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O. Sterck

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Why only one individual tests for HIV/AIDS among Sub-Saharan African Couples?

Olivier Sterck

IRES (Université catholique de Louvain) - 1348 Louvain-la-Neuve - Belgium

Abstract

Voluntary Testing and Counseling (VTC) is a popular method for fighting the epidemic of HIV/AIDS. The purpose of VTC is to reduce the incidence of the virus in a twofold manner. First, testing provides access to health care and antiretroviral therapies (ARV) that diminish the transmission rate of the virus. Second, counseling would encourage safer behavior for both individuals who test HIV-negative and want to avoid a dangerous disease, and altruistic individuals who test HIV-positive and want to protect the others. Surprisingly, empirical evidence from DHS surveys in Sub-Saharan Africa shows that testing services are underused. Moreover, it is rare that both partners of a couple test for HIV. In this paper, I construct a behavioral model explaining how misperceptions of the riskiness of HIV/AIDS may induce, at most, one individual in the couple to test. I show that the correction of wrong beliefs thanks to specific information campaigns may be sufficient to induce testing of both partners.

Keywords: HIV/AIDS, transmission rate, testing, prevention, risk perception, condom, beliefs, observability

JEL Classification: I10, I15, I18, O12

Email address: olivier.sterck@uclouvain.be (Olivier Sterck)

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1. Introduction

1.1. The question

Voluntary testing and counseling (VTC) is seen as one of the most powerful approaches to fight the HIV/AIDS epidemic. Indeed, testing may reduce the spread of the disease in a twofold manner. First, testing gives access to health care and antiretroviral therapies (ARV), provided that medicines are affordable. By diminishing the viral load in the blood, ARV reduces sharply the transmission rate of HIV/AIDS (Castilla et al., 2005). Second, receiving the test result may induce more cautious behavior either if the tested individual is altruistic and tested positive, or if he is fearful and tested negative. Empirically, it is not clear whether testing impedes the spread of the disease or encourage risky behavior (De Paula et al., 2010; Gong, 2010).

The sixth Millennium Development Goal requires to “achieve, by 2010, universal access to treatment for HIV/AIDS for all those who need it”. Nowadays, voluntary testing and counseling services and antiretroviral therapies are available for free in a large majority of African countries. Despite this increasing access to testing and therapies, the use of voluntary testing service remains low. Furthermore, even when people choose to test, many do not return for their results (see Obermeyer and Osborn (2007) for a good review).

While testing and counseling of couples appears to be more effective at altering risky behavior, joint testing for couple is much less popular than single testing (Glick, 2005). For example in Lesotho, between 58 and 83% of people with advanced HIV infection received antiretroviral therapies in 2006 (WHO statistics). Despite this large access to HIV treatment, couples where both partners tested and received the results represent only 1.98% of the total number of couples (Table A.1). For 13.99% of couples, just one of the partners tested and received the result of the test. The remaining 84.04% either did not test at all or did not get the result of the test. This picture is all the more surprising in the light of the fact that at least one partner want to test in 79.74% of the never-tested couples. Both people want to

\footnote{An advanced HIV infection corresponds to a CD4 count at or below 350 cells/mm$^3$ (WHO guidelines before 2009).}
test in 43.46% of the never-tested couples (Table A.2). A similar pattern of behavior is observed in other Sub-Saharan countries (Table A.3).

Along this paper, I propose a model explaining this surprising testing pattern. I focus on voluntary testing that is, client-initiated testing. In the few existing models related to the testing decision (Philipson and Posner, 1994; Boozer and Philipson, 2000), people at an intermediate risk of infection use testing as a signal to obtain access to the risky sex market (Philipson and Posner, 1994; Boozer and Philipson, 2000). In this paper, I take into account other possible rationales for testing: people may also want to test to have access to health care, to protect themselves if they are tested negative for HIV, or to protect their partner if they are tested positive. Furthermore, the seminal papers of Philipson and Posner (1994) and Boozer and Philipson (2000) are build on very restrictive assumptions: the per-act transmission rate of HIV/AIDS is assumed to be equal to one, and the result of the test is supposed to be observable by the partner. Neither the expectation about the transmission rate of HIV/AIDS, nor the observability of the test result by the partner were considered as important variables and included in existing formal analysis of the testing decision. The aim of this paper is to show that, taken together, these two variables may partly explain why people fail to test in a couple. I propose a model that take into account the overestimation of the per-act transmission of HIV/AIDS by individuals (Delavande and Kohler, 2009; Sterck, 2010). I consider both egoistic and altruistic individuals in a couple. I show that a high expected observability of partner’s test result combined with a high expected transmission rate may induce two Nash equilibria where only one partner tests in the couple. I then conclude that disclosing a lower transmission rate may encourage people to test if both subjective and objective costs of testing are low. If costs of testing are high, disclosing a lower expected transmission rate induces that “not testing” becomes a dominant strategies for both partners.

