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Référence bibliographique
Lobbying in Public Decision Making*

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Abstract

The lobbying process is modelled as an auction with externalities in which lobbies bid to get implemented their most-preferred policy. Furthermore, the government may influence the lobbying process itself by biasing the auction among organized interests. We identify the following trade-off: competition yields a higher transfer to the government, but the outcome of the game tends to be less efficient than what it is when lobbies negotiate. We extend and illustrate the model by means of a public good game involving several regions. Lobbying by regions may yield a quantity of public good that may vastly differ from that chosen by a majority of regions. This is so when the regions with the highest financing shares lie at the extremes of the distribution.

1 Introduction

The idea that public decision making is influenced by pressure groups is an old one in political science. However, it is only recently that the economics literature has studied the role of competition among special interests in the making of policy by using formal models based on rational behavior (see Mueller, 1989, for a survey of the public choice literature). In particular,

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economic models have focused on the gathering of information by special interests, which is then provided to the government, and on influence-seeking activities. In the latter, the work of Bernheim and Whinston (1986) has served as the main benchmark. Within this framework, the government is viewed as a common agency endowed with its own preferences, which can be influenced through contributions made by organized interests. More precisely, Bernheim and Whinston assume that each interest group offers a contribution schedule whose value is contingent on the policy implemented by the government. Given these schedules, the government then chooses its most-preferred policy.

This model has triggered further important developments such as those proposed by Grossman and Helpman (1994), in which the government maximizes a weighted sum of total welfare and special interests' contributions. This objective can itself be rationalized as the outcome of electoral competition (Grossman and Helpman, 1996). This leads to a view of the government that departs from the traditional social welfare-maximizing planner, although it retains the idea that the government is semi-benevolent by compromising between social welfare and the personal interests of politicians.

The present paper belongs to the influence-seeking tradition and departs in an even more radical way from the standard public economics framework. Specifically, we follow Bernheim and Whinston and model the process of lobbying as an auction. However, we do not retain the menu auction formulation. Instead, the lobbying game is modeled as a standard second price auction in which the winning lobby (the one who has offered the highest contribution) gets its most-preferred decision implemented (in exchange for the second highest contribution).\footnote{In the complete information setup considered in this paper, first price auctions would yield the same analysis as second price auctions.} Several reasons justify our choice. First, as is well known, in simple contexts without externalities, the second price auction leads agents to reveal (through their bids) the private (monetary) value they attach to a particular decision. In this context, the auction mechanism leads to the efficient decisions, the ones for which agents are ready to pay most.\footnote{This holds true even if agents have private information on their valuations as shown by Vickrey (1961).} Second, the second price auction is simpler to design than the menu auction and is easier to implement. This is appealing to us because we consider the lobby’s most-preferred action as the outcome of a (possibly long) bargaining process within the lobby itself to which it is committed. The fact that the
agreement of the lobby’s members is required for the lobby to be active often prevents it from offering a schedule specifying an offer that would vary with the action taken by the government. Indeed, such a fine-tuning policy would generate many practical difficulties within the lobby that seem to be out of reach, once the actions at stake have a major impact for the lobby’s members.

Our aim is to provide a new analysis of the political decision process based on the assumption that the lobbying activity takes the form of an auction. A key feature of our framework is that we allow for the more realistic case in which all organized interests are affected by the policy effectively chosen by the winning lobby. As a consequence, the selection of a winner generates externalities for the other pressure groups. This implies that the lobbying process is modeled as an auction with externalities (Jehiel and Moldovanu, 1996).

We wish to point out that contributions made by lobbies (whether in the menu auction of Bernheim and Whinston or in our second price auction formulation) need not be thought of as bribes paid to the government.3 In the political arena, the equilibrium transfers associated with the auction may be implemented through intertemporal and sophisticated systems of activities that eventually benefit the government. Within this context, the contribution made by a lobby should instead be viewed as a reduced form for all the indirect benefits generated by the lobby’s activities.

It should be noted that our modeling of lobbying activity may be interpreted as one in which the government delegates to the most influential lobby the right to decide. The winning lobby then selects its most-preferred policy because this is the policy it likes most (more below). With this (delegation) interpretation, we depart from Grossman and Helpman who assume that the final decision remains with the government. In a sense, our model departs from the standard lobbying models mentioned above very much as Besley and Coate (1997) as well as Osborne and Slivinski (1996) depart from the classical Downsian models of party competition: in our model, lobbies commit not to implement a compromise policy; in the new models of political competition, candidates commit not to compromise. Baye et al. (1993) also view the influence-seeking activity as a game with a single winner. However,

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3Although some analysts claim that “If they know the right people in Congress and in the White House, they can often get anything they want” (Time, 2 February 2000). The situation is worse in some countries that we choose not to mention.
Baye et al. model the competitive political process as an all-pay auction in which each pressure group pays its bid - generally interpreted as the cost of effort - whether or not it is the winner. By contrast, we model the influence process as an auction in which groups offer contributions that only the winner effectively pays. In the simplest case (i.e. without biases, see below), the winner is the group with the largest contribution.\(^4\)

In our model, we also allow the government to influence the lobbying process itself by biasing the auction among special interests. Different interpretations may be retained. In one of them, the government chooses biases such that the well-being of the unorganized people is accounted for. This assumption agrees with the approach taken by Grossman and Helpman. In this context, one may wonder how in practice the government knows the utility of these people. As shown by the example of “Ombudsman” in Sweden and other countries as well as the consumer association “Konsumentverket” sponsored by the Swedish government, one possibility for the government is to favor the launching of various associations whose aim is to collect information about unorganized people. However, other interpretations are also possible: for example, the government could bias the auction in favor of its constituency. This leads us to concentrate on a modeling strategy in which the biases are exogenously given to the lobbies. In the analysis, we make comparative statics with respect to these biases, although they should ideally be determined endogenously.

