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ABSTRACT

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What is the \( (\varepsilon'/\varepsilon)_{\text{exp}} \) telling us?

Andrzej J. Buras\(^a\), Jean-Marc Gérard\(^b\)

\(^a\) Technische Universität München, Physik Department, D-85748 Garching, Germany
\(^b\) Université catholique de Louvain, Physics Department, B-1348 Louvain-la-Neuve, Belgium

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Abstract

Nature might be kinder than previously thought as far as \( \varepsilon'/\varepsilon \) is concerned. We show that the recently obtained experimental value for \( \varepsilon'/\varepsilon \) does not require sizeable \( 1/N \) and isospin-breaking corrections. We propose to display the theoretical results for \( \varepsilon'/\varepsilon \) in a \( (P_{1/2}, P_{3/2}) \) plane in which the experimental result is represented by a \( (\varepsilon'/\varepsilon)_{\text{exp}} \)-path. This should allow to exhibit transparently the role of \( 1/N \) and isospin-breaking corrections in different calculations of \( \varepsilon'/\varepsilon \). From now on theorists are allowed to walk only along this \( (\varepsilon'/\varepsilon)_{\text{exp}} \)-path.

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1. Introduction

The totally unexpected observation [1] of a sizeable CP-violation in the \( K^0-\bar{K}^0 \) oscillations immediately triggered theoretical speculations about a new superweak interaction [2] obeying the strict \( |\Delta S| = 2 \) selection rule. The large value of the associated \( \varepsilon \)-parameter was then justified by the huge amplification due to the tiny \( K_L-K_S \) mass difference. Following this rather simple picture, it was absolutely unlikely that CP-violation would show up somewhere else in weak processes.

Almost exactly 37 years later, we know that superweak models have been definitely ruled out by the new generation of high-precision experiments on the \( |\Delta S| = 1 \) neutral \( K \)-decays. Indeed, the most recent measurements of the associated \( \varepsilon' \)-parameter that allows us to distinguish between \( \pi^+\pi^- \) and \( \pi^0\pi^0 \) final states in \( K_L \) decays give

\[
\text{Re}(\varepsilon'/\varepsilon) = \begin{cases} 
(15.3 \pm 2.6) \times 10^{-4} \text{ (NA48)} \ [3], \\
(20.7 \pm 2.8) \times 10^{-4} \text{ (KTeV)} \ [4].
\end{cases}
\]

Combining these results with earlier measurements by NA31 collaboration at CERN \((23.0 \pm 6.5) \times 10^{-4}\) [5] and by the E731 experiment at Fermilab \((7.4 \pm 5.9) \times 10^{-5}\) [6] gives the grand average

\[
\text{Re}(\varepsilon'/\varepsilon) = (17.2 \pm 1.8) \times 10^{-4}.
\]

The Standard Model for electroweak and strong gauge interactions accommodates, in principle, both \( \varepsilon \)- and \( \varepsilon' \)-parameters in terms of a single CP-violating phase. Rather early theoretical attempts [7] have predicted \( \varepsilon'/\varepsilon \) between \( 10^{-2} \) and \( 10^{-4} \). During the last decade a considerable progress in calculating \( \varepsilon'/\varepsilon \) has been done by several groups. These papers are reviewed in [8] where all relevant references can be found. The short distance contributions to \( \varepsilon'/\varepsilon \) are fully under control [9] but the presence of considerable long distance hadronic uncertainties precludes a precise value of \( \varepsilon'/\varepsilon \) in the Standard Model at present. Consequently, while theorists were able to predict the
sign and the order of magnitude of \( \varepsilon'/\varepsilon \), the range
\[
(\varepsilon'/\varepsilon)_{\text{th}} = (5 \text{ to } 30) \times 10^{-4}
\]
shows that the present status of \( (\varepsilon'/\varepsilon)_{\text{th}} \) cannot match the experimental one.

Though really expected this time, the non-vanishing value of a second CP-violating parameter has once again been determined by our experimental colleagues.

