted to labor, nor the fraction of people who become skilled. Hence, the only positive effect in terms of utility is generated by the increase of leisure on behalf of the unemployed. Because leisure is valued, some agents are indeed willing to renounce to potential labor earnings which are superior to the benefit they receive.

The first term in (32) represents the loss of utility due to the decrease in consumption of the individuals who are induced to become unemployed. This loss is, however, exactly offset by the accompanying utility gain due to the increased leisure enjoyed by the unemployed. The latter is represented by the second term. The cancellation of these two terms is to be expected since they result from the actions of the individuals implying marginal gain of getting unemployed (increased leisure) to equal its marginal loss (the decrease in consumption). As for the third term, it represents the loss of utility due to the decrease in aggregate consumption caused

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\(^7\)Utilitarianism belongs to the traditional welfarist criterion. It is nowadays criticized for representing a cardinal rather than an ordinal approach. See Pazner and Schmeidler (1978) and Varian (1974) for a discussion along these lines.
in turn by the taxes introduced to finance the benefit. Since taxes are considered as given in the individual decisions, the individuals have no grasp on this loss.

**Proposition 4** The introduction of a basic income increases aggregate utility at the steady state for low levels of $q$.

As far as the basic income is concerned, matters are again quite more complicated, as more effects play a role now. We have

$$
\lim_{q \to 0} \frac{dU}{dq} = m \log \left( \frac{b}{a} \right) [(1 + \rho)(1 + \beta^p (1 - \beta))
+ (1 + \rho)\beta^p (1 - \beta)(2 + \rho + \eta) \log \beta
- (1 + \rho)\beta^p (1 - \beta)(2 + \rho + \eta) \log \beta
+ (1 + \eta)(1 + \beta^p (1 - \beta))]
- \frac{(b - a)(2 + \rho + \eta)}{(1 - \beta(1 + \rho)(1 - \beta)).}
$$

The first term in expression (33) represents the increase of aggregate utility caused by an increase in consumption due to the introduction of a basic income. It reflects the fact that all individuals are given an extra income they can consume. This effect is net from the negative effect on consumption caused by the fact that a basic income reduces time devoted to work and hence labor earnings. The second term refers to the loss due to the decrease in consumption on behalf of those who, as a result of the basic income, decide not to become skilled, while they would have done so in its absence.

As (33) clearly shows, this effect is exactly offset by the third term representing the gain in utility due to a decrease in the disutility of educational effort, as less individuals become qualified. In short, the marginal increase in utility of not qualifying is equal to its marginal cost. The fourth term indicates an increase in utility generated by increased leisure. Finally, the last term represents the loss of utility due to the decreasing aggregate consumption because of the taxes levied. Expression (33) reduces to

$$
m(2 + \rho + \eta) \left[ \log \left( \frac{b}{a} \right) (1 + \beta^p(1 - \beta)) - \frac{b - a}{m(1 - \beta(1 + \rho)(1 - \beta))} \right],
$$

which is always positive (see appendix D for a proof of this statement). Despite the taxes and the decreased production, aggregate utility increases in the presence of a small basic income. It must be the case, therefore, that the decrease in aggregate consumption (reducing total welfare) is being more than compensated by an increase in equality (increasing total welfare as marginal utility is decreasing).
As we depart from this limiting case matters become more subtle. Again numerical exercises will shed some light on the compared effects of these systems when we consider higher unemployment benefits and a higher basic income. We use the same parameters’ setting as in the previous section and plot the aggregate utility for a range of levels of the unemployment benefit and of the basic income.

From figure 2 we see that our analytical results in the limit are confirmed. Whenever a sufficiently attractive unemployment benefit is introduced, it decreases aggregate utility (the plain curve). This is because of the drastic fall in production, and therefore consumption, generated by an unemployment benefit. As far as a basic income is concerned, we observe an initial increase in utility (the dashed curve). We also see from figure 2 that the optimal basic income in terms of aggregate utility corresponds to almost 10% of what would be the average wage income in the laissez-faire economy. Higher levels of basic income discourage too much the incentive to work reducing labor supply (λ reduces and eventually becomes 0), production and ultimately consumption which finally decreases aggregate utility. Indeed, considering too high a level of basic income or unemployment benefits in general is not of great interest, as they are likely to generate a considerable fall in production which cannot possibly be the objective of a social security system.