To sum up, our approach combines the idea that the influence-seeking game has a winner, as in Baye et al., and the idea that the winning group must make a monetary contribution, as in Bernheim and Whinston. The reduced form of the lobbying game is that of an auction with externalities, in which the lobby winning the influence-seeking game is able to implement its most-preferred action. We also allow for a system of biases that affects the lobbying process itself (yet it is not modeled as resulting from a strategic decision of the government).

In the first part of the paper, we analyze the two-lobby case. In absence of biases, we first observe that competition between lobbies leads to the efficient decision from the aggregate lobby viewpoint when the choice is restricted to the sole most-preferred policies of the two lobbies. However,

\(^4\)Insofar as contributions (whether direct or indirect) are more important than effort (this may depend on the context) in determining the outcome of the lobbying activity, our model seems more appropriate than the all-pay auction formulation.
negotiation opens the door to the possible implementation of alternative decisions that may be more efficient, not only from the standpoint of the two lobbies but also from the social viewpoint. This happens to be the case when the unorganized individuals have interests congruent with the lobbies’ aggregate viewpoint. Yet, negotiation between lobbies deters competition at the auction stage and, therefore, reduces the contribution received by the government as compared with the no negotiation case. This leads to the following trade-off between competition and negotiation: to the extent that unorganized interests have interests (more or less) congruent with the lobbies’ aggregate viewpoint, negotiation rather than competition is desirable from the total welfare viewpoint, but it reduces the contribution received by the government. Furthermore, when there is competition between the lobbies and when the government is interested in maximizing total welfare, it should bias the auction by accounting only for the interests of the unrepresented people. Even though the government cares about their welfare, it should ignore the organized interests in its biases for otherwise there would be double-counting.

In the second part of the paper, we consider a setting in which the public decision is given by the quantity of a public good supplied to an arbitrary number of regions, interpreted here as the pressure groups. These regions are heterogenous in both the benefit they receive from and the share they pay for this public good. We assume that there are no biases in the lobbying auction game. We show that there always exists an equilibrium in which the winner is the region with the largest financing share. When there are more than two regions, there may be several equilibria and some pressure groups may strategically choose not to participate to the lobbying game. This is so when two groups have similar interests and in which each one relies on the other to obtain the right of choosing how much public good is made available.

We also study how the distribution of the most-preferred quantity of public good affects the outcome of the game, and show that it may vastly differ from the (simple majority) voting outcome. When the regions with the highest financing shares lie at the extremes of the distribution, the winning region is

\footnote{Hence, we differ from the standard setting in which the public good is specific to each interest group (Persson and Tabellini, 1999).}

\footnote{This free riding problem may be illustrated by the recent attitudes of France and Germany regarding the embargo against British beef. France was considered as the main opponent, although Germany also opposed to it but abstained from taking a strong position.}
typically far from the median one.

2 A Model of Lobbying

2.1 The model

We consider an institutional setting with one government and \( n \) interest groups, also called lobbies. In our model, we take the view that the government is nonstrategic in that the lobbies are the only players in the game. However, we should make it clear from the outset that the government is not indifferent about the outcome of the game among lobbies since it has a utility that is increasing in both the total welfare and the revenue received from the lobbies. We further elaborate on this point below.

A public policy must be chosen and which policy is selected affects differently organized interests. Formally, the choice space is described by a set \( S \) of policies. Each lobby has a utility \( u_i(s, m_i) \) defined over \( S \times R \) where \( s \in S \) and \( m_i \in R \) stands for the numéraire held by \( i \). For simplicity, it is assumed that the utility is quasi-linear

\[
u_i(s, m) = v_i(s) + m_i
\]

and that each lobby has a single most-preferred policy denoted

\[
s_i^* = \arg \max_{s \in S} v_i(s) \quad i = 1, ..., n
\]

If lobby \( i \) gets the right of choosing the public policy, it should be clear that it will choose to implement \( s_i^* \) since this policy corresponds to its most-preferred choice. Indeed, it may be argued that the lobby has to choose \( s_i^* \) because this policy is precisely the outcome of a process of negotiation within the interest group itself.

We assume that not all interests are organized. More precisely, the preferences of the corresponding individuals are subsumed in the utility

\[
u_0(s, m_0) = v_0(s) + m_0
\]

where \( m_0 \) is the amount of the numéraire owned by unorganized individuals.

Since utility is transferable, the total welfare for all the lobbies is given by the sum of their utilities whereas the total welfare is given by this sum augmented by \( u_0(s, m_0) \).
Finally, we assume that the lobbies’ utilities are commonly known among lobbies.

We now describe the lobbying game, called bidding for deciding (in short BD), as an auction among special interests. Formally, lobbies simultaneously submit a nonnegative bid, where the bid of lobby \( i \) is denoted by \( b_i \geq 0 \). Since the government cares also about other interests than the lobbies, for example the well-being of unorganized individuals, the auction is modified by the government according to the following rescaling. For each lobby’s bid \( b_i \), let
\[
\hat{b}_i = b_i + \beta_i
\]
be the effective bid of lobby \( i \) where \( \beta_i \) is the bias imposed by the government on lobby \( i \)’s bid.

The lobby with the highest effective bid then has the right of choosing the public policy \( s \). If lobby \( i \) is the winner, it must pay to the government the second highest effective bid net of \( \beta_i \) to get the right of deciding.\(^7\) One must deduct \( \beta_i \) because the auction is based on effective bids \( \hat{b}_i \) that differ from the submitted bids \( b_i \).

