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Hal Berenson, President of True Mountain Group, LLC.1

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1 Introduction

There are countless examples of markets in which there are switching costs and network effects[2]. In the existing literature, there is a wealth of works in the dual areas of switching costs and two-sided markets, which, for instance, finds that large switching costs cause firms to charge a higher

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price to their locked-in customers (Klemperer, 1987b), and large network externalities cause platforms to charge a lower price (Armstrong, 2006), but very little is known or understood about how markets react to the interaction between the two concepts. This paper provides new insights on how switching costs and network externalities affect firm’s pricing strategies; I show that in the presence of interaction effects, welfare analyses that merely sum up the bargain effects of switching costs and network externalities in the introductory period are prone to error.

A useful example is the smartphone operating system market. Apple, Google and Windows are key players in the market. Each of them faces two groups of consumers, application users and application developers. While it is easy for consumers to migrate data from an older version of Windows Phone to a newer version, a consumer who switches from Android to Windows Phone incurs the cost of migrating—if not re-purchasing—a set of apps, media files, as well as contacts, calendars, emails and messages. As suggested by Hal Berenson, one of the problems faced by Windows Phone is its weak app library. Suppose now that Windows improves its library by introducing more Android apps. This not only raises the utility of users through network externality but also lowers their switching cost in terms of data migration. For instance, making some Android movie or music streaming apps available also for Windows Phone allows users to migrate their media files across devices more easily without the hassle of moving the data manually, which results in lower switching costs. Such change may seem to be welfare-improving because the extent to which platforms can exploit their locked-in customers is smaller. In a model incorporating both switching costs and network effects, however, I show that a decrease in switching costs of the user leads to an increase in the price for developers. Since developers value the participation of the user and a decrease in switching costs of the user makes attracting users easier, the platform can price higher to extract the increased value to developers. As a consequence, lower switching costs may not improve overall consumer welfare. This cross-group effect of switching costs is a novelty of this paper as it does not emerge from the classic Armstrong’s (2006) two-sided model due to the model’s static property and from the classic Klemperer’s (1987b) switching cost model due to the one-sidedness of his model. The existence of cross-group effects emphasizes that regulators need to consider the interaction between switching costs and network externalities carefully and avoid mechanical analysis of them by simply adding up their effects because the overall effect across all consumer groups, through feedback effects, can be larger than the sum of effects. In some cases, larger network externalities increase switching costs.

This paper analyzes the combined effect of switching costs and network effects on profits and pricing of platforms competing in two-sided markets, and explores the implications of my results from both regulatory (e.g. welfare concerns about switching costs) and managerial points of view (e.g. whether focusing on switching costs or network effects is more likely to increase the profits of platforms, which may lead to very different app/OS design strategies beyond pricing). The analysis also provides insight into other two-sided markets with switching costs.

\(^3\)In reality, there are also other types of switching costs (e.g. the cost of learning how to operate a new interface and the psychological inclination to stick with what we know). See Klemperer (1995) and the UK Office of Fair Trading (2003).
costs, such as media, credit cards, video games, and search engines.

I consider a two-period duopoly model, where platforms 0 and 1 sell their product to two groups of consumers. Each group is represented by a Hotelling line with unit length. Each consumer can purchase from either platform (single-homing). The penultimate section will extend the analysis to cover the multi-homing case. There are both switching costs, meaning consumers exhibit inertia in their product choice, and indirect network externalities, meaning participation of one group increases the value of participating for the other group. Moreover, consumers may be loyal/disloyal and myopic/farsighted, i.e. there are four types. Loyal consumers never switch once they are attached to a platform, while disloyal consumers can switch to another rival platform at a cost. Myopic consumers are short-sighted and care only about their utility in the current period, while farsighted consumers make decisions based on their lifetime utility. This model is flexible enough that it can collapse to either a pure switching cost model or to a pure two-sided model for extreme parameter values. When both effects are at work, I show that conventional results will change: the overall bargain effect in this model can be larger than the sum of effects in pure switching cost and pure two-sided models. I focus on symmetric equilibrium in which platforms charge the same price to each side. I also show that such equilibrium exists even when parameters on the two sides are not symmetric.

The main results can be summarized as follows. I show that in equilibrium switching costs do not affect second-period prices, whereas the impact of switching costs on first-period prices depends on the strength of two effects. The first is consumer’s anticipation effect: more patient consumers are less tempted by a temporary price cut because they understand that the price cut will be followed by a price rise in later periods. Their demand is therefore less elastic, and platforms will respond by charging higher prices. The second effect is firm’s anticipation effect: more patient platforms put more weight on future profits, and thus both compete aggressively for market share. When network externalities are weak, the first-period price is U-shape in switching costs: platform’s anticipation effect dominates when switching costs are small, and consumer’s anticipation effect dominates when switching costs are large. When externalities become sufficiently strong, however, consumer’s anticipation effect is reversed because consumers value the platform as the platform facilitates their interaction with the other side, even though they anticipate that the platform might exploit their reluctance to switch later. Consequently, switching costs with strong network externalities overturn the standard U-shape result and always intensify first-period price competition. See Proposition 2. This effect is new in the literature because consumer’s anticipation effect is absent from Armstrong’s (2006) model and goes in opposite direction in Klemperer’s (1987b) model. Furthermore, there is another new cross-group effect that is not present in either of these two models: an increase in switching costs on one side unambiguously decreases the price on the other side. The reason is that platforms can build market share on one side either by directly lowering price on this side or by indirectly increasing participation on the other side. When switching costs on the first side are large, an easier way to build market share is to focus on the indirect channel; consequently first-period competition is intensified on the other side. I call the new interaction effect between switching costs and network externalities an “indirect bargain effect”, as opposed
to the traditional “direct bargain effect” of switching costs in Klemperer’s (1987b) model, where feedback effects between customer groups are not present.

Adding consumer heterogeneity does not change the main result above, but provides more insights that do not emerge from standard two-sided analyses. In particular, I show that platforms offer lower first-period prices to one side if there are many myopic and loyal consumers. The reason is that loyal customers know that they will patronize the same platform for an indefinite period of time, and thus platforms need to provide a bigger carrot to lure these customers to join. The presence of myopic consumers, who care only about immediate cost and reward, gives even more incentive to platforms to compete aggressively. Moreover, an increase in the proportion of myopic consumers on one side causes the platform to charge a higher first-period price to the other side. This is because higher price elasticity on the first side with more myopic consumers reduces the opportunity cost of recruiting consumers on the other side. Therefore, it leads to less competitive behavior on the other side. See Proposition 4.

This paper’s contribution is twofold. First, it provides some general policy rules for studying how markets react to switching costs and network externalities in the absence of Coasian bargaining. While the existing literature has tended to focus on either switching costs or network externalities, I study the two concepts together and show that switching costs may work differently in two-sided markets compared to one-sided markets. The important policy rule is to recognize the role of the indirect bargain effect, and factor that into the overall assessment of the effects of switching costs. Regulators should not merely sum up the effects of switching costs and network externalities in traditional models because failing to account for the indirect bargain effect may underestimate the extent to which switching costs can enhance welfare. Furthermore, from a managerial perspective, although both switching costs and network externalities reduce the overall profits of platforms, I show that strategies that lead to lower switching costs (e.g. by offering guides on how to make a switch across platforms and introducing apps that smooth data migration) might be more effective at increasing overall profits than strategies that lead to lower network externalities (e.g. by improving the app library), depending on the relative magnitude of switching costs and network externalities.

The second contribution relates to the characterization of equilibrium pricing strategy in a model not only with switching costs and network externalities, but also with consumer heterogeneity, which is an issue that has been neglected in the two-sided market literature. More

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4The Coase (1960) Theorem, which states that bargaining will lead to an efficient outcome even with externality if there are no transaction costs, does not apply in a market with switching costs because the signing of long-term contracts to eliminate inefficiencies that arise from dynamic complementarity between goods bought across periods is not possible. Similarly, two-sided markets, where indirect network externalities are important, only exist in situations where Coase Theorem fails because there is cross-group complementarity between demands of different customer groups. As defined by Rochet and Tirole (2006), a necessary condition for a market to be two-sided is that Coasian bargaining cannot take place. In this model, the Coase Theorem fails because there are both dynamic complementarity on the firm’s side and cross-group complementarity on the consumer’s side.

5Heterogeneity in consumers is unusual in the two-sided literature except for a few recent papers that focuses on matching problem, e.g Gomes and Pavan (2013).
specifically, this model shows that the strategy of lowering prices in two-sided markets is not simply due to network externalities, but it also depends on the characteristics of consumers such as their level of loyalty and farsightedness. An interesting policy issue is that by re-interpreting “disloyal” consumers as “ignorant” consumers, I find that platforms may provide imprecise information about consumers’ preferences, so that these consumers are more ignorant, and they will switch more, which platforms can exploit later. Therefore, there is room for government intervention, particularly in achieving a greater transparency of information. Ignorant consumers would benefit from more information, so that they are able to make choices that are best aligned to their tastes, and save considerable switching costs. See Section 4.

1.1 Related Literature

There is a sizeable literature on switching cost, which broadly speaking, can be categorized into two main groups. One group of papers assumes that firms cannot discriminate between old and new consumers. Firms knowing that they can exercise market power in the second period over those consumers who are locked-in, they are willing to charge a lower price in the first period in order to acquire these valuable customers. This “bargains-then-ripoffs” pattern is the main result of the first-generation switching cost models (see for instance Klemperer (1987a, b)). A second group of works allows for price discrimination, so firms can charge a price to its old customers and a different price to new ones. Chen (1997) analyzes a two-period duopoly with homogeneous goods. Under duopoly, consumers who leave their current supplier have only one firm to switch to. Since there is no competition for switchers, this allows the duopolist to earn positive profits in equilibrium. Taylor (2003) extends Chen’s model to many periods and many firms. With three or more firms, there are at least two firms vying for switchers, and if products are undifferentiated, these firms will compete away all their future profits. More recent contributions include Biglaiser, Crémer and Dobos (2013), who study the consequence of heterogeneity of switching costs in an infinite horizon model with free entry. They show that even low switching cost customers are valuable for the incumbent.