Summarizing, the rapidly decreasing production in combination with the increased income inequality is responsible for the aggregate utility reducing effect of an unemployment benefit. The introduction of a reasonable basic income (< 10% of average earnings in the economy) increases aggregate utility despite the drop in production. This is due to the redistribution of income from the richer to the poorer.
Examining aggregate utility does not tell us anything about the distribution of the increased or decreased utility among the different individuals of the economy. In the next section we analyze the impact of both social security systems on individual utility.

### 4.4 Individual welfare

To analyze who is loosing in terms of utility and who is winning (if any), in this section we derive the conditions for each individual to benefit from the introduction of an unemployment benefit and a basic income. In order to do so we compare the utility each type of individual obtains in both social security systems with the one in the laissez-faire economy.

An unemployment benefit can positively affect the utility of individuals if they choose to give up work. Indeed, for the individuals who decide to keep on working, nothing changes except for the taxes they have to pay. The introduction of an unemployment benefit will consequently affect them exclusively negatively. In appendix E we prove that the individuals whose productivity is less than

\[
\phi m (1 - \tau) \left( \frac{1 + \eta}{1 + \rho} \right)^{2 + \eta + \rho} \equiv z^* \in (a, b)
\]

will benefit in terms of utility from an unemployment benefit; the others will all loose. Using (9) it can be deduced that that 

\[
z^* = \bar{z} (1 - \tau) < \bar{z} ;
\]

this is, the critical productivity level above which an individual will loose because of the benefit is inferior to the one below which an individual chooses to become unemployed. Stated differently, not all individuals who decide to become unemployed are better off in terms of utility compared to the laissez-faire situation. From the definition above we can further easily observe that if the unemployment benefit is too high, it would imply too high a tax which may finally decrease \(z^*\) below its lower bound \(a\). In that case an unemployment benefit will make everyone worse off in terms of utility due to the combination of the reduced production and high taxes. The critical level of \(\phi\) above which everyone is worse off is given by

\[
\left( \frac{a^2 + b^2}{(1 - \lambda)^{2(1+\eta)/(1+\rho)}} - a \right) \lambda (1 - \lambda)^{2(1+\eta)/(1+\rho)} \equiv \phi^*
\]

See appendix F for a proof. We also know because of (11) that in order to have any effect, \(\phi\) should be greater than \(a \lambda (1 - \lambda)^{(1+\eta)/(1+\rho)}/m\). In other words, only if \(\phi^* > a \lambda (1 - \lambda)^{(1+\eta)/(1+\rho)}/m\), the introduction of an unemployment benefit may increase the utility of some individuals. In appendix G we prove that the necessary
condition is given by
\[
\left( \frac{b}{a} \right)^2 > 1 + 2(1 - \lambda)\frac{1+\eta}{1+\rho}.
\] (34)

Expression (34) implies that when the disparity between individual productivities is not important enough, introducing an unemployment benefit will decrease the utility of all individuals. This is due to the fact that in that case, the benefit on one hand has to be relatively high (compared to \(b\)) in order to have some effect, which on the other hand, however, rapidly decreases labor supply and thus production.

As far as the case of a basic income is concerned, we divide all individuals in different classes according to their productivity endowment and effort needed to become skilled. We then analyze under which conditions the different individuals eventually gain from a basic income. These conditions, summarized in table 1, consist of the maximum level of the tax rate and hence the maximum amount of the basic income which still provides the individual with a higher utility compared to the laissez-faire (see the appendix H for the computation of these conditions). This implies that there exists in general some positive level of basic income that makes most of the individuals better off. However, this level is limited since a huge basic income implies too high a drop in production which finally makes the individual worse off compared to the laissez-faire economy.

From table 1 we observe that the maximum tax rate decreases with individual productivity and increases with effort required to become skilled. In other words, the less an individual is productive and the higher the effort, the higher can be the basic income without making him worse off compared to the laissez-faire. From table 1 we can also deduce that the utility of almost all individuals increases due to the basic income, provided the latter in not too high. The improvement is, however, not Pareto. Those individuals with the highest productivity (\(z = b\)) and the lowest effort (\(e = 0\)) will always loose when a basic income is introduced, no matter how small it is. Whether individuals characterized by a high productivity (\(z > z_2\)), but also by higher effort requirements \(e\) loose or win (the cases marked by an asterisk in table 1), depends on the other parameters like \(\beta\).