However, one should not forget the tremendous efforts made by theorists to calculate \( \varepsilon'/\varepsilon \) in the Cabibbo–Kobayashi–Maskawa paradigm [10] of the Standard Model. Simultaneously, one should not give up the hope that one day theorists will be able to calculate \( \varepsilon'/\varepsilon \) precisely. It is therefore important to have a transparent presentation of different theoretical estimates of \( \varepsilon'/\varepsilon \) in order to be able to identify the patterns of various contributions. On the other hand, having for the first time the definite precise number for \( (\varepsilon'/\varepsilon)_{\text{exp}} \) it is crucial to learn what Nature is trying to tell us about theory. In this note, we intend to make first steps in both directions.

2. Basic formulae

The standard parametrization for the hadronic \( K \)-decays into two pions:
\[
\begin{align*}
A(K^0 \to \pi^+\pi^-) &= A_0 e^{i\delta} + \frac{1}{\sqrt{2}} A_2, \\
A(K^0 \to \pi^0\pi^0) &= A_0 e^{i\delta} - \sqrt{2} A_2, \\
A(K^+ \to \pi^+\pi^0) &= \frac{3}{2} A_2,
\end{align*}
\]
contains the necessary ingredients to produce non-vanishing asymmetries. For illustration consider
\[
a_{\text{CP}} \equiv \frac{\Gamma(K^0 \to \pi^+\pi^-) - \Gamma(K^0 \to \pi^+\pi^-)}{\Gamma(K^0 \to \pi^+\pi^-) + \Gamma(K^0 \to \pi^+\pi^-)} = \frac{\sqrt{2} \sin \delta}{1 + \sqrt{2} \omega \cos \delta + \omega^2/2} \Im\left(\frac{A_2}{A_0}\right),
\]
where
\[
\omega \equiv \frac{\Re A_2}{\Re A_0}.
\]
In order that \( a_{\text{CP}} \) is non-vanishing the two partial isospin amplitudes \( A_0 \) and \( A_2 \) must have a relative CP-conserving phase (extracted from \( \pi\pi \) scattering) which turns out to be roughly equal to the phase of the \( \varepsilon \)-parameter:
\[
\delta \approx \Phi_\varepsilon \approx \pi/4,
\]
and a relative CP-violating phase
\[
\Im\left(\frac{A_2}{A_0}\right) \neq 0.
\]

These phases are nicely factorized in the physical parameter measuring direct CP-violation in hadronic \( K \)-decays
\[
\varepsilon' = \frac{i}{\sqrt{2}} e^{-i\delta} \Im\left(\frac{A_2}{A_0}\right),
\]
if one defines
\[
\begin{align*}
\eta_{+-} &\equiv \frac{A(K_L \to \pi^+\pi^-)}{A(K_S \to \pi^+\pi^-)} = \varepsilon' + \frac{\varepsilon'}{1 + \sqrt{2} \omega e^{-i\delta}}, \\
\eta_{00} &\equiv \frac{A(K_L \to \pi^0\pi^0)}{A(K_S \to \pi^0\pi^0)} = \varepsilon - \frac{2\varepsilon'}{1 - \sqrt{2} \omega e^{-i\delta}}.
\end{align*}
\]
This allows to measure \( \Re(\varepsilon'/\varepsilon) \) through
\[
\Re(\varepsilon'/\varepsilon) = \frac{1}{6} \left(1 - \frac{\omega}{\sqrt{2}} \cos \delta \right) \left(1 - \frac{\eta_{00}}{\eta_{+-}}\right)^2,
\]
where we have kept the small \( \mathcal{O}(\omega) \) correction usually dropped by experimentalists but kept by theorists in the evaluation of \( \varepsilon' \) using (9). Notice that the coincidence displayed in (7) implies an almost real \( \varepsilon'/\varepsilon \) so that, already at this level, Nature is kind to us.