The design of pricing strategies to induce consumers on both sides to participate has occupied a central place in the research on two-sided markets. The pioneering work is Caillaud and Jullien (2003), who analyze a model of imperfect price competition between undifferentiated intermediaries. In the case where all consumers must single-home, the only equilibrium involves one platform attracting all consumers and the platform making zero profit. In contrast, when consumers can multi-home, the pricing strategy is of a “divide-and-conquer” nature: the single-homing side is subsidized (divide), while the multi-homing side has its entire surplus extracted (conquer). Armstrong (2006) advances the analysis by putting forward a model of competition between differentiated platforms by using the Hotelling specification. He finds that the equilibrium price is determined by the magnitude of cross-group externalities and whether consumers single-home or multi-home. His approach is the closest to mine. However, he focuses

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on a static model of two-sided market without switching costs, while here with switching costs and different degrees of sophistication the problem becomes a dynamic one. Another closely related paper is Rochet and Tirole (2006), who combine usage and membership externalities (as opposed to the pure-usage-externality model of Rochet and Tirole (2003), and the pure-membership-externality model of Armstrong (2006)), and derive the optimal pricing formula. But they focus on the analysis of a monopoly platform.

Substantial studies have been separately conducted in the dual areas of switching costs and two-sided markets, but analysis is rarely approached from a unified perspective. This paper seeks to fill the gap. Besides this study, there is little literature that studies the interaction between switching costs and network externalities. Su and Zeng (2008) analyze a two-period model of two-sided competing platforms. Their focus is on the optimal pricing strategy when only one group of consumers has switching costs and their preferences are independent, and thus their model applies only to a limited subset of multi-sided markets, such as browsers, search engines, and shopping malls; whereas this paper studies a richer setting in which both sides bear switching costs, and consumers may be loyal/disloyal and myopic/farsighted. This seems a more natural feature of many markets, such as smartphone and video games. Biglaiser and Crémer (2011, 2014a, 2014b) and Biglaiser, Crémer and Dobos (2013), in a series of papers, compare the effect of switching costs and network effects on entry in a one-sided market. They show that switching costs and network effects together can have complicated effects on the profits of the incumbent, which depends on the relative importance of switching costs versus network effects. However, this model differs from theirs in that I focus on the interaction between “two-sided” network effects and switching costs, whereas they focus more on “one-sided” network effects. This means that in their model switching costs of one type of consumers do not affect prices of the other type, which is one of the key elements I study here. This is also one of the key forces that lead to the whole effect of switching costs and network effects being greater than the sum of its parts in this model.

2 Model

Consider a two-sided market with two periods. Each side \( i \in \{A, B\} \) of the market is characterized by a Hotelling line with unit length; and two platforms are located at the endpoints 0 and 1 on each side. That is, platform 0 is located at 0 and platform 1 at 1 on both sides of \( A \) and \( B \).\(^8\) Both platforms discount second-period profit at rate \( \delta_F \).

On each side \( i \), there is a unit mass of consumers, who are uniformly distributed on the Hotelling line. A consumer can be of four types: he can be either farsighted or myopic, and be either loyal or disloyal.\(^9\) Farsighted consumers on side \( i \) make decisions based on their lifetime utility, and they discount second-period utility at rate \( \delta_i \). Myopic consumers on side \( i \) make decisions based on their first-period utility, and therefore they have a discount factor \( \delta_i = 0 \).

\(^8\)This does not affect the analysis up to relabeling of the endpoints.

\(^9\)My main result (Proposition 5) does not depend on these sources of consumer heterogeneity, but heterogeneity sheds light on more general issues such as the design of loyal programs that arise in markets with switching costs (See Section 4).
Loyal consumers do not switch. Their preference, which is represented by their location on the Hotelling line, does not change across the two periods, and thus in the second period, they always stay with the same platform from which they have purchased in the first period.\footnote{I verify in Appendix B that platforms have no incentive to deviate to serve these loyal consumers under condition (B.2). Klemperer (1987b) makes a similar assumption, but he assumes that those consumers, who have fixed tastes, respond to prices in both periods.} Disloyal consumers’ preference is independent across the two periods, and more specifically, they are randomly relocated on the Hotelling line in the second period. A disloyal consumer on side $i$, whether they are farsighted or myopic, can switch to the other platform at a cost $s_i$, provided the other platform delivers a higher level of utility.\footnote{A survey published by Consumer Intelligence Research Partners (CIRP) reveals that 20% of Apple’s new iPhone customers were previous Android phone owners. The possibility of learning new information overtime could be one reason why consumers switch, as it is difficult for consumers to fully understand in advance their taste for apps and smartphones, which are constantly evolving. This quarterly survey was taken from data surveying 500 subjects in the US who had purchased a new mobile phone in the previous 90 days over the last four quarters, between July 2012 and June 2013.} The assumption that some consumers have independent preferences is needed for technical reason because it smoothes the demand function, which is a standard assumption in the switching costs literature. However, Ruiz-Aliseda (2013) shows that such assumption may have unintended consequences of price competition in the second period ending up being too soft. The presence of loyal consumers relaxes such assumption. This could also support the fact that in practice not all consumers have changing preferences. Assume that for some exogenous reasons in each period consumers choose to single-home. Section 5.1 will extend the analysis to cover the multi-homing case.

I assume that a proportion $\alpha_i$ of the consumers is myopic\footnote{This is different from Klemperer (1987b) because he does not consider the possibility of having a mixture of myopic and farsighted consumers. Consumers are either all myopic or all farsighted.} and a proportion $\mu_i$ of the consumers is loyal.\footnote{Loyalty in this model can be interpreted in two ways: First, it can be interpreted as exogenous. Loyal consumers are not able to switch because they have large switching costs. Second, loyalty can be interpreted as endogenous. Suppose that switching cost is drawn from a two-point distribution: $s$ is small with probability $1 - \mu$, and $s$ is big with probability $\mu$. In this case, the concept of loyalty is endogenized because it is determined by switching costs. Both interpretations lead to the same calculations, but for simplicity I adopt the first interpretation for the rest of the analysis.} Both $\alpha_i$ and $\mu_i$ are known by the platforms and by users on both sides.\footnote{Myopia is therefore defined as consumers ignoring the link between their utility in the two periods, even though they observe prices set by both platforms, and the proportions of loyal and farsighted consumers.}

I also assume that platforms charge uniform prices and they cannot price discriminate among their previous customers and customers who have bought the rival’s product in the previous period. Platforms and consumers have common knowledge about the value of switching cost $s_i$. Consumers know that they will not switch with probability $\mu_i$, and have switching cost $s_i$ with probability $1 - \mu_i$. The timing of the game is as follows:

- At the beginning of the first period, consumers are unattached to any platforms. They learn their initial preferences (but they do not learn their loyalty until they make their first-period purchase). Platforms set the first-period prices. Consumers choose which platform to join.
At the beginning of the second period, consumers learn whether they are loyal or not. If indeed consumers are disloyal, they also learn their second period preferences. Platforms set the second-period prices. Consumers decide to switch or not.

The solution concept for the game is subgame perfect equilibrium (SPE).

The utility of a consumer on side \(i\), who is located at \(x\), is

\[
v_i + e_i n^i_{k,t} - |x - k| - p^i_{k,t},
\]

where \(i, j \in \{A, B\}\) and \(i \neq j\). \(v_i\) is the intrinsic value of consumers on side \(i\) for using either platform. Assume that \(v_i\) is sufficiently large such that the market is fully covered. \(e_i\) is the benefit that consumer from side \(i\) enjoys from interacting with each consumer on the other side (for simplicity, I ignore the possibility that consumers also care about the number of people in the same group who joins the platform). \(n^i_{k,t}\) is the market share of platform \(k\) on side \(i\) in period \(t\), where \(k \in \{0, 1\}\), \(i \in \{A, B\}\) and \(t \in \{1, 2\}\). Thus, \(e_i n^i_{k,t}\) is the total external benefit from interacting with the other side of the market. \(|x - k|\) is the transport cost when a consumer purchases from platform \(k\), where the unit cost is normalized to one without loss of generality. Platform \(k\) charges a uniform price \(p^i_{k,t}\) on side \(i\) in period \(t\).

Platform \(k\)’s profit in period \(t\) is given by

\[
\pi_{k,t} = p^A_{k,t} n^A_{k,t} + p^B_{k,t} n^B_{k,t},
\]

which is the sum of revenues from side \(A\) and side \(B\). I make three assumptions. First, assume that the marginal cost of production is equal to zero for simplicity. Second, assume that \(s_i \in [0, 1]\), where one is the unit transport cost, so that at least some consumers will switch. Third, assume \(e_i \in [0, 1]\) in order to ensure that the profit function is well-defined, and the demand is decreasing in a platform’s own price and increasing in its rival’s price.\(^\text{15}\)

### 2.1 Second Period: the mature market

I work backward from the second period, where each platform has already established a customer base. Given the first-period market shares \(n^A_{0,1}\) and \(n^B_{0,1}\), a consumer on side \(i\), located at \(\theta^i_0\) on the unit interval, purchased from platform 0 in the first period is indifferent between continuing to buy from platform 0 and switching to platform 1 if

\[
v_i + e_i n^i_{0,2} - \theta^i_0 - p^i_{0,2} = v_i + e_i (1 - n^i_{0,2}) - (1 - \theta^i_0) - p^i_{1,2} - s_i.
\]

The indifferent consumer is given by

\[
\theta^i_0 = \frac{1}{2} + \frac{1}{2} [e_i (2n^i_{0,2} - 1) + p^i_{1,2} - p^i_{0,2} + s_i].
\]

Another consumer on side \(i\), positioned at \(\theta^i_1\), previously purchased from platform 1 is indifferent between switching to platform 0 and continuing to purchase from platform 1 if

\[
v_i + e_i n^i_{0,2} - \theta^i_1 - p^i_{0,2} - s_i = v_i + e_i (1 - n^i_{0,2}) - (1 - \theta^i_1) - p^i_{1,2}.
\]

\(^\text{15}\)More specifically, one represents the unit transport cost. Assuming \(e_i < 1\) ensures that in the symmetric equilibrium, both platforms serve some consumers.
The indifferent consumer is given by
\[ \theta_i = \frac{1}{2} + \frac{1}{2}[e_i(2n_{0,2}^i - 1) + p_{1,2} - p_{0,2} - s_i]. \]

We then substitute \( \theta_i^0 \) and \( \theta_i^1 \) into the following.
\[ n_{0,2}^i = \mu_i n_{0,1}^i + (1 - \mu_i)n_{0,1}^i \theta_i^0 + (1 - \mu_i)(1 - n_{0,1}^i)\theta_i^1. \] (2)

Consumers of platform 0 consist of three types, and similarly for platform 1. The first type is loyal customers (whose tastes do not change over time), who buy from platform 0 in both periods. The second type is disloyal customers (whose tastes change randomly over time), who stay with platform 0 despite being disloyal. The third type is also disloyal customers, who indeed switch away from platform 1 to platform 0.