Summarizing, a basic income is indeed likely to redistribute welfare from the richest to the poorest. In fact, the group of individuals characterized by the highest productivity and the lowest effort will always be worse off in the presence of a basic income. The extent to which other groups of individuals will benefit from a basic income depends on its level. The higher the basic income, the smaller has to be individual productivity and the higher effort required to become skilled in order to gain in terms of utility; up to the upper limit above which all individuals loose in utility because of the decreased production and high taxes. We can conclude that

\textsuperscript{8}If there were no leisure in the model (\(\lambda = 1\)), condition (34) would always be satisfied provided that \(b > a\)
an unemployment benefit also give rise to a certain degree of redistribution from the individuals who work towards the ones who decide to become unemployed. Two conditions have however to be fulfilled: the benefit may not be too high and a minimum disparity in individual productivities is required. Otherwise the introduction of an unemployment benefit will decrease the utility of all individuals.

5 Concluding remarks

In this paper two social security systems are compared: one based on unemployment benefits, the other based on a basic income. We concentrated on the effects both systems are likely to have on individual economic incentives and, when translating these into the aggregate, on economic performance, income inequality and social welfare in the long run. The effect of both systems on economic performance is negative, as they both tend to reduce labor supply and hence production. In the unemployment benefit case, it is the resulting unemployment which is the main responsible for this phenomenon. The basic income produces this effect through a decrease of the proportion of skilled individuals and a reduction of time devoted to work.
Income inequality initially worsens when an unemployment benefit is introduced, the cause of which is to be found in the valuation of leisure: the latter induces individuals to renounce to labor earnings that are higher than the unemployment benefit they receive. In the basic income case the valuation of leisure will induce all individuals to reduce their working time. This time reduction will, however, be relatively more drastic for unskilled individuals, and it is this asymmetric answer which may initially also generate more inequality. Nevertheless, for plausible parameters’ settings a basic income will decrease income inequality. In general, when further increasing the basic income or unemployment benefits, inequality unambiguously reduces. This occurs, however, at the expense of production confirming the trade-off between a more equal income distribution and economic performance.

As far as social welfare is concerned (measured as aggregate utility), it will always decrease when introducing an unemployment benefit because of the reduction in consumption. Although the latter is also observed in the case of a basic income, it is less severe and it can be more than compensated by the redistributional effect. It is this last effect that allows for a social welfare improvement, provided the basic income it is not too high as it otherwise also discourages too much productive activities.

A crucial difference between the basic income and the unemployment benefit advocating in favor of a basic income, is that a basic income is not incompatible with paid work. In contrast, the unemployment benefit implies that the beneficiary is unemployed by definition. It is clear that as long as a low skilled individual has to choose between doing a low-paid job or being unemployed and receiving a benefit which is superior (or not even that) to his potential wage, he will opt for the second. A basic income scheme might, however, induce many low skilled to also take a job or to not quit it, even if it is low paid. This may constitute a way to fight the unemployment trap.

To draw even preliminary conclusions on whether the basic income scheme is to be preferred to the actual social security system lies obviously beyond the scope of this paper. The present analysis is far from being complete, not in the least because of the underlying assumptions of a competitive economy, complete markets, and perfect information which prevent from taking into account other important implications of a basic income. Nevertheless, we have stand out some of the underlying mechanisms explaining the effects on individual choices of a basic income scheme as compared to actual unemployment insurance systems. Leisure appears to play an important role in these mechanisms, leading to results which seems to advocate in favor of a basic income. An interesting extension to be explored consists of the analysis of the transition from a system based on unemployment benefits to one with a basic income.
Appendix

A. Proof of proposition 1

First \( dC(\phi)/d(\phi) < 0 \). Aggregate net consumption, after deduction of taxes is given by

\[
C(\phi) = \frac{(1 + \alpha \rho)(1 - \alpha)^{-1}}{(1 + \rho)(b - a)}(\bar{v} + (1 - \bar{v})\beta)w_1 \int_{z}^{b} z\lambda \ dz
\]  

(35)

with \( \bar{z} = (\phi m)/(\lambda(1 - \lambda)((1+\eta)/(1+\rho))) \). When deriving (35) with respect to \( \phi \) we obtain

\[
\frac{dC(\phi)}{d(\phi)} = \frac{(1 + \alpha \rho)(1 - \alpha)^{-1}}{(1 + \rho)(b - a)}(\bar{v} + (1 - \bar{v})\beta)w_1 \frac{(-\phi m^2)}{(1 - \lambda)((1+\eta)/(1+\rho))},
\]

(36)

which is clearly negative.