1.2. Literature review

The economic literature on HIV/AIDS and testing mainly focuses on the decision to test and on the riskiness of behavior after testing. The majority of this literature does not take into account the potential biased knowledge of people. However, a wrong knowledge of the HIV risk of transmission may have strong implications on behavior (O’Donoghue and Rabin, 2001; Sterck, 2010). In this literature review, I first describe the existing literature on
the voluntary testing decision, and the impact of receiving the test result on sexual behavior. Then, I describe the first attempts to integrate biased beliefs into theoretical models of sexual behavior.

1.2.1. The decision to test:

The first model of testing choice and behavior after testing was set up by Philipson and Posner (1994). In their formal analysis, they assume no access to ARV, no altruism, full observability of the test result by the partner, and a transmission rate per sexual act of 1. With these very restrictive assumptions, testing is a signal needed for individuals at an intermediate risk of being infected to obtain a risky trade. Indeed, if the probability that the individual is infected is low, the partner will anyhow accept risky sex without requiring testing. If the individual is at an intermediate risk of being infected, the partner will require testing to accept risky sex. If the probability of being infected is high, the individual will not test because he is almost sure that he will not have access to risky sex after testing.

In the empirical part of their paper, Philipson and Posner (1994) use a dataset from homosexual and heterosexual unmarried couples in San Francisco. Their analysis suggests that an individual is more inclined to test if his probability of being infected is high, if his partner’s probability of being infected is low and if his partner tested for HIV/AIDS. Unfortunately, the econometric method used does not take into account the possible simultaneity problem between $i$ and $j$’s decisions to test.

The model of Philipson and Posner (1994) is a first step in the understanding of the testing choice. In this paper, I propose to relax some of their very restrictive hypotheses. I will consider both egoistic and altruistic individuals and I will include the availability of ARV. I will also assess the impact of the expected transmission rate on the testing decision. Finally, I will discuss the impact of the observability of the test result by the partner.

Empirically, many other studies assessed the causal factors of testing. In a recent review of the literature, Obermeyer and Osborn (2007) points out the role of gender, fear and stigma in shaping attitude towards testing. Glick (2005) indicates that agreeing to be tested may be tantamount to revealing extramarital affairs and hence hinders joint testing. Using rapid tests in places that are convenient to clients has been shown to increase the acceptance of testing (Obermeyer and Osborn, 2007; Angotti et al., 2009).
This suggests that people face high testing cost (objective and/or subjective costs), but do not fear learning their status. In a recent field experiment, Thornton (2008) randomized the distance to the health centers and the financial incentives to learn HIV result. She finds that small incentives double the share of people who learn the result.

1.2.2. Behavior after testing:

Several authors assessed the impact of testing on risky behavior. Using the same hypotheses as Philipson and Posner (1994), Boozer and Philipson (2000) concludes that testing is chosen only if the prior belief of infection is intermediate. Their model assumes no behavioral change if the revealed serostatus is equal to the expected serostatus. Within this framework, only individuals who are surprised by the result of the test will respond to it. Whether people will engage in safer or riskier behavior after testing depends on the relative importance of expectation revisions and altruism, and is ultimately an empirical question.

Several recent empirical papers assessed the behavioral response of individuals after receiving the test result. Their findings are contradictory. On the one hand, the empirical evidence gathered by Thornton (2008), Delavande and Kohler (2009) and De Paula et al. (2010) suggests that individuals are altruistic. After learning HIV-positive status, Thornton (2008) and Delavande and Kohler (2009) find that individuals are more likely to buy condoms and De Paula et al. (2010) and Delavande and Kohler (2009) find a decrease in extramarital affairs. After learning HIV-negative status, Thornton (2008) finds no significant effect on the purchase of condoms and De Paula et al. (2010) find an increase in extramarital affairs.

On the other hand, the findings of Gong (2010) suggest that selfish motivations and self-protecting behavior are more important than altruistic considerations after receiving the test result. Gong (2010) finds a six-fold increase in other sexually transmitted diseases (STIs) after a positive test and a 60% decrease in STIs after a negative test. The presence of STIs is a trustworthy proxy for extramarital relationships. This measure of risk is more reliable than the self-reported measures used by Thornton (2008), Delavande and Kohler (2009) and De Paula et al. (2010) because it is objective and direct. At the macro level, testing would decrease the incidence of HIV/AIDS for De Paula et al. (2010) and would increase the spread of the disease for Gong (2010).
Another important outcome after testing is the disclosure of the test result to the partner. In their literature review, Obermeyer and Osborn (2007) draw the attention on the fact that disclosure after testing is variable and generally low. In a meta-review, Medley et al. (2004) find that disclosure rates range between 16.7 and 86%. The fear of abandonment, discrimination and violence or accusations of infidelity are frequently highlighted to justify the non disclosure of the test result. Violent consequences of disclosure were reported in 3.5% to 14.6% of cases (Medley et al., 2004).

1.2.3. Behavior and risk perception:

Few empirical papers integrated risk perceptions in their analysis of condom use. First, Meekers and Klein (2002), Maharaj and Cleland (2005) and Lammers et al. (2009) find that the perception of risk is positively correlated to condom use, especially for casual encounters. Unfortunately, these papers do not distinguish the expected probability that the partner is infected from the expected transmission rate.