Then, we solve for the market shares, plug them into the profit functions, and solve for the second-period prices. The details are shown in Appendix A.

**Lemma 1.** Given first-period market share, on side \( i, i \in \{A, B\} \), the platform with a larger market share \( (n_{k,1} > 1/2) \), \( k \in \{0, 1\} \), increases the second-period price \( p_{k,2}^i \) as switching costs \( s_i \) increase; whereas the other platform with a smaller market share \( (n_{k,1}^i < 1/2) \) decreases the second-period price \( p_{k,2}^i \) as switching costs \( s_i \) increase. When platforms have equal market shares in the first period \( (n_{0,1}^i = n_{1,1}^i) \), switching cost \( s_i \) does not affect second-period prices \( p_{k,2}^i \).

**Proof.** See Appendix A.1

There are two possible strategies: On the one hand, the platform might want to exploit its locked-in customers with a high price due to its market power over these consumers. On the other hand, the platform might want to poach its rival’s customers with a low price. A larger market share means exploiting old customers is more profitable than attracting new customers. Notice that if both platforms have equal market share in the first period, these two effects offset each other, which means that switching cost does not affect second period prices. This is indeed what happens in the symmetric equilibrium (see Proposition 1). However, analyzing second-period pricing strategy is important because it determines the intertemporal effect of first-period pricing: a first-period price change will lead to a change in second-period profit and hence second-period price.

Moreover, there is a cross-group effect of switching cost, i.e. the pricing on one side also depends on the switching cost on the other side.

**Lemma 2.** Given first-period market share, the second-period price of platform 0, \( p_{0,2}^i \), is increasing in switching costs on the other side \( s_j \) if

(i) Consumers on side \( j \) are more valuable \( (e_i > e_j) \), and platform 0 has a larger market share on side \( j \) \( (n_{0,1}^j > 1/2) \), or

(ii) Consumers on side \( i \) are more valuable \( (e_i < e_j) \), and platform 0 has a smaller market share on side \( j \) \( (n_{0,1}^j < 1/2) \).
Proof. See Appendix A.2

The intuition behind Lemma 2 runs as follows. Part (i) shows that consumers on side \( j \) are more valuable to the platform because they exert stronger externalities on consumers on side \( i \) compared to externalities of side \( i \) on side \( j \). If the platform has a larger market share of the more valuable side, it can charge higher second-period prices to both sides compared to the case without switching costs. That is, \( \partial p_{0,2}/\partial s_j > 0 \) from Lemma 1 and \( \partial p_{0,2}/\partial s_j > 0 \) from (i) of Lemma 2.

By contrast, part (ii) shows that if the platform has a smaller market share of side \( j \), according to Lemma 1 it will focus more on poaching side \( j \) with a low price than exploiting them with a high price, that is, \( \partial p_{0,2}/\partial s_j < 0 \). It will then charge a higher second-period price to side \( i \) because decreasing the price on side \( j \) reduces the “opportunity cost” of recruiting consumers on side \( i \): the platform loses less revenue on side \( j \) by recruiting one less consumer on side \( i \).

Both platforms thus compete less aggressively for them. Consequently, higher switching costs on side \( j \) cause the platform to charge a higher price on side \( i \), that is, \( \partial p_{0,2}/\partial s_j > 0 \). Note that what platform 1 will do is just the opposite of platform 0 because the market is fully covered.

2.2 First Period: the new market

I now turn to the first-period equilibrium outcomes when consumers are unattached. All consumers have discount factor \( \delta_i \). However, on side \( i \), a proportion \( \alpha_i \) of consumers are myopic (\( N \)) with \( \delta_i = 0 \). They make decisions based on their first-period utility only. The remaining \( 1 - \alpha_i \) of side \( i \)’s population is farsighted (\( R \)) with \( \delta_i > 0 \). They make decisions based on their lifetime utility.

A myopic consumer on side \( i \) located at \( \theta^i_N \) is indifferent between buying from platform 0 and platform 1 if

\[ v_i + e_i n_{0,1}^j - \theta^i_N - p_{0,1}^i = v_i + e_i (1 - n_{0,1}^j) - (1 - \theta^i_N) - p_{1,1}^i, \]

which can be simplified to

\[ \theta^i_N = \frac{1}{2} + \frac{1}{2}[e_i (2n_{0,1}^j - 1) + p_{1,1}^i - p_{0,1}^i]. \]

As for sophisticated consumers, they also take into consideration their second-period utility. If a sophisticated consumer on side \( i \) located at \( \theta^i_R \) joins platform 0 in the first period, his expected second-period utility is given by

\[ U_{0,2}^i = \mu_i (v_i + e_i n_{0,2}^j - \theta^i_R - p_{0,2}^i) + (1 - \mu_i) \int_0^{\theta^i_0} (v_i + e_i n_{0,2}^j - x - p_{0,2}^i) dx \]
\[ + (1 - \mu_i) \int_{\theta^i_0}^1 (v_i + e_i (1 - n_{0,2}^j) - (1 - x) - p_{1,2}^i - s_i) dx. \]

\[ ^{16} \text{Rochet and Tirole (2003, 2006) explain that the difference between a one-side market and a two-sided market lies in the change in this opportunity cost. In particular, the standard Lerner formula becomes} \]
\[ \frac{p^i - (c - p^j)}{p^i} = \frac{1}{\eta^i} \]
\[ \text{in a two-sided market, where } c \text{ is the marginal cost and } \eta \text{ is the price elasticity.} \]
$U_{0,2}$ is the sum of three terms. With probability $\mu_i$ the consumer is loyal and chooses to join platform 0 in both periods; with probability $(1 - \mu_i)\theta^0_i$ he has independent preferences but still chooses to stay with platform 0; and with probability $(1 - \mu)(1 - \theta^0_i)$ he has independent preferences and he switches to platform 1.

Similarly, if he joins platform 1 in the first period, his expected second-period utility is given by

$$U_{1,2} = \mu_i (v_i + e_i (1 - n_{0,2}^i) - (1 - \theta^i_R) - p_{1,2}^i) + (1 - \mu_i) \int_{\theta^i_R}^1 (v_i + e_i (1 - n_{0,2}^i) - (1 - x) - p_{1,2}^i) dx \quad + (1 - \mu_i) \int_0^{\theta^i_R} (v_i + e_i n_{0,2}^i - x - p_{0,2}^i - s_i) dx.$$

A sophisticated consumer on side $i$ is indifferent between purchasing from platform 0 and platform 1 if

$$v_i + e_i n_{0,1}^i - \theta^i_R - p_{0,1}^i + \delta_i U_{0,2}^i = v_i + e_i (1 - n_{0,1}^i) - (1 - \theta^i_R) - p_{1,1}^i + \delta_i U_{1,2}^i.$$

After some rearrangement, this gives

$$\theta^i_R = \frac{1}{2} + \frac{1}{2} [e_i (2n_{0,1}^i - 1) + p_{1,1}^i - p_{0,1}^i + \delta_i (U_{0,2}^i - U_{1,2}^i)].$$

The indifferent farsighted consumer and the indifferent myopic consumer have different preferences ($\theta^i_R$ vs. $\theta^i_N$) because they have different expectations about future utility.

The first-period market share of side $i$ is

$$n_{0,1}^i = \alpha_i \theta^i_N + (1 - \alpha_i) \theta^i_R. \quad (3)$$

Then, we can derive the profit functions, and solve for the equilibrium prices. I focus on the platform-symmetric equilibrium: both platforms charge the same price to each side, that is, $p^A_{0,1} = p^A_{1,1}$ and $p^B_{0,1} = p^B_{1,1}$.

**Proposition 1.** The single-homing model has a symmetric equilibrium. The equilibrium prices are given by

$$p_{0,1}^i = \frac{1 - \kappa_i}{\tau_i} - \frac{\sigma_j}{\tau_j} - e_j - \delta_F \xi_i,$$

and

$$p_{0,2}^i = \frac{1 - e_j (1 - \mu_i)}{1 - \mu_i},$$

where $\tau_i$, $\sigma_i$ and $\xi_i$ are positive, $\kappa_i$ may be positive or negative, for $i, j \in \{A, B\}$, and $j \neq i$.

**Proof.** See Appendix B, where expressions for $\kappa_i$, $\tau_i$, $\sigma_i$, and $\xi_i$ are also given. 

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It is well-known that network externalities are a source of multiplicity of equilibria. However, rather than studying asymmetric equilibrium, this paper focuses on symmetric equilibrium and proves the existence of it.
A full characterization of the symmetric equilibrium is quite complex and lengthy, and hence it is left to Appendix [B]. Here, I will only mention the intuitions: the existence of a symmetric equilibrium requires that platforms’ profit to be concave, and there is not a profitable deviation for platforms to serve only the loyal customers. More precisely, the former condition requires that externalities are not too strong (see Equation (B.1)), and the latter implies that consumers’ preferences are sufficiently independent across periods (see equation (B.2)). Notice that although I focus on the analysis of symmetric equilibrium, the existence of it does not require all parameters on the two sides ($\alpha_A$ and $\alpha_B$, $\mu_A$ and $\mu_B$, $e_A$ and $e_B$, $s_A$ and $s_B$) to be symmetric, provided that consumers on each side view the platforms as symmetric. I discuss the case of asymmetric platforms where it is more costly to switch from one platform to the other ($s_0 \neq s_1$) in Section [5.2]

Both Klemperer (1987b) and Armstrong (2006) are special cases of this general model. More particularly, when there are no network externalities ($e_i = 0$), and all consumers are farsighted ($\alpha_i = 0$) and have independent preferences ($\mu_i = 0$), $i \in \{A, B\}$, the first-period equilibrium price becomes

$$p_i^0 = 1 + \frac{2}{3} \left( \frac{\delta_i s_i^2}{\text{consumer's anticipation}} - \frac{\delta_F s_i}{\text{firm's anticipation}} \right),$$

which is equivalent to Equation (18) in Klemperer (1987b).