We now prove that \( dC(q)/d(q) < 0 \). Net aggregate consumption \( C(q) \) is given by

\[
C(q) = \frac{(1 + \alpha \rho)(1 - \alpha)^{-1}}{(1 + \rho)(b - a)} \times
\]

\[
\left[ \int_{z_1}^{z_2} \left( 1 - \frac{(qm)^{1+\rho}(2 + \rho + \eta)^{2+\rho+\eta}z^{1+\eta}}{(1 + \rho)^{1+\rho}(1 + \eta)^{1+\eta}(z + qm)^{2+\rho+\eta}} \right) z\lambda_1(z) \ dz 
\]

\[
+ \int_{z_2}^{b} \left( 1 - \frac{z\beta + qm}{z + qm} \frac{2+\rho+\eta}{\beta^{1+\eta}} \right) z\lambda_1(z) \ dz 
\]

\[
+ \int_{z_2}^{b} \left( \frac{z\beta + qm}{z + qm} \frac{2+\rho+\eta}{\beta^{1+\eta}} \right) z\lambda_2(z) \ dz \]

with \( z_1 = qm(1 + \eta)/(1 + \rho) \) and \( z_2 = qm(1 + \eta)/((\beta(1 + \rho)) \). When deriving this expression with respect to \( q \) we obtain

\[
\frac{dC(q)}{d(q)} = \frac{m(1 + \alpha \rho)(1 - \alpha)^{-1}}{(1 + \rho)(b - a)} \times 
\]

\[
\left[ \int_{z_1}^{z_2} \left( \frac{z}{1+\eta} \right)^{1+\eta} \left( \frac{2 + \rho + \eta}{z + qm} \right)^{2+\rho+\eta} \left( \frac{qm}{1 + \rho} \right)^{\rho} \left( 1 + \frac{2 + \rho + \eta}{z + qm}(1 + \rho) \right) z\lambda_1(z) 
\]

\[
- \left( 1 - \frac{(qm)^{1+\rho}}{(1 + \rho)} \left( \frac{z}{1+\eta} \right)^{1+\eta} \left( \frac{2 + \rho + \eta}{z + qm} \right)^{2+\rho+\eta} \right) \frac{1 + \eta}{2 + \rho + \eta} \right] \ dz 
\]

\[
- \int_{z_2}^{b} \left( 2 + \rho + \eta \right) \left( \frac{1}{\beta} \right)^{1+\eta} \left( \frac{z\beta + qm}{z + qm} \right)^{1+\rho+\eta} \left( 1 - \frac{z\beta + qm}{z + qm} \right) \left( \frac{1 + \eta}{2 + \rho + \eta} \right) \left( \frac{1}{\beta} - 1 \right) 
\]

\[
+ \frac{1 + \eta}{2 + \rho + \eta} \left( \frac{1}{\beta} \right)^{1+\eta} \left( 1 - \left( \frac{z\beta + qm}{z + qm} \right)^{1+\rho+\eta} \left( 1 - \frac{1}{\beta} \right) \right] \ dz \]

, 27
which is again clearly negative, given that \( z \geq z_1 = qm(1 + \eta)/(1 + \rho) \).