More recently, Delavande and Kohler (2009) and Sterck (2010) studied the links between sexual behavior and the subjective evaluation of parameters such as the expected transmission rate of the HIV/AIDS virus, the expected prevalence or the expected length of life while infected. These parameters are the ones rational individuals should use in deciding whether to use condoms and to test or not. Delavande and Kohler (2009) investigated the impact of learning HIV status on HIV/AIDS-related expectations and behaviors. They find that learning an HIV positive status does not imply a revision of infection expectation two years after receiving the result of the test. HIV-positive tested allocated on average a probability of 28% of being infected. Delavande and Kohler (2009) explain this surprising result by a lack of trust in the test result, by forgetting and by the embarrassment vis-à-vis the interviewer. As a complementary explanation, Sterck (2010) suggests that some of them may not trust the positive result of the test because they think that the per-act transmission rate of HIV is high, and they did not infect their partner while they had risky encounters. While the transmission rate of HIV/AIDS is generally lower than 1% (Boily et al., 2009), the average per-act expected transmission rate in the sample considered by Delavande and Kohler (2009) is 86.8%. Sterck (2010) also finds that the expected transmission rate per-act is largely overestimated by students in Burundi. On average, students evaluate that the transmission occurs in 81.4% of the sexual encounters with
an infected partner. By simulating a theoretical model of rational choice, Sterck (2010) finds that the overestimation of the per-act transmission rate is not optimal for people engaged in a long-lasting relationship. Indeed, if an unexpected and unprotected encounter occurs and if the transmission rate is expected to be high, people may be fatalistic and not protect themselves for further encounters. Sterck (2010) calculates that the optimal transmission rate to disclose ranges between 4.44% and 24.5%.

1.3. My contribution:

I will improve the existing models (mainly Philipson and Posner (1994)) related to the HIV testing decision by integrating formally the expected transmission rate per-act and the expected observability of the test result into the decision process. I will show that an overestimated transmission rate combined with a high observability of the test result by the partner may imply that only one individual tests in couples. Disclosing a lower transmission rate may induce that both partners test if the overall testing cost is low enough.

In the next section, I set up a model of the testing decision for a rational agent in a couple. In section 3, I solve this model for an egoistic individual. Assuming the observability of partner’s test result, I show that an overestimation of the per-act transmission rate of HIV/AIDS may induce that only one individual tests in the couple. I show that disclosing a lower transmission rate may favor testing if the test result of the partner is observable and if the ratio of the testing cost to the value of health care is low compared to the probability of having been infected before any sexual relationships with the partner. In section 4, I extend this model to altruistic individuals. I show that disclosing a lower transmission rate may increase voluntary testing because both individuals may decide to test to protect the partner if they are tested positive. The last two sections discuss the findings and conclude.

2. A theoretical model

2.1. HIV and testing

I assume that an individual $i$ is in a couple with $j$. Both $i$ and $j$ have to choose whether to test or not. Without loss of generality, I focus on $i$’s decision process. The sequence of the decision process is depicted on figure A.4. Prior to the first testing decision, $i$ has $n$ risky sexual encounters with $j$. Then, $i$ and $j$ choose whether to test or not (T1). If $i$ or $j$ chooses to
test, the partner has a probability $q$ to observe the result of the test if it is positive. If $i$ or $j$ observes that the partner is infected, a test may again be done in order to have access to ARV treatments (T2). After the testing decisions T1 and T2, $i$ and $j$ have $N$ risky encounters.

Both $i$ and $j$ believe that a positive test result is observable by the partner with a probability $q$. Several reasons may explain that a positive test result is observable by the partner. First, learning a HIV-positive status implies a psychological shock that may be difficult to hide. Second, the treatment of HIV/AIDS requires frequent visits to health facilities, and antiretroviral therapies imply the daily uptake of medicines. Hiding such treatment to the partner may be difficult knowing the promiscuity of African households and the important side-effects of ARV treatments. Third, people may believe that their altruistic partner would advise them in case of infection. I assume that the expected observability $q$ is the same for $i$ and $j$.

The test costs $C$. $C$ includes both objective costs (the amount paid to the health facility) and subjective costs (indirect costs related to social stigma, fear of partner’s reaction, etc.). If infected, testing gives access to health care facilities and ARV, that are denoted $S$. HIV induces a disutility $d$. I assume that health care is more valued than the cost of testing: $0 < C < S$. With this assumption, someone infected for sure will always choose to test to have access to health care. Even with access to ARV, people prefer not being infected: $0 < S < d$.

### 2.2. Sexual life

Prior to their relationship, both $i$ and $j$ have had some risky encounters. I denote $p_{ii}$ the assessment of $i$ about his own risk of having been infected prior to his relationship with $j$, and $p_{ij}$ the assessment of $i$ about the probability that $j$ was infected at the beginning of their relationship.