Since the level of the first-period price is lower in a market with switching costs than without them, the literature calls it a “bargain”. However, the extent of the bargain depends on switching costs. More specifically, Klemperer (1987b) shows that the first-period price is U-shape in switching costs: whether the first-period price increases or decreases with switching costs depends on the relative strength of the consumer’s and the firm’s anticipation effects. On the one hand, farsighted consumers anticipate that if they are locked-in in the second period, the platform will raise its price. Thus, consumers are less responsive to a first-period price cut. This explains why consumers’ sophistication increases the first-period price through $\delta_i$. On the other hand, forward-looking platforms have strong incentive to invest in market share because they anticipate the benefit of having a larger customer base in the future. Platforms thus compete more aggressively to capture market share, and platforms’ sophistication decreases the first-period price through $\delta_F$.

When there are no switching costs ($s_i = 0$), all consumers have independent preferences ($\mu_i = 0$), and both the consumers and the firms do not care about the future ($\delta(1-\alpha_i)$, $\delta_F = 0$), $i \in \{A, B\}$, the first-period equilibrium price becomes

$$p^i = 1 - e_j,$$

\[\text{footnote 18: Other expressions favored in the literature are “anticipation effect” and “investment incentive”—e.g. Einav and Somaini (2013)—and “consumer elasticity effect” and “investment effect”—e.g. Rhodes (2013). I will simply call them consumer’s and firm’s anticipation effect because the mechanism goes through the discount factor. My paper is quite different from Einav and Somaini (2013) and Rhodes (2013): they examine the effect of switching costs in a dynamic setting without network externalities, while I discuss a model with both switching costs and network externalities.} \]
which is the same as in Proposition 2 of Armstrong (2006). This equation shows that platforms compete fiercely for the more valuable group, whose external benefit exerted on the other group of consumers is larger.

In what follows, I explore the implications of several variations of this model, focusing on homogeneous consumers with $\alpha_i \in \{0, 1\}$ and $\mu_i = 0$ and heterogeneous consumers with $\alpha_i \in (0, 1)$ and $\mu_i \in (0, 1)$ in turn.

3 Switching Costs in Two-sided Markets

The pattern of attractive introductory offers followed by higher prices to exploit locked-in consumers—the “bargains-then-ripoffs” pricing—is well-known in the literature. However, the standard “ripoff” effect in the literature, which means the second-period price paid by consumers is higher in a market with switching costs than in a market without switching costs, does not emerge in this model. Rather, the focus here is more on the “bargain” effect in the first period, and this analysis is the first to decouple a “direct” bargain from an “indirect” bargain. More specifically,

- A direct bargain means that the first-period price is lower with switching costs than without, as defined in Klemperer (1987b).
- An indirect bargain means increasing participation on one side increases the value of the platform to the other side, and such indirect network effects leads to an even bigger bargain effect of switching costs for consumers on the other side, which is a new interaction effect in the literature.

The indirect bargain effect will be made clearer in Proposition 2.

3.1 Farsighted Consumers

Let us first consider the case of homogeneous consumers in which all consumers are farsighted. In a two-sided market with switching costs, because of an additional indirect bargain, I find that the first-period price is always decreasing in switching costs when externalities are strong, which is different from Klemperer’s result that the first-period price is U-shape in switching costs. And such indirect bargain operates through the interaction between network externalities and switching costs of both sides (rather than entirely through network externalities as in Armstrong’s result). This variation is a good representation of markets such as smartphone and video games. Smartphone: switching from Apple’s iOS to Google’s Android system, application

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19Ruiz-Aliseda (2013) shows that, however, in a model in which the only source of differentiation comes from switching costs, “small” switching costs will result in “ripoffs-then-ripoffs” pricing rather than the traditional result of “bargains-then-ripoffs”. This paper follows traditional models, which focus on “large” switching costs. Reader, who is interested in the case of “small” switching costs, may refer to Ruiz-Aliseda (2013).

20There is the possibility to exploit loyal customers even though the standard ripoff effect of switching cost is absent. As shown in Proposition 1, the second-period equilibrium price is $p_{0, 2}^i = \frac{1 - e_j (1 - \mu_i)}{1 - \mu_i}$, which is larger than the price in a model without loyal customers, $p^i = 1 - e_j$. 
developers need to re-code their programs for different interfaces, as well as to create additional support and maintenance; whereas application users need to migrate and re-purchase their applications. Video games: switching from Sony’s PlayStation to Windows’ Xbox, gamers need to re-learn how to use the controller and lose the progress of their games, whereas developers have to buy a separate development kit to create games for different consoles.

**Proposition 2.** In the single-homing model, with all consumers and both platforms equally patient, \( \delta_i = \delta_F = \delta > 0 \) and \( \alpha_i = 0 \); independent preferences, \( \mu_i = 0 \); and symmetric externalities, \( e_i = e > 0 \), \( i \in \{A, B\} \), the first-period price is given by

\[
p_{0,1}^i = 1 - \underbrace{e}_{\text{Armstrong (2006)}} + \underbrace{\frac{2\delta}{3}(s_i^2 - s_i)}_{\text{Klemperer (1987b)}} - \underbrace{\frac{\delta}{3(1 - e^2)}(e^2s_i^2 + es_i s_j)}_{\text{Indirect bargain}}.
\]

i. If externalities are weak \( (e < \sqrt{2/3}) \), on each side the first-period price \( p_{0,1}^i \) is U-shape in switching costs \( s_i \).

ii. If externalities are strong \( (e \geq \sqrt{2/3}) \), on each side the first-period price \( p_{0,1}^i \) is decreasing in switching costs \( s_i \).

iii. The first-period price charged to side \( i \), \( p_{0,1}^i \), is decreasing in switching costs on side \( j \), \( s_j \).

**Proof.** See Appendix C.

Proposition 2 shows that the extent of the bargain depends not only on switching costs on one side (as in Klemperer) and the strength of network externalities (as in Armstrong), but also on switching costs on the other side, which operates through the indirect bargain.

More specifically, part (i) shows that when externalities are weak, we get the result of Klemperer: the bargain is inverted U-shape in switching costs. For small switching costs, farsighted consumers understand that they can easily switch in the second period, and are therefore more responsive to price cut in the first period. Platforms have strong incentive to compete for market share. Consequently, switching costs lead to more competition when they are small. By contrast, when switching costs are very large, farsighted consumers recognize that they will be exploited in the second period, and are therefore less tempted by a price cut. Their demand becomes less elastic, and platforms will respond by charging higher prices. This explains why switching costs lead to less competition when they are large.

Part (ii) describes the first term in the indirect bargain. It shows that strong externalities overturn the U-shape result: the first-period price is always decreasing in switching costs, and the positive relationship between first-period price and switching costs does not arise. The reason is that network externalities together with switching costs weaken the incentive of farsighted consumers to avoid being locked-in (consumer’s anticipation effect) because participation on the other side increases the value of the platform, even though farsighted consumers anticipate that the platform might exploit their reluctance to switch later. Consequently, switching costs always leads to more first-period price competition when externalities are strong. This effect
differs from Armstrong (2006): since he examines a static two-sided model, consumer’s anticipation effect is absent. It also differs from Klemperer (1987): the consumer’s anticipation effect in his model softens price competition, while the effect in this model intensifies competition through the indirect bargain.

Part (iii) describes the second term in the indirect bargain. It shows that an increase in switching costs on one side unambiguously decreases the first-period price charged to the other side. The reason is that platforms can build market share on side $j$ via two channels: directly through side $j$, and indirectly through side $i$. When switching costs on side $j$ are large, farsighted consumers are less responsive to price cuts because they expect a price rise to follow in the second period. An easier way to build market share on side $j$ is then to focus on the indirect channel, i.e. attracting side $i$. As a result, first-period competition is increased on side $i$. This cross-group effect of switching costs is absent from both models of Armstrong and Klemperer.

The results of (ii) and (iii) show that we cannot simply add up the bargain effects of switching costs and network externalities as combined together they will lead to an even bigger price reduction than the sum of price reductions in pure switching cost and pure two-sided models. This is complementary to the literature because it provides a formal explanation on the mechanism through which the interaction effects $e_{s_i}$ and $e_{s_i}s_j$ work.

### 3.2 Myopic Consumers

Let us now examine the case in which all consumers are myopic. A straightforward interpretation of myopic consumers is that these consumers only care about their utility in the current period. Or, alternatively, this could be interpreted as “new” consumers, who are different in every period. For example, a company buys some software for their workers in the first period. Some workers leave the company in the second period, and purchase their own software. These workers have a switching cost of learning the new software product that is different from that purchased by their former company, but the company will not take into consideration such switching cost when making its purchase in the first period.

**Proposition 3.** In the single-homing model, when all consumers are myopic, $\delta_i = 0$ and $\alpha_i = 1$; and have independent preferences, $\mu_i = 0$, $i \in \{A,B\}$, the first-period price $p_{i,1}$ is decreasing in switching costs $s_i$ regardless of the level of externalities.

**Proof.** See Appendix D

Myopic consumers do not anticipate that a first-period price cut will lead to a second-period price rise, and will therefore react more responsively to price cut in the first period. This increases the incentives of platforms to compete for market share, and thus results in more competition for myopic consumers. Strictly speaking, expectation about whether the others will switch play no role here because $\mu_i$ and $\alpha_i$ are common knowledge. In a broader sense, however, Proposition 3 is consistent with results in von Weizsäcker (1984) and Borenstein, MacKie-Mason and Netz (2000). They show that if consumers expect that a firm’s price cut is
more permanent than their tastes, which can be interpreted as consumers being myopic, then switching costs tend to lower prices.

3.3 Heterogeneous Consumers

Propositions 2 and 3 are derived under the assumption of homogeneous consumers. Let us now turn to discuss the consequence of heterogeneity in consumers. On each side, a fraction $\alpha_i$ of consumers is myopic, while $1 - \alpha_i$ of them is farsighted; and a proportion $\mu_i$ of consumers is loyal, while the remaining consumers have independent preferences.