**B. Proof of proposition 2**

The Atkinson index of inequality is given by

\[
I(\phi) = 1 - \exp \left[ \bar{\varepsilon} \left( \int_a^z \log \left( \frac{\phi m w_1(1 - \tau)}{Y w_1(1 - \tau)} \right) f(z) \, dz + \int_z^b \log \left( \frac{z \lambda w_1(1 - \tau)}{Y w_1(1 - \tau)} \right) f(z) \, dz \right) 
+ (1 - \bar{\varepsilon}) \left( \int_a^z \log \left( \frac{\beta \phi m w_1(1 - \tau)}{Y w_1(1 - \tau)} \right) f(z) \, dz + \int_z^b \log \left( \frac{z \lambda \beta w_1(1 - \tau)}{Y w_1(1 - \tau)} \right) f(z) \, dz \right) \right]
\]

where \( \bar{Y} w_1(1 - \tau) \) is the net total income in the economy with

\[
\bar{Y} = \left( \int_a^z \phi m f(z) dz + \int_z^b z \lambda f(z) dz \right) (\bar{\varepsilon} + (1 - \bar{\varepsilon}) \beta).
\]

Knowing that \( f(z) = (b - a)^{-1} \) we can rewrite the above expression as

\[
I(\phi) = 1 - \exp \left[ \frac{1}{b - a} \left( \int_a^z \log \left( \frac{\phi m}{Y} \right) \, dz + \int_z^b \log \left( \frac{z \lambda}{Y} \right) \, dz \right) + (1 - \bar{\varepsilon}) \int_a^b \log \beta \, dz - \int_a^b \bar{Y} \, dz \right] \tag{37}
\]

When deriving (37) with respect to \( \phi \), and taking the limit when \( \phi \) tends to \( a \lambda(1 - \lambda)(1+\eta)/(1+\rho) \), we obtain

\[
\lim_{\phi \to 0} \frac{dI}{d(\phi)} = -\exp[\ldots] \frac{m}{b - a} \frac{1}{\lambda(1 - \lambda)(1+\eta)/(1+\rho)} \times \left( \frac{1 + \eta}{1 + \rho} \log \left( \frac{1 + \rho}{2 + \rho + \eta} \right) + \frac{a}{m} \left( 1 - \left( \frac{1 + \eta}{2 + \rho + \eta} \right)^{\frac{1+\eta}{1+\rho}} \right) \right). \tag{38}
\]

The sign of the last factor is not clear at first sight; it depends, however, only on the value of two parameters \( \rho \) and \( \eta \) which are assumed to take values in respectively \((0, 1)\) and \((-1, 1)\). It can be proved numerically (figure 3) that this last factor is always negative, for all combinations of values of \( \rho \) and \( \eta \). Hence we can conclude that (38) is always positive.

**C. The effect of the introduction of a basic income on income inequality**

We have to determine the sign of \( \lim_{q \to 0} dI/d(\phi) \). In this case we have

\[
I(q) = \]
where we already simplified for the tax rate, and the wage.  \( \bar{Y} \) is the total and average income in the economy and equals

\[
\frac{1}{b-a} \left\{ (b-a) qm + \int_{z_1}^{z_2} \left( 1 - \frac{(qm)^{1+\rho}(2 + \rho + \eta)^2 + \rho + \eta z^{1+\eta}}{(1 + \rho)^{2+\rho+\eta} (1 + \eta)^{2+\rho+\eta}} \right) \log \left( \frac{(1 + \rho)(z + qm)}{2 + \rho + \eta} \right) dz \\
\times \left( \frac{(1 + \rho)(z - (1 + \eta)q)(2 + \rho + \eta)}{2 + \rho + \eta} \right) dz \\
+ \int_{z_2}^{b} \left( \frac{(1 + \rho)(z - (1 + \eta)q)^{2+\rho+\eta}}{2 + \rho + \eta} \right) \left( \frac{1}{\beta^{1+\eta}} \right) \left( \frac{(1 + \rho)(\beta + qm)}{2 + \rho + \eta} \right) dz \right\}.
\]
When deriving $I(q)$ with respect to $q$, and taking the limit when $q$ tends to 0, we obtain

$$
\lim_{q \to 0} \frac{dI}{d(\phi)} = -m \exp[... \left[ \log \left( \frac{b}{a} \right) (1 + \beta^p(1 - \beta)) + \beta^p(1 - \beta)(2 + \rho + \eta) \log \beta \\
- \frac{(b - a)(1 - (2 + \rho + \eta)(1 - \beta^p))}{m(1 - \beta^{1+p}(1 - \beta))} \right];
$$

simulations show that this expression is likely to be positive for low values of $\beta$ and negative for high values of $\beta$, independently from the values of $a$ and $b$. Hence its sign is ambiguous.