The outcome of the game is described as follows. Lobbies submit \( b_1, ..., b_n \). Without loss of generality, we may re-index lobbies such that \( \hat{b}_1 \geq \hat{b}_2 \geq ... \geq \hat{b}_n \). If \( W \) is the set of lobbies with the highest effective bid \( \hat{b}_1 \), one lobby \( w \) in \( W \) is selected with probability \( 1/\#(W) \) and pays the amount \( \hat{b}_2 - \beta_w \) to the government. The structure of payoffs is then as follows. The lobby \( w \) has a payoff given by \( v_w(s_w^*) - \hat{b}_2 + \beta_w \) whereas any lobby \( i \neq w \) receives \( v_i(s_w^*) \). In this case, the government obtains a transfer equal to
\[
\hat{b}_2 - \beta_w = b_2 + \beta_2 - \beta_w
\]
Observe that this payment is necessarily a subsidy when \( \hat{b}_2 - \beta_w < 0 \).

2.2 The two-lobby case

We start by studying the case of two lobbies and then show how the outcome described in the foregoing is affected when the two lobbies may negotiate before playing BD. The two lobbies are denoted \( i = A, B \) and their most-preferred policy by \( s^*_A \) and \( s^*_B \), respectively.

\(^7\)Hence, we consider a second price auction and it is readily verified that our results equally hold in the case of a first price auction in which the winner has to pay its own bid with a standard tie breaking rule.
2.2.1 The case of competition

We assume here that lobbies behave noncooperatively in BD.\(^8\)

**Proposition 1** When there are two lobbies, there exists a unique undominated pure strategy equilibrium. The equilibrium bids are respectively \(b^*_A = v_A(s^*_A) - v_A(s^*_B) > 0\) and \(b^*_B = v_B(s^*_B) - v_B(s^*_A) > 0\). If

\[
v_A(s^*_A) - v_A(s^*_B) + \beta_A \geq v_B(s^*_B) - v_B(s^*_A) + \beta_B
\]

then A’s payoff is given by

\[
v_A(s^*_A) - [v_B(s^*_B) - v_B(s^*_A) + \beta_B] + \beta_A
\]

whereas B’s payoff is equal to \(v_B(s^*_A)\).

**Proof.** See Jehiel and Moldovanu (1996, Proposition 2).

The intuition behind this result is fairly straightforward. The net value of winning BD for, say, lobby A is nonnegative and given by the difference of lobby A’s payoffs when A decides and receives \(\beta_A\) from the government and when B decides. Since A (resp. B) chooses \(s^*_A\) (resp. \(s^*_B\)) when it is the winner, the expressions for the equilibrium effective bids follow immediately. From these expressions, the equilibrium bids as given in the proposition may then be obtained.

In fact, condition (1) means that the winner has a differential surplus between the two policies \(s^*_A\) and \(s^*_B\) that (weakly) exceeds the surplus of the competing lobby. When the inequality is strict in (1), A is the winner. By contrast, when \(s^*_A = s^*_B\) the two equilibrium bids are zero because the lobbies have no incentives to compete. In this case, player A (resp. B) is the winner when \(\beta_A > \beta_B\) (resp. \(\beta_A < \beta_B\)).

Observe that condition (1) implies that

\[
v_A(s^*_A) + v_B(s^*_A) + \beta_A \geq v_A(s^*_B) + v_B(s^*_B) + \beta_B
\]

This proposition has the following interesting implication.

**Corollary 2** If the biases \(\beta_i\) are given by the values attached by the unorganized interests to the most-preferred policy of lobby i (i.e., \(\beta_i = v_0(s^*_i)\)), then the equilibrium of BD is efficient from the viewpoint of the society as a whole when the choice space is confined to the pair \(\{s^*_A, s^*_B\}\).
When the preferences $v_0(.)$ are known to the government, a *benevolent* government will choose biases equal (up to a constant) to $\beta_i = v_0(s^*_i)$. By contrast, this property ceases to hold when the biases $\beta_i$ are different from the utility derived by the unorganized interests each time $s^*_i$ is implemented. Hence, if its aim is to maximize a utilitarian social criterion, the government *should not account for the welfare of the organized interests* when biasing the auction. For example, choosing $\beta_i = v_A(s^*_i) + v_B(s^*_i) + v_0(s^*_i)$ would in general lead to a suboptimal policy, as it would induce a double-counting of the lobbies’ welfare, a point that bears close resemblance to Coase (1960) in a different, but related, context.

### 2.2.2 The case of collusion

Since the equilibrium outcome is not necessarily efficient from the aggregate lobby viewpoint, lobbies have incentives to collude since they may guarantee to themselves a (weakly) higher surplus by maximizing the sum of their payoffs and by agreeing on the corresponding policy.

Let us elaborate on this point. Suppose there is a pre-play stage in which the two lobbies freely negotiate on the public policy to implement and on the bids to submit at the BD game. Since utility is transferable, any efficient mechanism will yield an outcome that maximizes $v_A(s) + v_B(s)$. Let

$$s^* \in \arg \max_{s \in S} [v_A(s) + v_B(s)]$$

be such a solution. It then follows immediately that:

**Proposition 3** Assume there is a pre-play negotiation between the two lobbies, and that the biases are given by $\beta_A$ and $\beta_B$. Then, in equilibrium both lobbies agree on pursuing the same objective $s^*$ and the government pays a subsidy equal to $|\beta_A - \beta_B|$ to the lobbies.