Before the testing decision T1, I assume that $i$ and $j$ have $n$ risky sexual encounters. These $n$ risky encounters gives a utility $U_n$. If $j$ is infected, and if $\beta$ stands for the expected transmission rate per sexual act, the probability $P_n$ that the virus is transmitted throughout these $n$ encounters follows the binomial law$^3$: $P_n = 1 - (1 - \beta)^n$.

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$^3$Kaplan (1990) and Rottingen and Garnett (2002) argued that the binomial law is not
Similarly, I assume that $i$ and $j$ have $N$ risky encounters after the second testing decision $T_2$. These $N$ risky encounters gives a utility $U_N$. If $j$ is infected, the probability $P_N$ that the virus is transmitted during these $N$ encounters is equal to: $P_N = 1 - (1 - \beta)^N$.

2.3. The four states of infection

Prior to the first testing decision $T_1$, there exist four states of infection. First, both individual in the couple may be infected. This state would occur if both individual were infected before their relationship, or if only $i$ or $j$ was infected and transmitted the virus to the partner throughout the $n$ risky encounters. The subjective probability $s_{i11}$ that this state occurs is given by:

$$s_{i11} = p_{ii}p_{ij} + p_{ii}(1 - p_{ij})P_n + (1 - p_{ii})p_{ij}P_n.$$  (1)

Second, only $i$ may be infected. This state would occur if only $i$ was infected prior to his relationship with $j$ and if the virus was not transmitted throughout the $n$ risky encounters. The subjective probability $s_{i10}$ that this state occurs is given by:

$$s_{i10} = p_{ii}(1 - p_{ij})(1 - P_n).$$  (2)

Third, only $i$ may be infected before the testing decision $T_1$. This state would occur if only $j$ was infected prior to the relationship between $i$ and $j$ and if the virus was not transmitted throughout the $n$ risky encounters. The subjective probability $s_{i01}$ that this state occurs is given by:

$$s_{i01} = (1 - p_{ii})p_{ij}(1 - P_n).$$  (3)

Fourth, $i$ and $j$ may be uninfected before the testing decision $T_1$. This state would occur if neither $i$ nor $j$ were HIV-positive before their relationship. The subjective probability $s_{i00}$ that this state occurs is given by:

$$s_{i00} = (1 - p_{ii})(1 - p_{ij}).$$  (4)

consistent with empirical data on the transmissibility of HIV/AIDS. However, I assume that people cannot distinguish the different HIV/AIDS stages and that they act having in mind a constant per-act probability of transmission.
2.4. Behavior after testing

Behavioral choice after testing mainly depends on altruism and self-protecting behavior. As shown in the introduction, the existing studies about behavioral changes after testing have conflicting conclusions (Thornton, 2008; De Paula et al., 2010; Delavande and Kohler, 2009; Gong, 2010). Because of these contradictory findings, I will not focus as much on behavior after testing. I will assume that sexual behavior is similar before and after testing. However, if an individual observes that his partner is infected, I assume that he will have the $N$ following encounters with a new partner. The expected probability that the new partner is infected is also assumed to be $p_{ij}$.

In section 3, I consider an egoistic individual and I discuss the self-protecting case where the individual choose safe sex if he tests negative. In section 4, I consider an altruistic individual whose utility is affected negatively if he transmits the disease to his partner. I denote $\alpha$ the propensity of being altruistic that is, the disutility that an altruistic individual faces if he transmits the virus.

3. The solution without altruism ($\alpha = 0$)

3.1. No observability ($q=0$)

I will first solve the model assuming that the result of the test is not observable by the partner. If $i$ does not test in T1, the utility of $i$ is given by:

$$V(T_{1i} = 0) = U_n + U_N + (s_{i11} + s_{i10})(-d) + s_{i01}P_N(-d). \quad (5)$$

The sum $U_n + U_N$ stands for the utility of sex. $(s_{i11} + s_{i10})(-d)$ is the loss of utility if $i$ is infected in T1. $s_{i01}P_N(-d)$ stands for the possibility that $i$ was not infected in T1, and will be infected by $j$ throughout the $N$ risky encounters that will occur after the testing choice.

Similarly if $i$ chooses to test in T1, $i$ has access to health care (S) if he is positive. The utility of $i$ is then given by:

$$V(T_{1i} = 1) = U_n + U_N + (s_{i11} + s_{i10})(S-d) + s_{i01}P_N(-d) - C. \quad (6)$$

The equation (6) takes into account the cost of testing C, and the fact that $i$ has access to health care and ARV if he is tested positive. Let us
define $A$ as the net utility of testing that is, the difference between equations (6) and (5):

$$A \equiv V(T1_i = 1) - V(T1_i = 0) = (s_{i11} + s_{i10})(S) - C$$

$$= [p_{ii} + (1 - p_{ii})p_{ij} P_n]S - C.$$  

The individual will test if $A$ is positive. $A$ may be interpreted as the utility of having access to health care if he is infected in $T1$, minus the testing cost.