In the Standard Model, CP-violation only arises from the arbitrary quark mass matrices. A straightforward diagonalization shifts then the unique physical phase into the Cabibbo–Kobayashi–Maskawa (CKM) unitary mixing matrix \( V \) associated with the \( V - A \) hadronic charged current
\[
J_{\mu}^{ab} = \bar{q}^a \gamma_\mu (1 - \gamma_5) q^b \equiv (\bar{q}^a q^b).\]

In this physical basis, we therefore have to start with the classical current–current \( \Delta S = 1 \) Hamiltonian
\[
\mathcal{H}^{\Delta S = 1} = \sum_{q = u, c, t} \lambda_q J^{\mu ij} J_{\mu qr} \left(\lambda_q \equiv V_{qs} V_{qd}^* \right)
\]
\[
= \lambda_d \left[ (\bar{s}t)(\bar{u}d) - (\bar{s}c)(\bar{c}d) \right] + \lambda_t \left[ (\bar{s}t)(\bar{t}d) - (\bar{s}c)(\bar{c}d) \right] + \lambda_u (\bar{s}i)(\bar{d}i) \Delta t = 1/2, 3/2
\]
(13)
to estimate the \( A_0 \) and \( A_2 \) partial decay amplitudes.
The $\Delta I = 1/2, 3/2$ current–current operator involving only the light $u, d$ and $s$ quarks is just proportional to $\lambda_u$. A tree-level hadronization into $K$ and $\pi$ mesons fields would therefore imply $A_0 = \sqrt{2} A_2$, i.e., a vanishing $\varepsilon'$-parameter (see (9)). In other words, a non-zero $\varepsilon'$-parameter is a pure quantum-loop effect in the Standard Model. Notice that these loop effects are also welcome to explain the empirical $\Delta I = 1/2$ rule:

$$\omega_{\text{exp}} \approx \frac{1}{22} \ll \frac{1}{\sqrt{2}} \quad (14)$$

The quantum transmutation of the heavy $t\bar{t}$ and $c\bar{c}$ quark paires into light $u\bar{u}$ and $d\bar{d}$ ones which, eventually, hadronize into final pion states allows now the pure $\Delta I = 1/2$ current–current operator proportional to $\lambda_t$ to contribute to the $\Delta S = 1$ $K$-decays. In the most convenient CKM phase convention, we have

$$\operatorname{Im} \lambda_t = 0 \quad (15)$$

such that CP-violation only appears in the $A_0$ partial amplitude as long as isospin is strictly respected in the “heavy-to-light” transmutation process. But in the Standard Model, neutral transmutations are possible through heavy quark annihilations into gluons, $Z^0$ or photon that are represented by the so-called penguin diagrams. While the latter electroweak contributions obviously break isospin symmetry, the former may also do so by producing first an off-shell iso-singlet mesonic state (mainly $\eta$ or $\eta'$) which then turns into an iso-triplet pion. These lowest order $\Delta I = 1$ isospin-breaking (IB) effects respectively induced by the electric charge difference $\Delta e = e_u - e_d$ and the mass splitting $\Delta m = m_u - m_d$ between the up and the down quarks are usually expected to show up at the percent level in weak decays. However, a CP-violating $\Delta I = 3/2$ amplitude turns out to be enhanced by the famous $\Delta I = 1/2$ rule factor $\omega^{-1}$ since

$$\operatorname{Im} \left( \frac{A_2}{A_0} \right) = -\omega \operatorname{Re} A_0 \left( \operatorname{Im} A_0 - \frac{1}{\omega} \operatorname{Im} A_2 \right). \quad (16)$$

From these quite general considerations, one concludes that

$$\langle \varepsilon'/\varepsilon \rangle_{\text{th}} = \operatorname{Im} \lambda_t \left[ P^{1/2} - \frac{1}{\omega} P^{3/2} \right] \quad (17)$$

with $P^{1/2}$ and $P^{3/2}$, two separately measurable quantities defined with respect to the CKM phase convention defined in (15). Formally, $P^{1/2}$ and $P^{3/2}$ are given in terms of short distance Wilson coefficients $y_i$ and the corresponding hadronic matrix elements as follows

$$P^{1/2} = r \sum y_i \langle Q_i \rangle_0, \quad (18)$$

$$P^{3/2} = r \sum y_i \left[ \langle Q_i \rangle^{1/2} + \omega^{\Delta m} \langle Q_i \rangle_0 \right], \quad (19)$$

where $r$ is a numerical constant and

$$\omega^{\Delta m} = \frac{(\operatorname{Im} A_2)^{\Delta m}}{\operatorname{Im} A_0}. \quad (20)$$