**Proof.** If biases $\beta_A \neq \beta_B$ prevail at the BD-game, lobbies may agree on implementing $s^*$ while making bids that allow them to share a subsidy equal to $|\beta_A - \beta_B|$. For example, if $\beta_A > \beta_B$ then $A$ will submit a positive but arbitrarily small bid while $B$ will submit a zero bid.

If the government is aware that there is collusion between the two lobbies, it will choose not to bias the auction because it understands that such a choice has no influence on the final outcome and allows it to avoid a negative revenue.
Since there is a priori no reason for $s^*$ to coincide with $s_A^*$ or $s_B^*$, this proposition does not necessarily mean that the government is worse off than under competition between the two lobbies. Indeed, the policy $s^*$ might well result in a much higher level of total welfare. This is likely to be so when (1) $s^*$ improves (over $s_A^*$ and $s_B^*$) the welfare of the lobbies, and (2) all the interests involved in the implementation of a particular policy are represented by the two lobbies, i.e. $v_0(.)$ is independent of $s \in S$ or the unorganized individuals find that $s^*$ is a “good” policy (as compared with $s_A^*$ and $s_B^*$).

The foregoing discussion suggests two interesting institutional recommendations. First, the government may well prefer collusion to competition between lobbies. Second, the government should foster the formation of lobbies representing all the parties concerned with a particular policy. Of course, these recommendations are valid only if the government cares about total welfare. If the government cares more about revenue, collusion should be avoided.

2.3 The strategic choice of policy by lobbies

So far, we have assumed that each lobby was committed in implementing its ideal policy $s_i^*$. However, strategic considerations may induce lobbies to move away from their ideal policies and to choose others, depending on when players take action (the timing of the game). In fact, the $BD$-game assumes that each lobby chooses its best policy after having gained the right of deciding. One could think of the reverse order and assume that each lobby must declare the policy it wishes to implement before playing the corresponding auction subgame. Such a game may be described as follows. In the first stage, lobbies choose simultaneously the policies $s_1\ldots s_n$ that each would implement if it were the winner of the auction; the second stage is then identical to our $BD$ game under the proviso that lobby $i$ is committed to choose $s_i$, which may now differ from its utility-maximizing policy. Clearly, in such a game, the equilibrium outcome depends on the available information at the time the policy $s_i$ is chosen. In the sequel, we restrict ourselves to the case of two players.

Consider first the case in which the preferences of the lobbies are commonly known by all agents. In the first stage, lobbies announce their policies $s_A$ and $s_B$; second, the $BD$-game is played assuming that the biases are given by the schedules corresponding to the welfare of the unorganized interests,
\[ \beta(s_A) = v_0(s_A) \mbox{ and } \beta(s_B) = v_0(s_B). \] We then have:

**Proposition 4** When there are two lobbies and when biases are given by the schedules \( \beta(s_i) = v_0(s_i) \), there exists a unique subgame perfect Nash equilibrium in undominated pure strategies. This equilibrium is such that

\[ s_A = s_B = s_{\text{opt}} \]

where

\[ s_{\text{opt}} \in \arg \max_{s \in S} [v_A(s) + v_B(s) + v_0(s)] \]

while the equilibrium transfer is zero.

**Proof.** Assume that \( B \) chooses \( s_B \). If \( A \) chooses \( s \) in the first stage, its payoff is equal to

\[ \max \{ v_A(s_B), v_A(s) - [v_B(s_B) - v_B(s) + \beta(s_B)] + \beta(s) \} \]

Since \( v_A(s_B) \) is constant, a (weakly) dominant strategy for \( A \) is to select the policy that maximizes

\[ v_A(s) - [v_B(s_B) - v_B(s) + \beta(s_B)] + \beta(s) \]

that is, the policy \( s_{\text{opt}} \) maximizing \( v_A(s) + v_B(s) + v_0(s) \), and so regardless of \( s_B \). □

It is worth noting that the outcome of this multi-stage game is the one that maximizes total welfare. Unfortunately, the optimality property of this game holds only in the case of two players. Indeed, the choice of \( s_i \) rests only upon pairwise comparisons and will not involve the maximization of the sum of all utilities.

**Remark.** Using an argument similar to that of Proposition 2, it is readily verified that the solution in Proposition 4 is the same as that of the game in which both lobbies agree on negotiating before playing \( BD \) when the biases schedules \( \beta_i(s_i) = v_0(s_i), i = A, B \), are announced prior to the policies \( s_A \) and \( s_B \).

We now consider the polar case in which each lobby \( i \) knows only its utility \( v_i \) in the first stage game at the time where they must choose their policy. However, in the second stage, the policy selected by the other lobby as well as its type are commonly known before playing the auction game, so that this subgame is described by a complete information game. This
is a reasonable assumption if the debates within each lobby that leads to
the choice of a policy also allow outside participants to figure out what the
preferences of the lobby are.

For ease in exposition, we assume that $v_0(.)$ is independent of $s \in S$.
The utility $v_i(s, t_i)$ of lobby $i$ is parametrized by $t_i$, which is distributed
according to a given probability distribution. The outcome of the first stage
is then given by a perfect Bayesian equilibrium that satisfies the following
conditions: for $i = A, B$ and all $t_i$

$$s_i(t_i) \in \arg \max_{s_i} \mathbb{E}_{t_j} \max \{v_i(s_j(t_j), t_i), v_i(s_i, t_i) - [v_j(s_j(t_j), t_j) - v_j(s_i, t_j)]\}$$

In such a context, it is plausible to expect that, when a player does not
know the type of the other, its choice is pulled toward the outcome of the
$BD$-game as described in Proposition 1 in which the policy chosen by lobby
$i$ is given by its most-preferred policy.