**Proposition 3.1.** An egoistic individual who believes that the partner’s test result is not observable will test iff:

$$A > 0 \iff C < p_{ii} + (1 - p_{ii})p_{ij} P_n,$$

that is, if the relative cost of testing $C$ compared to health care value $S$ is lower than the probability of being infected at the time of the testing choice.

**Proof** The proof is in the text. \qed

Hence, $i$ will choose to test if the probability to be infected in $T1$ is high that is, if $p_{ii}$, $p_{ij}$ and $P_n$ are high. Within this framework, the sole motivation for being tested is the access to health care. Because the expected transmission rate $\beta$ is positively correlated to the probability that the virus was transmitted during the $n$ first encounters $P_n$, we obtain the following corollary.

**Corollary 3.2.** For an egoistic individual who believes that the partner’s test result is not observable, the overestimation of the transmission rate favors testing.

**Proof** The derivative of $P_n$ with respect to the expected transmission rate $\beta$ is positive. The derivative of the condition $A > 0$ with respect to $P_n$ is positive and equal to $(1 - p_{ii})p_{ij} S$. Hence, the derivative of the condition $A > 0$ with respect to $\beta$ is positive. \qed
3.2. Full observability ($q=1$)

With full observability, $i$ believes that he will observe a positive test result if his partner tests in T1 and is infected. With full observability, incentives to test are modified because an individual may bet on the fact that his partner will test in T1 and that the test result will be observed if it is positive. Then, $i$ may choose to test in T2 to have access to health care. The decision to test is then strategic and depends on partner’s choice.

On the one side, if $j$ does not test, the condition for $i$ to test is not modified: $i$ will test if the condition $A > 0$ is satisfied.

On the other side, if $j$ tests in T1, $i$ believes that he will observe the test result if it is positive. If $i$ observes that his partner is HIV-positive, he may choose to test in T2 if he believes that there is a large probability to be himself infected. Hence, if $j$ does the test, we have to compare 3 cases: $i$ tests neither in T1, nor in T2, $i$ does not test in T1, and tests in T2 if $j$ was infected, and $i$ tests in T1. The following statements show that the condition $A > 0$ is not sufficient anymore for testing to be optimal.

**Proposition 3.3.** We assume full observability. Testing in T1 is a dominant strategy iff $A > O_p$, where the opportunism $O_p$ is defined as:

$$O_p \equiv [p_{ii} + p_{ij}(1 - p_{ij})P_n + (1 - p_{ii})p_{ij}P_n]S - [p_{ij} + p_{ii}(1 - p_{ij})P_n]C.$$ 

If the condition $A > 0$ is satisfied and the condition $A > O_p$ is not satisfied, testing in T1 is optimal for $i$ iff his partner does not test.

Not testing in T1 is a dominant strategy for $i$ iff $A < 0$.

**Proof** The proof is in appendix. □

The condition $A > 0$ is equivalent to the inequality (8) that is, the condition for testing to be optimal if test results are not observable. The condition $A > O_p$ may be rewritten:

$$\frac{C}{S} < \frac{p_{ii}(1 - P_n)}{1 - p_{ii}P_n}$$

(9)

The interpretation of these two conditions is as follows. When the result of the test is observable, the condition $A > 0$ fulfilled for both $i$ and $j$ does not
imply that testing is a dominant strategy. If they are opportunist \((O_p > A)\), both players may bet on the fact that the partner will test, and that they will observe the result of the test if it is positive. If conditions \(A > 0\) and \(O_p > A\) are satisfied for both individuals, only one person to test in the couple is a Nash equilibrium. This will occur if the expected probability that the virus was transmitted between \(i\) and \(j\) is high \((P_n \gg 0)\).

The figure A.5 and A.6 shows graphically the relative cost of testing \((C/S)\) and the conditions \(A > 0\) and \(A > O_p\) for \(p_{ii} = p_{ij} = 0.05\). If the relative cost of testing is low (Figure A.5) both partners may bet on the fact that the partner will test if \(P_n\) is high that is, if they expected a high probability that the virus was transmitted if \(i\) or \(j\) were infected. Diminishing the expected transmission rate may encourage \(i\) to test if the probability that \(i\) is infected before the relationship is high enough compared to the relative cost of testing. If the relative cost of testing is high (Figure A.6) both individuals may also bet on the fact that the partner will test if \(P_n\) is high. However, diminishing the expected transmission rate may discourage testing.

The figure A.7 shows the conditions \(A > 0\) and \(A > O_p\) when \(P_n\) is replaced by his functional form \(P_n = [1 - (1 - \beta)^n]\). The interpretation is similar. Disclosing a lower transmission rate may favor that both partners test if the relative testing cost \((C/S)\) is low compared to the probability of being infected before any sexual encounters with the partner. The higher \(n\), the lower the \(\beta\) required for testing to be a dominant strategy. The following proposition summarizes this reasoning.

**Corollary 3.4.** Let us assume that the transmission rate \(\beta\) is overestimated \((\beta \approx 1)\). If \(n = 0\), testing is a dominant strategy iff \(\frac{C}{S} < p_{ii}\). Testing does not depend on the expected transmission rate \(\beta\).