**Proposition 4.** In the single-homing model,

i. On each side, the first-period price $p_{i,0}$ is decreasing in the proportion of myopic consumers $\alpha_i$ if

$$\frac{\mu_i + 2\mu_i(1 - \mu_i)s_i + (1 - \mu_i)^2s_i^2}{\mu_i^2 + 3\mu_i(1 - \mu_i)s_i + (1 - \mu_i)^2s_i^2} > \frac{9 - (1 - \mu_i)(1 - \mu_j)(e_i + 2e_j)(e_j + 2e_i)}{3}.$$  

(ii). The first-period price on side $i$, $p_{0,i}$, is increasing in the proportion of myopic consumers on side $j$, $\alpha_j$.

(iii). The first-period price $p_{0,i}$ is decreasing in the discount factor of the platform $\delta_F$.

**Proof.** See Appendix E.

A sufficient condition under which Equation (4) holds is the proportion of loyal consumers $\mu_i$ is high. Part (i) therefore shows that on each side, if there are many loyal consumers, the first-period price is lower with myopic consumers than without. The reason is that after loyal consumers make their first-period purchase, they know that they will patronize the same platform for an indefinite period of time. Platforms therefore need to provide a bigger carrot to lure these loyal customers to join. Myopic consumers, who care only about today, are more attracted by a price cut. Therefore, increasing the proportion of consumers who are loyal and myopic makes the market more competitive in the first period. However, if consumers’ tastes change ($\mu_i$ is small), it nullifies the competitive effect of myopia partially.

Part (ii) shows that an increase in the proportion of consumers who are myopic on one side softens price competition on the other side. Intuitively, the demand of myopic consumers on side $j$ is more elastic, and platforms will react by charging lower prices. This, in turn, reduces the opportunity cost of recruiting consumers on side $i$. Platforms thus compete less aggressively for market share on side $i$. Consequently, consumers’ myopia on one side mitigates the ferocity of first-period competition for market share on the other side.

Part (iii) shows that first-period prices are lower when platforms are more patient. Platforms compete harder on prices because they foresee the advantage of having a large customer base in the future.

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21 Gabszewicz, Pepall and Thisse (1992) also explore the consequences of heterogeneity in consumers’ brand loyalty, but they consider the pricing strategy of a monopoly incumbent, who anticipates the entry of a rival in the subsequent period, and focus on the effect of loyalty on entry, a different issue from this model.
More generally, Propositions 2 and 4 show that the strategy of lowering price is not simply due to network externalities in a two-sided market, a view that is central to the work of Rochet and Tirole (2003), and Armstrong (2006). But in my model whether the platform will act more aggressively also depends on switching costs of different consumer groups (as in Proposition 2) and the characteristics of these consumers (as in Propositions 4). This has important implications for analyzing the impact of switching costs and loyalty rate on social welfare, which will be explored more fully in Section 4.

3.4 Asymmetric Sides

The model also covers the case of asymmetric sides, where consumers on one side, say side $B$, do not incur any switching costs in the second period ($s_B = 0$). Examples of such a market include browsers, search engines, and shopping malls. Browsers: Internet users can switch relatively more easily between Internet Explorer, Chrome, and Firefox than content providers because when content providers switch, they need to rewrite the codes so that they are compatible with the new browser. Search engines: customers can switch easily between Google, Bing and Yahoo in as little as one click, but there are switching costs for top-listed publishers, who want their website to appear on the top list of another search engine. Shopping malls: shoppers are free to go to any shopping malls, but there are high transaction costs for shops in terminating the old contract and initiating a new one.

For simplicity, assume that consumer preferences are independent, $\mu_i = 0$; all consumers are farsighted, $\alpha_i = 0$; and they have the same discount factor as the firm, $\delta_i = \delta_F = \delta, i \in \{A, B\}$.

**Corollary 1.** If only one side of consumers has switching costs, then switching costs only affect the price on this side but not the other side.

**Proof.** Under the assumptions above,

\[
p^B_{0,1} = 1 - e_A.
\]

The intuition is that since preferences of consumers on side $B$ in the two periods are unrelated and they do not have switching costs, every period’s choice is independent. This means that the first-period price is not affected by the second-period price. Consequently, although side $A$’s switching costs affect side $B$’s second-period price in the subgame equilibrium, it does not affect side $B$’s first-period price.

3.5 Effect of Switching Costs on First-period Profit

In a platform-symmetric equilibrium, the two platforms share consumers on each side equally, that is $n^A_{0,1} = n^B_{0,1} = 1/2$. Therefore, the expected profit of platform 0 is

\[
\pi_0 = \frac{1}{2} p^A_{0,1} + \frac{1}{2} p^B_{0,1} + \delta \pi_{0,2},
\]

where $\pi_{0,2}$ is the second-period profit.
Differentiating $\pi_0$ with respect to $s_i$, we obtain

$$\frac{\partial \pi_0}{\partial s_i} = \frac{1}{2} \frac{\partial p_{0,1}^i}{\partial s_i} + \frac{1}{2} \frac{\partial p_{0,1}^j}{\partial s_i}$$

because the profit in the second period, $\pi_{0,2}$, is not affected by $s_i$ in equilibrium.

As in Klemperer (1987b), switching costs do not affect the second-period profit of the platform, but they lead to a decrease in overall profit because the presence of market power over locked-in consumers intensifies price competition in the first period. More interestingly, I identify a new channel—the indirect bargain—through which switching costs can reduce overall profit. The indirect bargain has two effects. First, network externalities together with switching costs weaken consumer’s anticipation effect because consumers value participation on the other side. This effect distinguishes this model from Armstrong (2006) because such effect is absent from his model, and from Klemperer (1987b) because consumer’s anticipation effect goes in opposite direction. As a result, switching costs on side $i$ intensify price competition on side $i$ (see (ii) of Proposition 2). The second effect of the indirect bargain, which is absent from both Armstrong (2006) and Klemperer (1987b), is that higher switching costs on side $i$ also lead to more competitive behavior on side $j$ because capturing more consumers on side $j$ is a cheaper way to build market share on side $i$. Side $i$ consumers are harder to attract as they have strong incentives to avoid being locked-in and thus paying large switching costs in the second period (see (iii) of Proposition 2). Consequently, higher switching costs lower prices on both sides, which result in lower overall profit.

Since both switching costs and network externalities reduce the overall profit of platforms, from a managerial perspective it would be desirable for platforms to lower both of them. However, depending on the relative magnitude of switching costs and network externalities, reducing switching costs might be more effective at increasing overall profits than reducing network externalities because a decrease in switching costs lowers prices on both sides, whereas a decrease in network externalities lowers the price on one side only. Therefore, strategies that lead to lower switching costs such as providing guides on how to make a switch across platforms and introducing apps that help data migration, and thus allows users to assess the same media content across platforms (e.g. Google Play Movies & TV on iTunes and cloud computing technology), would benefit the platforms more than strategies that lead to lower network externalities such as improving their own app library relative to their rivals’.

4 Welfare and Policy Implications

There are in general two welfare criteria for analyzing the impact of switching costs on welfare. The first one concerns social welfare.

**Proposition 5.** Social welfare decreases with switching costs.

**Proof.** The first-period social welfare is constant in switching costs, while the second-period welfare is decreasing in switching costs. More specifically, the second-period welfare loss is the
sum of two deadweight losses:

\[ 2(1 - \mu_i)[\left( \frac{1 - s_i}{2} s_i \right) + \frac{s_i^2}{4}]. \]

\[ \text{DWL from switchers} \quad \text{DWL from non-switchers} \]

The first-period social welfare is constant in switching costs because all consumers buy one unit of good, the size of the two groups is fixed, and the whole market is served. There are no demand-expansion and demand-reduction effects of switching costs as the total demand is fixed. However, the second-period welfare is decreasing in switching costs. The second-period welfare loss is the sum of two deadweight losses. Considering consumers who have independent preferences, since their tastes will change in the second period, for those who have previously bought from platform 0, consumers whose tastes change a lot will switch to platform 1 with probability \((1 - s_i)/2\) and each pays \(s_i\); consumers whose tastes change a little will continue to buy from platform 0 even though they prefer platform 1. This happens with probability \(s_i/2\) and each suffers an average loss of mismatch with an inferior product \(s_i/2\). A similar distortion arises for consumers who have previously bought from platform 1, and for both groups of consumers. As for loyal consumers, they do not suffer any loss because first, they do not switch; second, their preferences do not change, and hence there is no deadweight loss associated with a mismatch problem.

Another criterion concerns consumer welfare. Although switching costs lower social welfare, consumers may enjoy a net gain when the benefit from a lower first-period price is larger than the sum of the two deadweight losses. Focusing on disloyal consumers, we can show that

**Proposition 6.** Considering consumer welfare, in the symmetric case of \(e_i = e, s_i = s\) and \(\delta_i = \delta_F = \delta\), and when all consumers have independent preferences, \(\mu_i = 0, i \in \{A, B\}\),

- When all consumers are myopic, consumer surplus increases with switching costs.
- When all consumers are farsighted, consumer surplus increases with switching costs if network externalities are strong \((e \geq 3/7)\); and consumer surplus is inverted U-shape in switching costs if externalities are weak \((e < 3/7)\).

**Proof.** See Appendix F.

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22 The gain in better match is not relevant to the analysis of the effect of switching costs on social welfare because such gain is not affected by switching costs. To see why, suppose that there are no switching costs, consumers can switch if they prefer, and thus get a better match of product with their tastes. Suppose now that switching costs are positive, consumers who switch will still gain from a better match. On the contrary, the loss in mismatch is affected by switching costs, as switching costs may prevent some consumers from switching.

23 Myopia does not affect social welfare. The only thing that matters for social welfare is whether consumers’ preferences change or not. When myopic consumers’ preferences do not change, they make the right product choice and do not switch. When myopic consumers’ preferences change, switchers have to bear switching costs; and some of the non-switchers are forced into buying an inferior product that does not match their tastes.
Farsighted/myopic and disloyal consumers are included in the analysis of consumer surplus, but not loyal consumers because loyal consumers do not switch and therefore care about price only. Since the first-period price is always lower with switching costs than without, considering loyal consumers in the analysis would only increase the welfare-enhancing effect of switching costs.