To illustrate this claim (and with no claim of generality), consider the
(idealized) case in which the policy space is the circle of unit length $C$ and
each $t_i$ is distributed over $C$ independently of $t_j$ ($i, j = A, B$ and $i \neq j$).
The utility of $i$ is given by a linear and decreasing function of the distance
between the type and the implemented policy:

$$v_i(s, t_i) = \frac{1}{2} - |t_i - s|$$

where $|s - t_i|$ stands for the length of the shorter arc linking $t_i$ and $s$. Clearly,
in such a setting, the most-preferred policy of lobby $i$ whose type is $t_i$ is given
by $t_i$. Using the definition of $v_i(s, t_i)$, it can be shown that $s_i(t_i) = t_i$ for all
$i$ and $t_i$ is an equilibrium outcome. Indeed, when $s_j = t_j$, the definition of
$v_i(s, t_i)$ implies that $s_i(t_i) = t_i$ since any point $s_i$ between $t_i$ and $t_j$ does not
make $i$ strictly better off than $t_i$, while any point situated outside $t_i$ and $t_j$
yields a strictly lower payoff for $i$.

\section{The Choice of Public Good through Lobbying}

The purpose of this section is to further develop to the more than two lobby
case the analysis of the lobbying game proposed above. Specifically, we are
interested in the application in which lobbies are regions, the public policy
is a public good supplied by a federal government that affects possibly all regions. To do so, we consider a fairly popular model used in industrial organization and describing heterogeneous agents (Tirole, 1988).

Consider an economy formed by $n$ regions. A federal government must choose the amount of a pure public good $z$ whose cost $C(z)$ is covered by the regions. Each regional government has a specific utility function given by

$$v_i(z) = \theta_i z - \alpha_i C(z) \quad i = 1, \ldots, n$$

where $\theta_i$ is the valuation of the public good in region $i$ and $\alpha_i$ its share in the cost of providing the public good to the whole set of regions. Regions are therefore heterogeneous in two respects: (i) they do not equally value the public good ($\theta_i$) and (ii) they must contribute differently to the cost of the public good according to some prespecified weights ($\alpha_i$) whose sum equal one. By assumption, all regions involved in the consumption of this good are considered as lobbies so that there is no unrepresented interests ($v_0(z) \equiv 0$).

As a consequence, it is natural to describe the $BD$-game as in Section 2.1 in which all $\beta_i$'s are zero.

For simplicity, we assume that $C(z) = z^2$ so that the most-preferred quantity of public good by region $i$ is uniquely defined by

$$z_i = \frac{\theta_i}{2\alpha_i}$$

We first re-state our foregoing results for two regions $A$ and $B$. Proposition 1 yields the following equilibrium bids:

$$b^*_A = \alpha_A (z_A - z_B)^2 \quad \text{and} \quad b^*_B = \alpha_B (z_A - z_B)^2$$

so that

**Proposition 5** When there are two regions, the winning region is the one with the larger share in financing the public good.

Intuitively, the reason seems to be that the region with the larger share is more concerned with the policy effectively implemented. It is also interesting to observe that, in the present setting, the intensity of preferences for the public good has no influence on the determination of the winning region. We find this simple result to be fairly plausible in that the most influential
regions involved in the choice of the level of a public good are often those with the largest contributions to the federal budget.

When $\alpha_A = \alpha_B = 1/2$, the winner is undetermined but the structure of payoffs is uniquely determined.

It remains to investigate how the outcome changes when the two regions negotiate before playing $BD$. In this case, both regions will choose to implement the socially optimal quantity $z^* = (\theta_A + \theta_B)/2$ regardless of the way the cost of the public good is shared between them. In the special case where both regions have identical preferences ($\theta_A = \theta_B$), lobbying leads to under-provision in the public good since $\theta_i/2\alpha_i < 2\theta_i$, $i$ being the winning region. In general, the comparison depends on a fairly subtle interplay between the four parameters $\alpha_i$ and $\theta_i$.

This conclusion is similar to that investigated in the fiscal competition literature that similarly asked whether decentralization in the provision of local public goods leads to under-provision. In particular, we concur with Williams’ (1966) for whom whether under- or oversupply holds depends on the shape of the reaction curves. We must stress the fact, however, that our setting is quite different from the mechanism of fiscal competition developed since the work of Williams.

3.1 Existence of an equilibrium
We now return to the case of $n$ regions and show that an equilibrium always exists. Let

$$\Delta_{ij} = (z_i - z_j)^2 = \frac{1}{4} \left( \frac{\theta_i}{\alpha_i} - \frac{\theta_j}{\alpha_j} \right)^2$$

so that $\Delta_{ij} = \Delta_{ji}$.

**Proposition 6** In the $BD$ game with $n$ regions, there always exists an undominated pure strategy equilibrium. Furthermore, there is an equilibrium such that the winner is a region with the highest share in financing the public good.

**Proof.** Let

$$w \in \arg \max_{i=1,\ldots,n} \alpha_i$$

and

$$l \in \arg \max_{i \neq w} \alpha_i \Delta_{lw}$$

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An equilibrium is then as follows: (i) \( w \) bids \( \alpha_w \Delta_{wl} \), (ii) \( l \) bids \( \alpha_l \Delta_{wl} \) and (iii) all \( k \neq l, w \) bid \( \alpha_k \Delta_{kw} \). Indeed, region \( w \) is the winner because it has the highest bid so that every region \( j \neq w \) bids \( \alpha_j \Delta_{jw} \); by definition of \( l \), any region \( k \) different from regions \( w \) and \( l \) bids below region \( l \) so that it is optimal for region \( w \) to bid \( \alpha_w \Delta_{wl} \). The corresponding payoff structure is: \( w \) is the winner and pays \( \alpha_l \Delta_{lw} \) to the federal government for having the right of implementing its most-preferred quantity, notwithstanding its share in the public good cost; the remaining regions only contributes to the cost \( C(z^*_w) \).