If \(n \geq 1\), diminishing the expected transmission rate to the value \(\beta^+ = 1 - \left(\frac{(1-p_{ii})C}{p_{ii}(S-C)}\right)^{1/n}\) would induce that testing becomes a dominant strategy iff \(\frac{C}{S} < p_{ii}\).

If \(\frac{C}{S} > p_{ii} + (1 - p_{ii})p_{ij}\), diminishing the expected transmission rate below \(\beta^- = 1 - \left(\frac{C - S p_{ii}}{S (1 - p_{ii}) p_{ij}}\right)^{1/n}\) would induce that not testing becomes a dominant strategy.
**Proof** The proof is in appendix.

3.3. The intermediate case: $0<q<1$

Both individuals think that they have a probability $q$ to observe the result if the partner get a positive test. The condition $A > 0$ stands for the case where $i$ believes that $j$ does not test. Hence, the condition $A > 0$ is not modified by the fact that observability is not perfect. Indeed, if $j$ does not test, $i$ knows that he will not observe the test result, whatever value takes the observability $q$.

Conversely, if $i$ believes that $j$ will test, the condition $A > O_p$ is modified. Betting on the fact that the partner does a test is more risky because the test result is not fully observable. With a probability $q$, the result of the test is observable and the condition $A > O_p$ prevails. With a probability $1 - q$, the result of the test is not observable and the condition $A > 0$ prevails. When $i$ chooses whether to test or not, he does not know the result of the lottery $q$. Assuming that the expected utility theory holds, the condition for testing is a weighted average between the conditions $A > O_p$ and $A > 0$.

**Proposition 3.5.** The individual $i$ observes with a probability $0<q<1$ a positive test result of his partner. Within this framework, testing in $T1$ is a dominant strategy iff $A > qO_p$.

If the condition $A > 0$ is satisfied and the condition $A > qO_p$ is not satisfied, testing in $T1$ is optimal for $i$ iff the partner does not test.

Not testing in $T1$ is a dominant strategy for $i$ iff $A < 0$.

**Proof** The proof is in appendix.
The figure A.8 shows the conditions \( A > 0 \) and \( A > qO_p \) as a function of the observability \( q \) and the transmission probability \( P_n \). If \( q \) is low, conditions \( A > 0 \) and \( A > qO_p \) are similar and increasing in \( P_n \). If \( q \) is high, the condition \( A > 0 \) is increasing in \( P_n \) and the condition \( A > qO_p \) is decreasing in \( P_n \). As in the full observability case, diminishing the expected transmission rate may deter people to rely on the observation of partner’s test result. If the cost of testing is low, disclosing a lower transmission rate may encourage testing of both partner. The following proposition formalizes this statement.

**Corollary 3.6.** Let us assume that the transmission rate \( \beta \) is overestimated (\( \beta \approx 1 \)). If \( n = 0 \), testing is a dominant strategy iff \( \frac{CS}{S} < p_{ii} \). Testing does not depend on the expected transmission rate \( \beta \).

If \( n \geq 1 \), diminishing the expected transmission rate to the value \( \beta^+ \) would induce that testing becomes a dominant strategy iff \( \frac{CS}{S} < p_{ii} \) and \( q > \frac{p_{ij}}{p_{ii} + p_{ij}(1-p_{ii})} \), with \( \beta^+ \) given by:

\[
\beta^+ = 1 - \left( \frac{C + [p_{ij} + p_{ii}(1-p_{ij})][qC + (1-q)S]}{(1-q)(1-p_{ii})p_{ij}S - qp_{ii}(1-p_{ij})(S-C)} \right)^{1/n} \tag{10}
\]

If \( \frac{CS}{S} > p_{ii} + (1-p_{ii})p_{ij} \), diminishing the expected transmission rate below \( \beta^- = 1 - \left( 1 - \frac{C - Sp_{ii}}{S(1-p_{ii})p_{ij}} \right)^{1/n} \) would induce that not testing becomes a dominant strategy.

**Proof** The proof is in appendix. \( \square \)

This corollary is interpreted as follows. If observability is high and if the relative testing price is low, disclosing a lower transmission rate will induce people to test instead of relying on the observation of the partner’s test result. Indeed, if both the observability and the transmission rate are expected to be high, the individual \( i \) may think: “I don’t have to test in T1. I just have to observe the test result of my partner and to test in T2 if she is infected. Because I think that the transmission rate is high, if I am infected, I’m almost sure that she is infected and I’m almost sure to observe the result”. Conversely, if the observability or transmission rate are expected to be low, the individual \( i \) will choose to test in T1 because the observation of partner’s test result is not much informative on his own status.
3.4. Self-protecting behavior

Up to now, we assumed that the individual $i$ does not change behavior if he tests negative and if he does not observe the test result of the partner. In this section, we will relax this hypothesis. We define self-protecting behavior as the willingness of $i$ to protect himself if he knows that he is negative and if he does not know the status of his partner.