This model thus provides two general policy rules in two-sided markets with switching costs. First, policymakers need to consider demand interdependencies more carefully. When there are such interdependencies, switching costs of one group of consumers are likely to affect the prices of another group. However, since most of the theoretical models that study the welfare effects of switching costs rely on the assumption that the market is one-sided (see Section 2.9 in Farrell and Klemperer (2007)), they are generally not applicable to studying two-sided markets. Many papers in the two-sided literature, for instance, Wright (2004) and Evans and Schmalensee (2014), have pointed out various policies (without focusing on switching costs) that apply one-sided results to two-sided markets are prone to commit errors. The welfare analysis of this paper is complementary to their view in the sense that I show in Proposition 6 that switching costs can be welfare-increasing, depending on the strength of demand interdependencies between two groups of consumers. In this model, demands can be interdependent in two ways: through network externalities, which is the traditional channel in the two-sided literature, and through the interaction between switching costs and network externalities, which is the main novelty of this paper. In evaluating consumer protection policy, for instance, because it is common to have bargains-then-ripoffs pricing in one-sided markets with switching costs, attractive introductory offers may call for consumer protection in later periods through compatibility or standardization policies that lower switching costs. In two-sided markets, however, I show in Proposition 2 that lowering switching costs of one group will unambiguously raise the first-period price of the other group, and hence such change will benefit consumers on one side while making consumers on the other side worse off. Accurate welfare analysis should account for this cross-group effect of \( s_j \) on \( p_i \), which operates through demand interdependencies.

Second, policymakers need to assess carefully the value of the indirect bargain. This paper derives new insights on the bargain effect of switching costs: I show in Proposition 2 that the interaction between switching costs and network externalities may lead to a yet bigger bargain effect of switching costs than the sum of effects in traditional models. Policies should avoid mechanical analysis of simply adding up the effects of switching costs and network externalities because this may underestimate the extent to which switching costs can enhance welfare.

One can also draw some lessons from Proposition 4. In this model, all consumers know their initial preference before their first-period purchase. While loyal customers always know their first- and second-period tastes, disloyal customers, given that their tastes change randomly in the second period, only learn their second-period taste after their first-period purchase. This is equivalent to receiving a random signal in the first period and discovering their taste later. Therefore, one could alternatively interpret \( \mu_i \) as the fraction of consumers who know their preference, while \( 1 - \mu_i \) as the remaining consumers who do not know theirs. These “ignorant” consumers receive a signal about their taste, and learn their taste after first-period purchase.
In this case, all consumers’ locations on the line are fixed across periods. The previous results remain valid with this reinterpretation, provided the signal received by ignorant consumers is uniformly distributed. This allows us to assess the impact of information transparency policy. In particular, Proposition 4 shows that an increase in $\mu_i$ makes it more likely that consumers’ myopia will hurt the platform. Thus, platforms may lack incentive to enhance consumers’ understanding of their own preferences. They might try to provide imprecise information about consumers’ tastes, so that consumers are more ignorant, and they will switch more, which platforms can exploit later. Therefore, there is room for government intervention. In particular, increasing transparency of information would enable ignorant consumers to make choices that are best aligned to their tastes and save switching costs.

5 Extensions

The analysis so far is based on a single-homing model, but this is not the only market configuration in reality. There are various ways to extend the model, for instance, one may consider the case where one group single-homes while the other group join both (commonly termed as “competitive bottlenecks”). It might also be interesting to consider asymmetric compatibility between platforms’ products. I will sketch these extensions in turn.

5.1 Competitive Bottlenecks

Suppose that side $A$ continues to single-home, while side $B$ may multi-home. Competitive bottleneck framework is typical in markets such as computer operating systems, and online air ticket and hotel bookings. Operating systems: users use a single OS, Windows OS, Apple’s Mac OSX platform or Linux-based OS, while engineers write software for different OS. Travel bookings: consumers rely on one comparison site such as skyscanner.com, lastminute.com or booking.com, but airlines and hotels join multiple platforms in order to gain access to each comparison site’s customers. Since side $B$ can join both platforms, switching costs and loyalty on this side are not relevant, so that $s_B, \mu_B = 0$. The main difference from the single-homing model lies in the market share of consumers on side $B$, which can be described as follows. In period $t, t \in \{1, 2\}$, a consumer on side $B$ located at $\theta_{B,t}$ is indifferent between buying and not buying from platform 0 if

$$v_B + e_B n_{A,t} - p_{B,t}^{A} = 0,$$

Note that the concept of multi-homing is not compatible with switching costs in the current framework. I use two examples to illustrate. First, think of the smartphone market. If the option to multi-home means consumers are able to use both iPhone and Android systems, then it is not reasonable to impose an additional learning cost on them if they switch platform. Another example is the media market. If multi-homing means that advertisers are free to put ads on either or both platforms, then it does not make sense to impose an additional switching cost on these advertisers. One may argue that we can distinguish between learning switching costs (incurred only at a switch to a new supplier) and transactional switching costs (incurred at every switch), as in Nilssen (1992), but switching costs are not relevant on the multi-homing side because learning costs and transaction costs are equivalent in a two-period model. This also explains why it is not useful to consider the case in which both sides multi-home.
which can be simplified to
\[ \theta_{0,t}^B = v_B + e_B n_{0,t}^A - p_{0,t}^B. \]

Similarly, a consumer on side \( B \) located at \( \theta_{1,t}^B \) is indifferent between buying and not buying from platform 1 if
\[ v_B + e_B (1 - n_{0,t}^A) - (1 - \theta_{1,t}^B) - p_{1,t}^B = 0, \]
which can be simplified to
\[ \theta_{1,t}^B = v_B + e_B (1 - n_{0,t}^A) - p_{1,t}^B. \]

We solve the game by backward induction as before. Consider symmetric equilibrium. Appendix [G] proves the existence of it. We can then derive the equilibrium prices.

**Proposition 7.** In the multi-homing model, with all consumers and both platforms equally patient, \( \delta_i = \delta_F = \delta > 0 \) and \( \alpha_i = 0 \); independent preferences, \( \mu_i = 0 \); and symmetric externalities, \( e_i = e > 0 \), \( i \in \{A,B\} \),

i. For the single-homing consumers, if externalities are weak \( (e < \sqrt{2/3}) \), the first-period price \( p_{0,1}^A \) is U-shape in switching costs \( s_A \). If externalities are strong \( (e \geq \sqrt{2/3}) \), the first-period price \( p_{0,1}^A \) is decreasing in \( s_A \).

ii. First-period prices tend to be higher on the multi-homing side and lower on the single-homing side with respect to the single-homing model in Section 3.4 if the market is fully covered \( (i.e. \ e + v_B/2 > 1) \).

**Proof.** See Appendix [G].

Part (i) implies that for single-homing consumers stronger externalities make it more likely that first-period equilibrium prices decrease with switching costs, which is consistent with Proposition 2 in the single-homing model. As for multi-homing consumers, both switching costs and the degree of sophistication do not affect the price paid by them because each period’s choice is independent. This case and the previous case of asymmetric sides have similar intuition because \( s_B, \mu_B = 0 \). Part (ii) is different from results in the single-homing model. Since side \( B \) multi-homes, there is no competition between the two platforms to attract this group. Compared to the case of asymmetric sides, the higher first-period price faced by the multi-homing side is a consequence of each platform having monopoly power over this side, and the large revenue is used in the form of lower first-period price to convince the single-homing side to join the platform.

Before, in the single-homing model, Proposition 5 shows that switching costs do not affect the first-period social welfare, but lower the second-period (and hence overall) social welfare. However, in the multi-homing model switching costs also affect first-period social welfare through participation, which is, in turn, determined by the price. In the second period, switching cost has no effect on price because platforms have an equal share of the market, and their incentives to exploit old customers offset their incentives to poach new customers. However, if switching costs reduce first-period price (especially when externalities are strong, as shown in
(i) of Proposition 7), it is possible that switching costs increase overall social welfare. This is because lower price induces more consumers to multi-home, and more multi-homing consumers increases the utility of single-homing consumers through network externalities. Therefore, an increase in switching costs could improve social welfare if such gain outweighs the deadweight losses associated with switching costs.

5.2 Asymmetric Compatibility

Let us now consider competition between platforms of products with asymmetric compatibility: the cost of switching from platform 0 to 1, denoted $s_0$, is different from the cost of switching from platform 1 to 0, denoted $s_1$. As an example, some say that “iPhones are more expensive than most Samsung smartphones.” Can we attribute the difference in the pricing of devices between Apple and Samsung to the fact that Apple has successfully built an ecosystem that makes users hard to switch?

In this case, a platform-symmetric equilibrium would no longer exist because the platforms become asymmetric. Therefore, in each period, instead of having two different prices in equilibrium, we have four, which makes the analysis less tractable. I consider the following numerical example to illustrate how asymmetric compatibility influences the equilibrium pricing: $\delta_A = \delta_B = \delta_F = 0.8$, $\mu_A = \mu_B = 0$, $e_A = e_B = 0.5$, $s_1 = 0.5$, and $s_0 \in [0, 1]$. In addition, assume that only consumers on side $A$ bear switching costs, and that all consumers single-home. In Figure 1 I illustrate the equilibrium pricing of the platforms on side $A$. The analysis of the effects of switching costs on side $B$ is irrelevant for similar reasons in Corollary 0.

![Figure 1: Equilibrium Pricing with Asymmetric Compatibility.](image-url)

(a) First-period Prices

(b) Second-period Prices

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25 If there is quality choice as in Anderson et al. (2013), then welfare effects are less clear-cut: platform’s investment in quality may change depending on whether multi-homing is allowed.


27 Since this paper focuses on the effect of switching costs in the symmetric equilibrium, I will leave the full-scale analysis of asymmetric equilibrium for future research.
Panel (a) presents the first-period pricing on side $A$, and panel (b) shows the second-period pricing on side $A$ as functions of switching costs $s_0$. Pricing of platform 0 is shown with a solid line, and that of platform 1 is drawn as a dotted line. It is shown that if $s_0 < s_1$, platform 1 charges a lower price than platform 0 in the first period, but a higher price in the second period. The intuitive reason is that since platform 1 is relatively more expensive to switch away from in the second period, it is willing to charge a lower price in the first period in order to acquire more customers whom it can exploit later. On the contrary, if $s_0 > s_1$, platform 1, knowing that consumers will easily switch away tomorrow, will raise its price today. This result holds as long as externalities are not too strong.\textsuperscript{28}

6 Conclusion

This paper has characterized the equilibrium pricing strategy of platforms competing in two-sided markets with switching costs, which can be applied to a wide range of industries, ranging from traditional industries such as shopping malls and credit cards to high-tech industries such as smartphones and video games. The main contribution is that it has provided a useful model for generalizing, and extending beyond, the traditional results in the switching cost and the two-sided literature. In line with earlier research, there are some conditions under which the first-period price is U-shape in switching costs (à la Klemperer); and prices tend to be lower on the side that exerts stronger externalities (à la Armstrong). However, this model also provides new insights by proving that in a dynamic two-sided market—as opposed to a merely static one—under strong network externalities, the standard U-shape result does not emerge and the first-period price always decreases with switching costs. This is due to the existence of cross-group effects of switching costs that do not emerge in either Armstrong’s or Klemperer’s models. Recognizing the importance of these additional effects is critical for ensuring that consumer protection policies do not cause unintended consequence of reducing overall consumer welfare by causing more harm to one side than good to the other side.