This proof exhibits an equilibrium that has the same features as the equilibrium obtained in the two-region case: the winning region is the region with the largest contribution. However, with more than two regions, there may exist other equilibria in which the winning region is not the one with the largest share. To show this, consider the following example.

**Example 7** There are three regions in which \( \alpha_1 > \alpha_2 > \alpha_3 \) and \( \theta_1/\alpha_1 \approx \theta_2/\alpha_2 \neq \theta_3/\alpha_3 \).

Applying Proposition 6, there is an equilibrium in which region 1 is the winner and pays \( \alpha_3 \Delta_{31} \). Another equilibrium arises when region 1 bids \( \alpha_1 \Delta_{12} \) which is very small, while region 2 (resp. 3) bids \( \alpha_2 \Delta_{23} \) (resp. \( \alpha_3 \Delta_{23} \)). The region 2 wins and pays \( \alpha_3 \Delta_{23} \) to the federal government. Hence, the two equilibria differ in all respects.

In this example, regions 1 and 2 are very similar in their most-preferred policies. The two equilibria we have exhibited are such that either of these two regions free rides on the other, explaining the existence of two equilibria depending on the identity of the free rider. Clearly, there exists a third equilibrium in mixed strategies that resembles a war of attrition between regions 1 and 2, the winner of which may sometimes be region 3. We believe that such a characterization of our equilibria provides a good description of the opportunistic behavior displayed by some regions inside federations.

We now provide a general characterization of the equilibria of the \( n \)-region case. To this end, let

\[
j(i) \in \arg \max_j \alpha_j(z_j - z_i)^2
\]

A similar war of attrition is discussed in a dynamic bargaining context by Jehiel and Moldovanu (1995) who consider a reduced form analogous to ours.
We assume from now on that the lobby $j(i)$ is uniquely defined. Hence, if $i$ is the winning lobby, then $j(i)$ is the lobby that incurs the largest loss from not being the winner.

**Proposition 8** In any undominated pure strategy equilibrium of the BD game, the winning lobby $i$ is such that

$$\alpha_i \geq \alpha_{j(i)}$$

Furthermore, when this inequality holds, there exists an equilibrium in undominated pure strategies in which $i$ is the winner.

**Proof.** Consider first an undominated pure strategy equilibrium in which $i$ is the winner. Every bidder $j \neq i$ has a single best reply in undominated strategies which is given by $b_j = \alpha_j(z_j - z_i)^2$. Thus, bidder $j(i)$ makes the highest bid among the competing lobbies and the unique best reply of $i$ against these bids is to bid

$$b_i = \alpha_i(z_i - z_{j(i)})^2$$

This defines an equilibrium to the extent that, given these bids, $i$ is indeed the winner, implying that $b_i \geq b_{j(i)}$ or $\alpha_i \geq \alpha_{j(i)}$.

Conversely, assume that $\alpha_i \geq \alpha_{j(i)}$. Then, the bids as just defined constitute an undominated pure strategies equilibrium in which $i$ is the winner.

3.2 Is the winning lobby moderate or extremist?

In order to gain more insights about what the equilibria might be when there are several lobbies, we now explore particular, but relevant, classes of examples. Without loss of generality, the $n$ lobbies can be ranked in increasing order of $z_i = \theta_i/2\alpha_i$ along a linear segment. In the first class of examples, we assume that the distribution of the $\alpha_i$’s is single-peaked and we re-label the lobbies for the one with the largest share to be lobby 0, so that the others are re-indexed in a way such as those on the left (right) of 0 have a negative (positive) index, a lobby with a larger (smaller) index being more distant from lobby 0 on the right (left) side.

Assume that $-i < 0$ is the winning lobby and let $j(-i)$ be the lobby that incurs the largest loss from not being the winner, namely

$$j(-i) = \arg\max_j \alpha_j(z_j - z_{-i})^2$$
Then, it must be that $j(-i) > 0$ since we have

$$\alpha_0(z_0 - z_{-i})^2 > \alpha_{-j}(z_{-j} - z_{-i})^2$$

for all $j > 0$.

A symmetric property holds when $i > 0$ is the winning lobby, yielding $j(i) < 0$.

**Proposition 9** Assume that the distribution of the $\alpha_i$'s is single-peaked with a maximum reached at $z_0$. There exist two nonnegative numbers $i^*$ and $i^{**}$ such that set of possible winners of the BD-game is given by

$$\{i; -i^* \leq i \leq i^{**}\}$$

**Proof.** Let

$$I^- \equiv \{-i; \alpha_{j(-i)} \leq \alpha_{-i}\}$$

By Proposition 8, $I^-$ is the set of lobbies with a negative index that can win in equilibrium. Let $-i \in I^-$ and $-i' > -i$. Then, we show that $-i' \in I^-$. To ease the burden of notation, set $j = j(-i)$ and $j' = j(-i')$. By definition of $j(i)$, we have

$$\alpha_j(z_j - z_{-i})^2 \geq \alpha_{j'}(z_{j'} - z_{-i})^2$$

Similarly,

$$\alpha_{j'}(z_{j'} - z_{-i'})^2 \geq \alpha_j(z_j - z_{-i'})^2$$

Taking the square root of each side of (2) and (3) and adding the resulting inequalities, we obtain

$$\left(\sqrt{\alpha_j} - \sqrt{\alpha_{j'}}\right)(z_{-i'} - z_{-i}) \geq 0$$

Since $-i' > -i$, it follows that $z_{-i'} - z_{-i} > 0$. Consequently, by the previous inequality we have

$$\alpha_{j'} \leq \alpha_j$$

Since $-i \in I^-$, we have

$$\alpha_j \leq \alpha_{-i}$$

Moreover, using the single-peakness of the distribution implies that

$$\alpha_{-i} \leq \alpha_{-i'}$$
Adding these three inequalities, we get
\[ \alpha_{j'} \leq \alpha_{-i'} \]
thus implying that \(-i' \in I^-\).