Let us focus on the no observability case ($q = 0$). The expected utility of testing in $T_1$ is modified because $i$ will choose safe sex if he tests negative. Denoting $U_S$ the utility of safe sex after testing, the expected utility of $i$ if he tests at time $T_1$ becomes:

$$V_{sp}(T_1i = 1) = U_n + (s_{i11} + s_{i10})(S - d + U_N) + (s_{i01} + s_{i00})U_S - C.$$  (11)

A simple way of measuring self-protecting behavior is given by the difference:

$$B_{sp}^{T_1j=0} = V_{sp}(T_1i = 1) - V(T_1i = 1)$$  (12)

$$= (1 - p_{ii})p_{ij}(1 - P_n)P_Nd - (1 - p_{ii})(1 - p_{ij}P_n)(U_N - U_S).$$

This difference represents the utility of avoiding risky behavior if $i$ is tested negative and if the serostatus of $j$ is unknown. $B_{sp}^{T_1j=0}$ is made up of two terms. The first is positive and stands for the probability to avoid the HIV/AIDS disease by avoiding risky behavior. The second is negative and represents the disutility of choosing safe sex compared to risky sex. The individual will protect himself after a negative test if $B_{sp}^{T_1j=0}$ is positive.

Assuming self-protecting behavior ($B_{sp}^{T_1j=0} > 0$), the individual $i$ will test if $V_{sp}(T_1i = 1) > V_{sp}(T_1i = 0)$. This condition is equivalent to $A + B_{sp}^{T_1j=0} > 0$.

The derivative of the condition $A + B_{sp}^{T_1j=0} > 0$ with respect to $P_n$ is equal to $(1 - p_{ii})p_{ij}(S - dP_N + U_N - U_S)$. This derivative is negative if $P_Nd = 1 - (1 - \beta)^N d$ is high. Hence, if both the number of risky encounters after the testing decision, $N$, and the cost of HIV/AIDS, $d$, are high, disclosing a low transmission rate would favor testing in the case of self-protecting behavior. High $N$ and high $d$ seem reasonable assumptions.
When the test result of the partner is observable \((q = 1)\), the analysis is similar with self-protecting behavior. On the one hand, if \(i\) thinks that \(j\) will not test, then \(i\) chooses to test in T1 if \(A + B_{T1j}^{sp} > 0\). On the other hand, if \(i\) believes that \(j\) will test in T1 and that the test is observable, he will be able to deduce the status of his partner and self-protecting behavior plays no role.

4. The solution with altruism \((\alpha > 0)\)

When we introduce altruism, the main results are modified because incentives to test are higher. As in the egoistic case, \(i\) will choose to test if the probability of being infected in T1 is high compared to the relative cost of testing \((A > 0)\). With altruism, \(i\) will also test if the probability to transmit the virus to \(j\) after T1 is high. Indeed, with altruism, we assume that \(i\) has a disutility \(\alpha\) if he transmits the virus to \(j\). Hence, if \(\alpha\) is high, the individual will test and choose safe sex if the test result is positive, to avoid transmitting the virus to the partner. I focus on the no observability and the full observability cases. As in the egoistic case, the intermediate case \(0 < q < 1\) is a blend between the no observability and the full observability cases.

4.1. No observability \((q=0)\)

With altruism, the individual faces a disutility \(\alpha\) if he transmits the virus after T1. Hence, if \(i\) is infected and \(j\) uninfected in T1 (the state of infection \(s_{i10}\)), \(i\) has a probability \(P_N\) to suffer from a loss in utility \(\alpha\). In order to avoid the transmission of the virus, \(i\) may adopt safe behavior if he receives a positive test result in T1.

The individual \(i\) has 3 possible choices. First, if altruism is high \((\alpha \gg 0)\), he may choose to test and to adopt safe behavior if he is infected. The total expected utility of this choice is given by:

\[
V_{\alpha,S}(T1_i = 1) = U_n + (s_{i11} + s_{i10})(U_S + S - d) + s_{i01}(U_N - P_N d) + s_{i00}(U_N).
\]

Second, if altruism is low and if the expected probability that \(i\) is infected is high, \(i\) may choose to test to enjoy health care if he is infected. If altruism is low enough, he may engage in risky behavior if he tests positive. \(i\) will then suffer from a loss in utility \(\alpha\) if the virus is transmitted to \(j\). The total expected utility of this choice is given by:
$V_{\alpha,R}(T_{1i} = 1) = U_n + U_N + s_{i11} (S - d) + s_{i10} (S - d - \alpha P_N) + s_{i01} (-P_N d)$.

Finally, the individual may not test if both altruism and the expected probability of being infected are low. The total expected utility is then given by:

$V_{\alpha}(T_{1i} = 0) = U_n + U_N + s_{i11} (-d) + s_{i10} (-d - \alpha P_N) + s_{i01} (-P_N d)$.