**D. The effect of the introduction of a basic income on aggregate utility**

The effect of a basic income as $\phi \to 0$ is given by

$$
m(2 + \rho + \eta) \left[ \log \left( \frac{b}{a} \right) (1 + \beta^p(1 - \beta)) - \frac{b - a}{m(1 - \beta^{1+p}(1 - \beta))} \right]. \quad (39)
$$

We will first show that this expression is increasing in $b/a$. Its derivative with respect to $b/a$ (making abstraction of the term $m(2 + \rho + \eta)$) is given by

$$
\frac{a}{b} (1 + \beta^p(1 - \beta)) - \frac{2}{(1 - \beta^{1+p}(1 - \beta))} \left[ \frac{1}{1 + (b/a)} - \frac{(b/a) - 1}{(1 + (b/a))^2} \right].
$$

After rearranging and simplifying terms this is equal to

$$(1 - (b/a))^2 \beta^p(1 - \beta)(1 - \beta(1 - \beta)(1 - \beta))]
$$

which is always positive. We further know that (39) is zero when $b/a$ tends to its minimum value 1. Hence we can conclude that (39) is always greater than or equal to 0.

**E. Computation of $z^*$**

The level $z^*$ is the productivity level which equals the utility of a skilled individual in the laissez-faire with the one he would obtain being unemployed in the economy with an unemployment benefit $\phi$:

$$
\log(1 - e) + (1 + \eta) \log(1 - \lambda) + (1 + \rho) \log(z \lambda) = \log(1 - e) + (1 + \rho) \log(\phi m(1 - \tau)).
$$
when we isolate $z$ from the above expression, we obtain

$$z^* = \phi m (1 - \tau) \left( 1 + \frac{1}{1 + \rho} \right) \left( 1 + \frac{2 + \eta + \rho}{1 + \eta} \right)^{2 + \eta + \rho}. \quad (40)$$

For the unskilled individuals we have the same $z^*$, since the level of the unemployment benefit depends on whether you are skilled or not.

**F. Computation of $\phi^*$**

In order that at least the very poorest are better of in terms of utility, $z^*$ should be greater than $a$, or when substituting for $1 - \tau$ in (40), and after rearranging terms we should have that

$$- \frac{(\phi m)^2}{\lambda (1 - \lambda)^{2 + \rho}} - 2a \phi m + \lambda b^2 > 0,$$

which is a concave function in $\phi$. This polynomial has two roots, a negative and a positive one. Hence given concavity, the value of the polynomial will be positive for levels of $\phi$ up to its upper (positive) root ($\equiv \phi^*$) which can easily be computed and which is given by

$$\left( a^2 + \frac{b^2}{(1 - \lambda)^{2 + \rho}} - a \right) \lambda (1 - \lambda)^{2 + \rho} \quad \equiv \phi^*. \quad m$$

**G. Condition on the level of productivity disparity**

In order to have some effect on individual behavior, an unemployment we should have that

$$\phi > \frac{a \lambda (1 - \lambda)^{2 + \rho}}{m}.$$ 

We now compute under which conditions the maximum unemployment benefit $\phi^* mw$ may satisfy the above inequality. We should have that

$$\phi^* = \left( a^2 + \frac{b^2}{(1 - \lambda)^{2 + \rho}} - a \right) \lambda (1 - \lambda)^{2 + \rho} > \frac{a \lambda (1 - \lambda)^{2 + \rho}}{m}.$$ 

After developing terms it is straightforward to derive the following condition

$$\frac{b^2}{a^2} > 1 + 2(1 - \lambda)^{2 + \rho}.$$
H. Computation of conditions on the basic income

After dividing the individuals in different groups according to their productivity and effort required to become skilled, we compare the utility each individual would get in the laissez-faire economy with the one obtained in an economy with basic income. We thereby take into account the effect of the basic income on the individual decisions concerning whether to become skilled or not and how much time to devote to work. The utility each individual obtains in both economies obviously depends on his individual productivity, his effort needed to become skilled and on the tax rate. We substitute individual productivity $z$ by its upper and lower bound respectively in each group and distill from these comparisons maximum tax rates (and hence basic income) which provide each type of individual with a utility that is superior to that a same individual would get in the laissez-faire economy. This is represented by table 1. It is easy to check that the maximum tax rates decrease in $z$ and increase in $e$.

Bibliography