3. The $(\varepsilon'/\varepsilon)_{\text{exp}}$-path

Having all these formulae at hand, we can ask ourselves what the result in (2) is telling us. The answer is simple. It allows us to walk only along a straight path in the $(P^{1/2}, P^{3/2})$ plane, as illustrated in Fig. 1. The standard unitarity triangle analyses [11] give typically

$$\operatorname{Im} \lambda_s = (1.2 \pm 0.2) \times 10^{-4} \quad (21)$$

and, combined with (2), already allow us to draw a rather thin $(\varepsilon'/\varepsilon)_{\text{exp}}$-path in the $(P^{1/2}, P^{3/2})$ plane (see Fig. 1). This path crosses the $P^{1/2}$-axis at $(P^{1/2})_0 = 14.3 \pm 2.8$.

We are of course still far away from such a precise calculation of $P^{1/2}$ and $P^{3/2}$. These two factors are dominated by the so-called strong $Q_6$ and electroweak $Q_8$ penguin operators. The short-distance Wilson coefficients $y_6$ and $y_8$ of these well-known density–density operators are under excellent control [9]. In

![Fig. 1. $(\varepsilon'/\varepsilon)_{\text{exp}}$-path in the $(P^{1/2}, P^{3/2})$ plane.](image)
particular, the $\Delta I = 3/2$ $Z^0$-exchange contribution to $\epsilon'/\epsilon$ exhibits a quadratic dependence on the top quark mass which makes it compete with the $\Delta I = 1/2$ gluon-exchange one. Unfortunately, the resulting destructive interference between $P^{1/2}$ and $P^{3/2}$ strongly depends on the various hadronic matrix elements. Long-distance effects are therefore at the source of the large theoretical uncertainties illustrated by (3). Consequently, we advocate to adopt (temporarily) a different strategy to learn something from the new precise measurements of $\epsilon'/\epsilon$. The proposed exposition of $\epsilon'/\epsilon$ in the $(P^{1/2}, P^{3/2})$ plane turns out to be useful in this context.

4. A simple observation

It is well-known that isospin-symmetry and large-$N$ limit represent two powerful approximations to study long-distance hadronic physics. Here, these well-defined approximations would allow us to neglect $P^{3/2}$ and to express the hadronic matrix elements of the surviving strong penguin operators responsible for $P^{1/2}$ in terms of measured form factors. Earlier attempts [12] to go beyond such a zero-order approximation provided us already with some insight about the sign of the $1/N$ and IB corrections to $\epsilon'/\epsilon$. Recent works including further $1/N$ [13] and IB [14] corrections confirm their tendency to increase $P^{1/2}$ and $P^{3/2}$ respectively. We illustrate these generic trends

$$ (\epsilon'/\epsilon)_{\text{th}} = (\epsilon'/\epsilon)_{0} \left\{ 1 + \mathcal{O}(1/N) - \frac{1}{\omega} \mathcal{O}(\text{IB}) \right\} $$

as $1/N$ and (IB) arrows in Fig. 1), being aware of the fact that $1/N$ corrections can also affect the size of IB corrections. A systematic calculation of all $1/N$ and IB corrections is not yet available, but a direct comparison between the measured value $(\epsilon'/\epsilon)_{\text{exp}}$ and the zero-order approximation $(\epsilon'/\epsilon)_{0}$ should already tell us something about their magnitudes within the Standard Model. Indeed, if the experimental value quoted in (2) is larger than the zero-order theoretical approximation, one needs $1/N$ corrections along the $P^{1/2}$ axis:

$$ (\epsilon'/\epsilon)_{\text{exp}} > (\epsilon'/\epsilon)_{0} \Rightarrow 1/N \text{ corrections.} $$

On the other hand, an experimental value smaller than the zero-order approximation would be an indication for sizeable IB corrections along the $P^{3/2}$ axis:

$$ (\epsilon'/\epsilon)_{\text{exp}} < (\epsilon'/\epsilon)_{0} \Rightarrow \text{IB corrections.} $$

And here comes the surprise! It turns out that $(\epsilon'/\epsilon)_{0}$ lies on the $(\epsilon'/\epsilon)_{\text{exp}}$-path in Fig. 1. It is the crossing of this path with the $P^{1/2}$ axis.