In terms of managerial implications, this paper provides some guidance to platforms for analyzing the effect of switching costs and network externalities on profits. Recently, Windows has been trying to improve the value of its services by focusing more on network externalities. For example, the new Windows 10 allows users’ data to integrate more smoothly from phones and tablets to PCs and Xbox game consoles.\textsuperscript{29} However, my results suggest that since switching costs lower first-period prices on both sides, while network externalities lower first-period price on one side only, delivering services across a breadth of OS (between Windows and Android as well as between Windows and iOS), which lowers switching costs, might be more effective at increasing profits than delivering services across a breadth of devices of Windows’ ecosystem only.

\textsuperscript{28} When network externalities are strong ($e \to 1$), there is coordination problem, and all consumers might want to join one platform only.

The literature on the interaction between switching costs and network externalities is relatively thin and does not provide a solid basis for evaluating their effects. This paper is a first attempt to analyze the impact of such interaction, but much work remains to be done: First, this paper has taken switching costs as an exogenous feature of the market. Future research could consider endogenous switching costs. Second, this paper has focused on a two-period model, and it would be useful to understand the extent to which the results carry over to a multi-period model. Finally, this paper has explored consumer heterogeneity such as loyalty and farsightedness, but one can think of other forms of heterogeneity. For example, within-group switching costs may be different between the technologically advanced customers and the less advanced ones. Within-group externalities may also be different: youngsters use applications more heavily, and therefore care more about network externalities than their older counterparts, many of whom only use their smartphones for phone calls and text messages. However, including these forms of heterogeneity will complicate the analysis considerably.

The current model captures a lot of ingredients in reality, yet is sufficiently tractable to allow for a complete characterization of the equilibrium. This seems to be a reasonable first step to extend a literature that has not fully explored the consequences of consumer heterogeneity.

Appendices

A Second Period Equilibrium

Solving for $n_{0,2}^A$ and $n_{0,2}^B$ in Equation (2) simultaneously, we obtain the second-period market shares as follows:

$$n_{0,2}^i = \frac{\gamma + \beta_i + (1 - \mu_i)(p_{1,2}^i - p_{0,2}^i) + e_i(1 - \mu_i)(1 - \mu_j)(p_{1,2}^j - p_{0,2}^j)}{2\gamma},$$

where

$$\gamma = 1 - (1 - \mu_A)(1 - \mu_B)e_Ae_B,$$

$$\beta_i = (2n_{0,1}^i - 1)(\mu_i + (1 - \mu_i)s_i) + (2n_{0,1}^j - 1)(1 - \mu_i)e_i(\mu_j + (1 - \mu_j)s_j).$$

Because $e_i < 1$, we have $\gamma > 0$.

Substituting the market shares into the profit function in Equation (1), and differentiating it with respect to the prices, we obtain the following equations.

$$\frac{\partial \pi_{0,2}}{\partial p_{0,2}^i} = n_{0,2}^i - \frac{p_{0,2}^i}{2\gamma}(1 - \mu_i) - \frac{p_{0,2}^j}{2\gamma}e_j(1 - \mu_i)(1 - \mu_j),$$

$$\frac{\partial \pi_{1,2}}{\partial p_{1,2}^i} = 1 - n_{0,2}^i - \frac{p_{1,2}^i}{2\gamma}(1 - \mu_i) - \frac{p_{1,2}^j}{2\gamma}e_j(1 - \mu_i)(1 - \mu_j).$$

See Ambrus and Argenziano (2009) for a model with heterogeneous network effects, where platforms can also price discriminate, but with no switching costs.
Solving the system of first-order conditions, one finds the following second-period equilibrium prices.

\[
\begin{align*}
p_{0,2}^i &= \frac{1 - e_j(1 - \mu_i)}{1 - \mu_i} + \frac{\eta_i \lambda_i + \epsilon_i \lambda_j}{(1 - \mu_i)\Delta}, \\
p_{1,2}^i &= \frac{1 - e_j(1 - \mu_i)}{1 - \mu_i} - \frac{\eta_i \lambda_i + \epsilon_i \lambda_j}{(1 - \mu_i)\Delta}.
\end{align*}
\]  

(A.1)

where

\[
\Delta = 9 - (1 - \mu_A)(1 - \mu_B)(e_A + 2e_B)(e_B + 2e_A) > 0,
\]

\[
\lambda_i = (2n_{0,1}^i - 1)(\mu_i + (1 - \mu_i)s_i),
\]

\[
\eta_i = 3 - e_j(e_j + 2e_i)(1 - \mu_i)(1 - \mu_j) > 0,
\]

\[
\epsilon_i = (1 - \mu_i)(e_i - e_j).
\]

A.1 Proof of Lemma 1

Differentiate Equation (A.1) with respect to \(s_i\), we have

\[
\text{sign} \frac{\partial p_{0,2}^i}{\partial s_i} = \text{sign}(n_{0,1}^i - \frac{1}{2}),
\]

\[
\frac{\partial p_{0,2}^i}{\partial s_i} = - \frac{\partial p_{1,2}^i}{\partial s_i}.
\]

A.2 Proof of Lemma 2

Differentiate Equation (A.1) with respect to \(s_j\), we have

\[
\text{sign} \frac{\partial p_{0,2}^i}{\partial s_j} = \text{sign}(e_i - e_j)(n_{0,1}^j - \frac{1}{2}),
\]

\[
\frac{\partial p_{0,2}^i}{\partial s_j} = - \frac{\partial p_{1,2}^i}{\partial s_j}.
\]

B First Period Equilibrium

The indifferent farsighted consumer is given by

\[
\theta_i^R = \frac{1}{2} + \frac{e_i(2n_{0,1}^i - 1) + p_{1,1}^i - p_{0,1}^i + \delta_i(\mu_i + (1 - \mu_i)s_i))(1 - \mu_i)(e_i + 2e_j)(1 - \mu_i)\lambda_i + (3 - \Delta)\lambda_J}{2(1 + \delta_i\mu_i)}.
\]

Substitute \(\theta_i^N\) and \(\theta_i^R\) into Equation (3), and solve simultaneously for \(n_{0,1}^A\) and \(n_{0,1}^B\):

\[
n_{0,1}^i = \frac{1}{2} + \frac{e_i(1 - \kappa_j)(p_{1,1}^i - p_{0,1}^i) + \tau_j(e_i\tau_i + \sigma_i)(p_{1,1}^i - p_{0,1}^i)}{2[(1 - \kappa_i)(1 - \kappa_j) - (e_i\tau_i + \sigma_i)(e_j\tau_j + \sigma_j)]},
\]

\[
\theta_i^N = \frac{1}{2} + \frac{e_i(2n_{0,1}^i - 1) + p_{1,1}^i - p_{0,1}^i + \delta_i(\mu_i + (1 - \mu_i)s_i))(1 - \mu_i)(e_i + 2e_j)(1 - \mu_i)\lambda_i + (3 - \Delta)\lambda_J}{2(1 + \delta_i\mu_i)}.
\]
that platform
I derive the sufficient condition for the existence of such symmetric equilibrium, which requires
where
\begin{align*}
\tau_i &= \alpha_i + \frac{1 - \alpha_i}{1 + \delta_i \mu_i}, \\
\kappa_i &= \frac{\delta_i (\mu_i + (1 - \mu_i) s_i) (3 - \Delta)(1 - \alpha_i)(\mu_i + (1 - \mu_i) s_i)}{(1 - \mu_i) \Delta (1 + \delta_i \mu_i)}, \\
\sigma_i &= \frac{\delta_i (\mu_i + (1 - \mu_i) s_i) (\epsilon_i + 2 \epsilon_j)(1 - \alpha_i)(\mu_i + (1 - \mu_j) s_j)}{\Delta (1 + \delta_i \mu_i)}.
\end{align*}

The expected profit of platform 0 is
\[
\pi_0 = p_{0,1}^A n_{0,1}^A + p_{0,1}^B n_{0,1}^B + \delta_F \pi_{0,2}.
\]

The first-order conditions for maximizing \(\pi_0\) with respect to \(p_{0,1}^A\) and \(p_{0,1}^B\) are given as follows.
\[
\frac{\partial \pi_0}{\partial p_{0,1}^i} = n_{0,1}^i - p_{0,1}^i \frac{\tau_i (1 - \kappa_i)}{2 \varphi} - p_{0,1}^j \frac{\tau_j (1 - \kappa_j)}{2 \varphi} + \delta_F \left[ \frac{\partial \pi_{0,2}}{\partial n_{0,1}^i} \frac{\partial n_{0,1}^i}{\partial p_{0,1}^i} + \frac{\partial \pi_{0,2}}{\partial n_{0,1}^j} \frac{\partial n_{0,1}^j}{\partial p_{0,1}^i} \right],
\]
where
\[
\varphi = (1 - \kappa_i) (1 - \kappa_j) - (\epsilon_i \tau_i + \sigma_i)(\epsilon_j \tau_j + \sigma_j),
\]
\[
\frac{\partial \pi_{0,2}}{\partial n_{0,1}^i} = \left[ \frac{6}{(1 - \mu_i) \Delta} + \frac{(\epsilon_i - \epsilon_j) - (\epsilon_i + \epsilon_j)(\epsilon_j + 2 \epsilon_i)(1 - \mu_j)}{\Delta} \right] (\mu_i + (1 - \mu_i) s_i) \equiv \xi_i.
\]

Similarly, there are two first-order conditions for platform 1.

I focus on the platform-symmetric equilibrium, where \(p_{0,1}^A = p_{1,1}^A = p^A\) and \(p_{0,1}^B = p_{1,1}^B = p^B\).