A symmetric argument holds for the set
\[ I^+ \equiv \{ i; \alpha_{j(i)} \leq \alpha_i \} \]

When the \(\alpha\)-distribution is single-peaked, the winning lobby must be in the vicinity of the peak, thus suggesting that extreme lobbies cannot make the public decision.

Let us now consider our second class of examples in which the distribution of the \(\alpha_i\)'s is \(\cup\)-shaped. We re-label the lobbies for the one with the smallest share to be lobby 0, so that the others are re-indexed in a way such as those on the left (right) of 0 have a negative (positive) index, a lobby with a larger (smaller) index being more distant from lobby 0 on the right (left) side. The first and the last lobbies are respectively denoted \(-m\) and \(r\). If \(i\) is the winning lobby, then the \(\cup\)-shaped property implies that the lobby that incurs the largest loss from not being the winner is either \(r\) or \(-m\):
\[ j(i) \in \{-m, r\} \]

**Proposition 10** Assume that the distribution of the \(\alpha_i\)'s is \(\cup\)-shaped and that \(\alpha_r > \alpha_{-m}\). There exists a nonnegative number \(i^*\) such that set of possible winners of the BD-game is given by
\[ \{ i; i^* \leq i \leq r \} \]

**Proof.** First, any \(-i\) cannot be a winner because \(j(-i) = r\) and \(\alpha_r > \alpha_{-i}\).
Second, consider the case of \(i \geq 0\). Let
\[ I^+ \equiv \{ i \geq 0; \alpha_{j(i)} \leq \alpha_r \} \]
By Proposition 8, \(I^+\) is the set of possible winners. We now show that \(i \in I^+\) and \(i' > i\) imply that \(i' \in I^+\). Since \(i \in I^+\) and the \(\alpha\)-distribution is \(\cup\)-shaped, it must be that \(j(i) = -m\). Both \(i' > i\) and \(j(i) = -m\) imply that \(j(i') = -m\). Indeed
\[ \alpha_m(z_i - z_{-m})^2 \geq \alpha_r(z_i - z_r)^2 \]
implies that
\[ \alpha_m (z_i' - z_m)^2 \geq \alpha_r (z_i' - z_r)^2 \]
since \( z_i' > z_i \). Finally, \( \alpha_m < \alpha_i \) because \( \alpha_i < \alpha_i' \) (by the \( \cup \)-shaped assumption) and \( \alpha_m < \alpha_i \) (since \( i \in I^+ \)). ■

A symmetric result holds when \( \alpha_m > \alpha_r \). It thus appears that, in the case of \( \cup \)-shaped distribution, lobbying yields an outcome corresponding to the ideal policy of one the two extremist interest groups. Moderate lobbies are therefore excluded from the decision process, showing that the outcome of lobbying critically depends on the distribution of the \( \alpha_i \)'s. By contrast, it is interesting to observe that, in a similar context, voting would yield the same result regardless of the distributions of the \( \alpha_i \)'s, provided that they have the same median. Consequently, it is fair to conclude that lobbying and voting tend to generate very different social outcomes.

3.3 When to participate to a lobbying game?

So far, we have assumed that each interest group was involved in lobbying. Yet, one may wonder whether strategic considerations may lead some groups not to participate to the \( BD \) game. The yields a new two-stage game in which all potential interest groups, first, decide simultaneously whether or not to act as a lobby and, then, after having observed which groups have chosen to do so, \( BD \) is played.

The interesting point is that some groups may strictly prefer to drop out rather than participating with a zero bid. This is illustrated using Example 7.

**Proposition 11** In Example 7, there exists no subgame perfect equilibrium in pure strategies in which all three lobbies participate to \( BD \).

**Proof.** The proof is by contradiction. If all three lobbies participate, there are two pure strategy equilibria of the corresponding \( BD \)-subgame, which have been described in the foregoing. In these equilibria, either 1 or 2 wins and pays a strictly positive equilibrium bid. Without loss of generality, consider the case in which 1 is the winner. In this case, it would have been strictly preferable for 1 not to participate. Indeed, the \( BD \)-subgame is now between 2 and 3 and the winner is 2. Clearly, 1 strictly better off by not paying and enjoying \( z_2^* \) than by paying \( \alpha_1 \Delta_{13} \) and enjoying \( z_1^* \). ■
In fact, this game has three equilibria. Two are in pure strategies and the first stage equilibria are (1, 3) and (2, 3); the third equilibrium is in mixed strategies in which 1 and 2 randomize in the first stage. This situation is akin to the war of attrition discussed in the section above.