Indeed, $(\epsilon'/\epsilon)_{0}$ can easily be estimated. In the large-$N$ limit, the non-perturbative parameter $\hat{B}_K$ relevant for the usual analysis of the unitarity triangle equals $3/4$ [15]. This implies

$$ \text{Im} \lambda_t = (1.24 \pm 0.06) \times 10^{-4} $$

to be compared with (21) that uses $\hat{B}_K = 0.85 \pm 0.15$. Moreover, in the large-$N$ limit the hadronic matrix element of the strong penguin density–density operator $Q_6$ factorizes ($B_6 = 1$). A simple dependence on the inverse of the strange quark mass squared arises then to cancel the scale dependence of $y_S$ [16]. Taking the central values of the strange quark mass $m_s(2 \text{ GeV}) = 110 \text{ MeV}$ and of the QCD coupling $\alpha_s(M_Z) = 0.119$ relevant for $y_S$, we obtain

$$ (\epsilon'/\epsilon)_{0} = (17.4 \pm 0.7) \times 10^{-4}, $$

where the error results from the error in Im $\lambda_t$. In obtaining (26) we have taken also into account the contribution of the other ($Q_4$) surviving QCD penguin operator in the large-$N$ limit. Without this contribution we would find $18.4 \pm 0.7$, still within the $(\epsilon'/\epsilon)_{\text{exp}}$-path. Clearly, as $(\epsilon'/\epsilon)_{0}$ is roughly proportional to $(\Lambda_{\text{QCD}}^{(4)} 0.8/m_s^2) \Lambda_{\text{MS}}^{(4)} = 340 \pm 40 \text{ MeV}$ and $m_s(2 \text{ GeV}) = (110 \pm 20) \text{ MeV}$, improvements on these input parameters are mandatory.

Although this rather intriguing coincidence between (2) and (26) seems to indicate small $1/N$ and IB corrections, one cannot rule out a somewhat accidental conspiracy between sizeable corrections canceling each other

$$ \mathcal{O}(1/N) - \frac{1}{\omega} \mathcal{O}(\text{IB}) \approx 0. $$

The latter equation describes the walking along the $(\epsilon'/\epsilon)_{\text{exp}}$-path.

At this point, it is also worth noticing that CP-violation in the simplest extensions of the Standard Model, the models with minimal flavour-violation, might behave just like an IB correction along the $P^{3/2}$ axis. The reason is that the $Z^0$-penguin maximally violates the decoupling theorem. Consequently, it
depends quadratically on the top quark mass and is also quite sensitive to new physics [17]. If such is the case, one will have a hard time to disentangle new sources of CP-violation beyond the Standard Model from ordinary IB corrections.

Finally the \((\varepsilon'/\varepsilon)_{\exp}\)-path can be shifted vertically in the \((P_1/2, P_3/2)\) plane by new physics contributions to the quantities used for the determination of Im \(\lambda_t\) but this is a different story.

5. Conclusion

Nature might be kinder than previously thought as far as \(\varepsilon'/\varepsilon\) is concerned. Indeed, present data do not require sizeable \(1/N\) and IB corrections. Improvements on the input parameters \(\alpha_s(M_Z)\) and \(m_t\) leading to our estimate of \((\varepsilon'/\varepsilon)_0\) are mandatory.

We have proposed to display the theoretical results in a \((P_1/2, P_3/2)\) plane in which the experimental result is represented by a \((\varepsilon'/\varepsilon)_{\exp}\)-path. This plot should allow to exhibit transparently the role of \(1/N\) and isospin-breaking corrections in different theoretical results for \(\varepsilon'/\varepsilon\).

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