I derive the sufficient condition for the existence of such symmetric equilibrium, which requires that platform \(k\)’s profit is concave in its prices. The concavity condition is as follows.

\[
1 - \kappa_A > \epsilon_A \tau_A + \sigma_A > 0; \quad 1 - \kappa_B > \epsilon_B \tau_B + \sigma_B > 0.
\]

In addition to Equation (B.1), to ensure that the platform does not deviate from the equilibrium price, we need the following condition:
\[
v_i + \frac{1}{2} \epsilon_i - \frac{1}{2} > \frac{1}{1 - \mu_i} - \epsilon_i > (v_i + \frac{1}{2} \epsilon_i - \frac{1}{2}) \mu_i, \quad i \in \{A,B\}.
\]

The first inequality means that we need \(v_i\) to be big enough such that the market is covered. The second inequality means that we need \(\mu_i\) to be small enough and \(v_i\) to be big, but not too big, in order to guarantee that the platform does not deviate to serve only loyal consumers in the second period. For example, Equations (B.1) and (B.2) are satisfied when \(\alpha_i\) is big and/or \(\mu_i = 0\) is small.\(^{31}\)

Under symmetric equilibrium, the first-period equilibrium prices for side \(A\) and side \(B\) are given respectively by
\[
p_{0,1}^A = \frac{1 - \kappa_A}{\tau_A} - \frac{\sigma_B}{\tau_B} - \epsilon_B - \delta_F \xi_A; \quad p_{0,1}^B = \frac{1 - \kappa_B}{\tau_B} - \frac{\sigma_A}{\tau_A} - \epsilon_A - \delta_F \xi_B,
\]
and the second-period equilibrium prices are given by
\[
p_{0,2}^A = \frac{1 - \epsilon_B (1 - \mu_A)}{1 - \mu_A}; \quad p_{0,2}^B = \frac{1 - \epsilon_A (1 - \mu_B)}{1 - \mu_B}.
\]

\(^{31}\)When \(\alpha_i = 1\), we obtain the same existence condition for a symmetric equilibrium as in Armstrong (2006). I show that the equilibrium exists for a wider range of parameters.
C Proof of Proposition 2

If $\delta_A = \delta_B = \delta_F = \delta > 0$, $\alpha_A = \alpha_B = 0$, $\mu_A = \mu_B = 0$, and $e_A = e_B = e > 0$, Equation (B.3) becomes

$$p_{0,1}^i = 1 - e + \frac{2\delta}{3} (s_i^2 - s_i) - \frac{\delta}{3(1 - e^2)} (e^2 s_i^2 + e s_i s_j).$$

Differentiating $p_{0,1}^i$ with respect to $s_i$, we obtain

$$\frac{\partial p_{0,1}^i}{\partial s_i} = \frac{\delta}{3(1 - e^2)} \left[ 2(2 - 3e^2) s_i - 2(1 - e^2) - e s_j \right],$$

$$\frac{\partial^2 p_{0,1}^i}{\partial s_i^2} = \frac{2\delta(2 - 3e^2)}{3(1 - e^2)} \begin{cases} > 0 & \text{if } e < \sqrt{2/3}, \\ < 0 & \text{if } e \geq \sqrt{2/3}, \end{cases}$$

$$\left. \frac{\partial p_{0,1}^i}{\partial s_i} \right|_{s_i=0} = \frac{\delta}{3(1 - e^2)} [-2(1 - e^2) - e s_j] < 0.$$

Therefore, $p_{0,1}^i$ is U-shape in $s_i$ if $e < \sqrt{2/3}$, and decreasing in $s_i$ if $e \geq \sqrt{2/3}$.

Differentiating $p_{0,1}^i$ with respect to $s_j$, we get

$$\frac{\partial p_{0,1}^i}{\partial s_j} = -\delta e s_i \frac{\delta}{3(1 - e^2)} < 0.$$

Therefore, $p_{0,1}^i$ is decreasing in $s_j$.

D Proof of Proposition 3

If $\delta_A = \delta_B = 0$, $\alpha_A = \alpha_B = 1$, and $\mu_A = \mu_B = 0$, Equation (B.3) becomes

$$p_{0,1}^i = 1 - \delta_F \left[ \frac{6 + e_i - e_j - (e_i + e_j)(e_j + 2e_i)}{\Delta} \right] s_i - e_j.$$

Differentiating it with respect to $s_i$, we obtain

$$\frac{\partial p_{0,1}^i}{\partial s_i} < 0.$$

E Proof of Proposition 4

Differentiating Equation (B.3) with respect to $\alpha_A, \alpha_B$ and $\delta_F$, we obtain the following:

$$\frac{\partial p_{0,1}^i}{\partial \alpha_i} \begin{cases} \leq 0, & \text{if } \mu_i \to 1 \text{ or } e_i, e_j \to 0, \\ > 0, & \text{if } \mu_i \to 0 \text{ and } e_i, e_j \to 1, \end{cases}$$

since

$$\frac{\partial p_{0,1}^i}{\partial \alpha_i} > 0 \text{ if } \frac{\mu_i + 2\mu_i(1 - \mu_i)s_i + (1 - \mu_i)^2 s_i^2}{\mu_i^2 + 3\mu_i(1 - \mu_i)s_i + (1 - \mu_i)^2 s_i^2} > \frac{\Delta}{3}.$$

$$\frac{\partial p_{0,1}^i}{\partial \alpha_j} \geq 0.$$
F Proof of Proposition 6

Since second-period prices are not affected by switching costs, the impact of switching costs on consumer surplus is equal to the price reduction consumers enjoy with switching costs relative to without minus the discounted second-period deadweight losses.

When consumers are myopic, the change in consumer surplus is

\[
\Delta W = W(0) - W(s) = \delta \left[ \frac{2s}{3} - \left( \frac{s}{2} - \frac{s^2}{4} \right) \right]
\]

price reduction deadweight loss

\[
= \delta \left( \frac{s}{6} + \frac{s^2}{4} \right),
\]

which is increasing in \( s \).

When consumers are farsighted, the change in consumer surplus is

\[
\Delta W = W(0) - W(s) = \delta \left[ \frac{(3e^2 - 2)s^2 + 2(1 - e^2)s + es^2}{3(1 - e^2)} - \left( \frac{s}{2} - \frac{s^2}{4} \right) \right]
\]

price reduction deadweight loss

\[
= \delta \left( \frac{(9e - 5)s^2}{12(1 - e)} + \frac{s^2}{6} \right),
\]

which is a quadratic function. Thus, consumer surplus increases with switching costs for all \( s \in [0, 1) \) if \( \Delta W|_{s=1} > 0 \), which is satisfied if \( e \geq 3/7 \). If \( e < 3/7 \), then there exists a \( \bar{s} \in (0, 1) \) such that \( \Delta W|_{s \in (0, \bar{s})} > 0 \) and \( \Delta W|_{s \in (\bar{s}, 1)} < 0 \).

G Proof of Proposition 7

The first-order conditions of \( \pi_k, k \in \{0, 1\} \), with respect to \( p^A_{0,1} \) and \( p^B_{0,1} \) are, respectively,

\[
n^A_{k,1} - \frac{1}{2\omega}p^A_{k,1} - \frac{e}{2\omega}p^B_{k,1} - \frac{\delta}{2\omega} \frac{\partial \pi_{k,2}}{\partial n^A_{0,1}} = 0,
\]

\[
n^B_{k,1} - (1 + \frac{e^2}{2\omega})p^B_{k,1} - \frac{e}{2\omega}p^A_{k,1} - \frac{\delta e}{2\omega} \frac{\partial \pi_{k,2}}{\partial n^A_{0,1}} = 0,
\]

where

\[
\omega = 1 - e^2 - \frac{\delta s_A^2 (e^2 - 2\gamma)}{3\gamma}.
\]

Using similar proof as in the single-homing model, the symmetric equilibrium exists in the multi-homing model. The existence conditions are as follows. First, platform \( k \)'s profit is concave in its prices if \( \omega \geq 0 \), which means that \( \delta, s_A \) and \( e \) are not too big.

Second, we need to ensure that the platform does not deviate to sell only to loyal consumers on side \( A \).

\[
v_A + e \left( \frac{v_B}{2} + \frac{e}{2} \right) - \frac{1}{2} > 1 - (1 - \mu_A) \frac{e^2}{2} - \frac{ev_B}{2} > \left[ v_A + e \left( \frac{v_B}{2} + \frac{e}{2} \right) - \frac{1}{2} \right] \mu_A,
\]

or equivalently, \( \mu_A \) is small, and \( v_B \) is big, but not too big.
The first-period equilibrium prices are as follows.

\[
p^A_{0,1} = 1 - e^2 - \frac{\delta(3e^2 - 2)s^2_A}{3(1 - e^2)} - \frac{2\delta s_A}{3} - \frac{v_B e}{2},
\]

\[
p^B_{0,1} = \frac{v_B}{2}.
\]

For part (i), differentiate \( p^A_{0,1} \) with respect to \( s_A \).

\[
\frac{\partial p^A_{0,1}}{\partial s_A} = -\frac{2\delta}{3} - \frac{2\delta(3e^2 - 2)s_A}{3(1 - e^2)},
\]

\[
\frac{\partial^2 p^A_{0,1}}{\partial s_A^2} = -\frac{2\delta(3e^2 - 2)}{3(1 - e^2)} \begin{cases} > 0 & \text{if } e < \sqrt{2/3}, \\ < 0 & \text{if } e \geq \sqrt{2/3}, \end{cases}
\]

\[
\frac{\partial p^A_{0,1}}{\partial s_A} \bigg|_{s_A=0} = -\frac{2\delta}{3} < 0.
\]

Therefore, \( p^A_{0,1} \) is U-shape in \( s_A \) if \( e < \sqrt{2/3} \), and decreasing in \( s_A \) if \( e \geq \sqrt{2/3} \).

For part (ii), we compare the first-period prices paid by consumers who bear switching costs (side A) and those who do not (side B) in the multi-homing model (denoted \( mh \)) with that in the single-homing model in Section 3.4 (denoted \( sh \)).

For side A,

\[
p^A_{mh} < p^A_{sh} \text{ if } e + \frac{v_B}{2} > 1.
\]

For side B,

\[
p^B_{mh} > p^B_{sh} \text{ if } e + \frac{v_B}{2} > 1.
\]

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