### 3.4 The formation of lobbies

We want discuss here the possibility for different regions to join efforts and to form a single lobby. Hence, here again, we supplement the BD-game by adding a lobby formation stage, prior to BD. We start with the case in which homogeneous regions, with the same most-preferred public good to, form a lobby. To illustrate, consider the special, but relevant, case in which there are two groups of regions $I$ and $J$ that wish the same amount of public good:

$$ I = \left\{ i : \frac{\theta_i}{2\alpha_i} = z_I \right\} $$

$$ J = \left\{ j : \frac{\theta_j}{2\alpha_j} = z_J \right\} $$

with $z_I \neq z_J$. However, within $I$ ($J$) the regions $i$ ($j$) may have different shares $\alpha_i$ ($\alpha_j$); for simplicity, we assume that the $\alpha_i$ and $\alpha_j$ are such that the equality never holds between any two sums of these coefficients. In such a context, the relevant question is to determine who will belong to each lobby $A_I$ ($A_J$) since regions differ in their contributions. Of course, for a region $i$ to decide to join $A_i$, it must know what will be its contribution to the equilibrium payment when $A_i$ is the winner of BD. To fix ideas, consider the following proportional rule: if $P$ is the payment, then the share of region $i \in A_I$ is given by

$$ \frac{\alpha_i}{\sum_{k \in A_I} \alpha_k} P $$

and similarly for $j$. Thus, all the constitutive elements of the game are described.

Let

$$ \alpha_I \equiv \sum_{i \in I} \alpha_i $$

and

$$ \alpha_J \equiv \sum_{j \in J} \alpha_j $$

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Proposition 12 Assume that $\alpha_I > \alpha_J$. Then, there exists several undominated pure strategy equilibria but all of them are such the lobby $A_I$ is winner. Furthermore, in all equilibria we have $A_J = J$ and $A_I$ is such that

$$\sum_{i \in A_I} \alpha_i > \alpha_J \quad \text{and} \quad \sum_{i \in A_I \setminus \{k\}} \alpha_i < \alpha_J \quad \text{for any } k \in A_I \quad (4)$$

Since $\alpha_I > \alpha_J$, the members of $I$ have the potential to form a winning lobby by Proposition 1. Thus, (4) means that any member of the winning lobby $A_I$ is essential in that taking her away from the lobby would change the outcome of BD in favor of $J$.

**Proof.** Members of $A_J$ are indifferent between being in the lobby or staying out (however, staying out is a weakly dominated strategy). By contrast, the winning lobby $A_I$ contains only essential members since an inessential member would be strictly better off by dropping out, the public good being the same while her contribution would be zero. Conversely, given our payment sharing rule, the payoff of each member of $A_I$ is proportional to $\alpha_i$, which guarantees that each member of $A_I$ is better off by being in the lobby than dropping out that would result in the failure of lobby $A_I$. \[\blacksquare\]

A few remarks are in order. First, the essentiality of the members of the winning lobby is independent of the payment sharing rule chosen and holds as long as the payment of each participant is positive. This reflects the idea that an inessential member participating to the winning lobby would be strictly better off by being outside of the lobby since the same public policy would be implemented (because it is inessential) while avoiding any payment. Second, even though there are several equilibria, the quantity of public good supplied is the same for all equilibria and equal to $z_I$, while the revenue of the government is also the same and equal to $\alpha_J \Delta_{ij}$ where $i \in I$ and $j \in J$. Third, the essentiality of the members of each $A_I$ implies that the total contributions of two lobbies $A_I$ and $A_J$ to the financing of the public good are similar enough.

Another interesting question to investigate is the desirability of such large lobbies formed by regions with identical (similar) tastes. If they do not form, all the $i$ and $j$ compete within the BD game.

Proposition 13 Whatever the equilibrium, the revenue of the government and the total welfare are never lower when two homogeneous lobbies are formed than when all regions play the BD game separately.
**Proof.** Without loss of generality, assume that $\alpha_I > \alpha_J$. Then, the revenue of the government is

$$\alpha_I (z_I - z_J)^2$$

(5)

when the two lobbies are formed and the quantity of public good is $z_I$. Consider now the $BD$-game in which all lobbies play independently. If the winner $w \in I$, the government’s revenue is

$$\max_{j \in J} \alpha_j (z_I - z_J)^2$$

which is strictly smaller than (5). If the winner $w \in J$, the revenue is

$$\max_{i \in I} \alpha_i (z_I - z_J)^2$$

Furthermore, it must be that

$$\alpha_w \geq \max_{i \in I} \alpha_i$$

and, since

$$\alpha_w < \alpha_J$$

the revenue is again smaller than (5).

Regarding efficiency, the outcome when the two large lobbies are formed is the same as if the two complete lobbies were formed. So, by Proposition 1, the outcome is efficient between $\{z_I, z_J\}$ so that the game in which all lobbies play separately cannot be more efficient. ■

To conclude, consider the case in which there are (at least) two very heterogeneous lobbies. As discussed in Section 2.1, allowing these lobbies to negotiate is equivalent to assuming that they merge into a single lobby. In this case, Proposition 13 no longer holds because there is now a trade-off between the transfer the government receives from the winning lobby and the welfare level. Indeed, if there is a single lobby the welfare level is maximized across all participating lobbies but the government does not receive any payment. This suggests an interesting institutional recommendations that supplement those made in the foregoing: the government should foster the formation of homogeneous lobbies but prevent that of very heterogeneous ones.
4 Concluding Remarks

Our model allows for several possible extensions that are left for further research. First, in the spirit of Besley and Coate (1999), one could graft our lobbying model onto a voting one in which voters anticipate the behavior of the making of public policy through our lobbying technology. Second, when the government has a poor information about lobbies’ preferences, the trade-off between competition and negotiation is likely to be affected in ways to be determined. Last, the biases used by the government should themselves be subject to external influence and be, therefore, the outcome of a game played prior to bidding for deciding.

References