Let us introduce the kindness $K$ that stands for the willingness to avoid risky sex to protect the partner. I define the kindness $K$ when $q = 0$ as:

\begin{align*}
K_{q=0} &\equiv V_{\alpha,S}(T_{1i} = 1) - V_{\alpha,R}(T_{1i} = 1) \\
&\equiv \alpha p_{ii}(1 - p_{ij})(1 - P_n)P_N - [p_{ii} + (1 - p_{ii})p_{ij}P_n](U_N - U_S).
\end{align*}

The kindness $K_{q=0}$ is made up of two terms. The first term is positive and depends on altruism $\alpha$. It represents the utility of protecting the partner. The second term is negative and depends on the difference in utility between risky and safe sex $U_N - U_S$. It represents the disutility of choosing safe sex if the test result is positive. The following proposition describes the choice of an altruistic individual $i$ when $q = 0$.

**Proposition 4.1.** An altruistic individual $i$ who believes that the partner’s test result is not observable will test and choose safe sex if he is infected iff:

\begin{align*}
\begin{cases}
A + K_{q=0} > 0 \\
K_{q=0} > 0.
\end{cases}
\end{align*}

The individual $i$ will test and choose risky sex if he is infected iff:

\begin{align*}
\begin{cases}
A > 0 \\
K_{q=0} < 0.
\end{cases}
\end{align*}

The individual will not test iff:

\begin{align*}
\begin{cases}
A < 0 \\
A + K_{q=0} < 0.
\end{cases}
\end{align*}
The proof is in appendix

This proposition is interpreted as follows. If altruism, and hence kindness, are important, the individual tests to protect the partner. If altruism is low, the equilibrium is similar to the "no altruism" case. The figure A.9 represents the choice of $i$ as a function of both altruism $\alpha$ and transmission probability $P_n$. The plane which is increasing in $P_n$ represents the condition $A > 0$. The curved plane decreasing in $P_n$ stands for the condition $A + K_{q=0} > 0$. If altruism is low or if the probability that the virus was already transmitted is high, the decision to test depends only on the condition $A > 0$. If altruism is high and $P_n$ is low, $i$ will test in order to protect his partner if he receives a positive test result. Because the expected transmission rate $\beta$ is positively correlated to $P_n$, we conclude that disclosing a lower transmission rate may favor testing if altruism is high enough.

The figure A.10 shows the conditions $A > 0$ and $A + K_{q=0} > 0$ as a function of $\beta$ and $n (\alpha = 5, U_N - U_S = 1, S = 1, N = 100)$. $A$ is flat except for very small values of $\beta$. $A + K_{q=0}$ is incurved and maximal for some low $\beta$. We observe that $A + K_{q=0}$ is bigger than $A$ if $\beta$ and $n$ are low. Hence, for a given value of altruism $\alpha$, disclosing a lower transmission rate may favor testing if the number of sexual encounters before testing $n$ is not too high. Indeed, if $n$ is high, $i$ will think that the virus was anyhow transmitted. Conversely, if $n$ is low, he thinks that the virus was not necessarily transmitted. Hence, he will test and choose safe sex if he is infected.

There exist no an explicit equivalent of the corollaries 3.4 and 3.6 when altruism is introduced. Indeed, it is not possible to find an explicit formula for the thresholds $\beta^+$ and $\beta^-$. However, assuming that the transmission rate $\beta$ is overestimated ($\beta \approx 1$), it is easy to see on the figure A.10 that a diminution of the transmission rate to some low value $\beta^+$ would favor testing if there exists a $\beta^+$ given by the solution of the implicit inequality $A(\beta^+) + K_{q=0}(\beta^+) > 0$ and such that:

$$
\begin{cases}
K_{q=0}(\beta^+) > 0 \\
A(1) < 0,
\end{cases}
$$

(17)

The functions $A(\beta)$ and $K_{q=0}(\beta)$ are the functions $A$ and $K_{q=0}$ where $P_n$ and $P_N$ are replaced by their functional forms in terms of $\beta$ and $n$. 

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4.2. Full observability ($q=1$)

We follow the same reasoning than for the egoistic case. With full observability, $i$ believes that he will observe the test result if his partner tests positive in $T_1$. Hence, $i$ may bet on the fact that his partner will test in $T_1$ and that the test result will be observed if it is positive. Then, $i$ may choose to test in $T_2$ in order to have access to health care. The decision to test is strategic and depends on beliefs about partner’s testing choice.

On the one side, if $j$ does not test, the conditions for $i$ to test are not modified and the proposition 4.1 holds.

On the other side, if $j$ tests in $T_1$, $i$ believes that he will observe the test result if it is positive. If $i$ observes that his partner is HIV-positive, he may choose to test in $T_2$ if he believes that there is a large probability to be himself infected. If $j$ does the test, $i$ has 3 possible testing choices: $i$ tests neither in $T_1$, nor in $T_2$, $i$ does not test in $T_1$, and test in $T_2$ if $j$ was infected, and $i$ tests in $T_1$. Furthermore, if $i$ chooses to test in $T_1$, he has two possible choices of riskiness if he is tested positive. If altruism is high and if the probability that the virus was already transmitted, $P_n$, is low, he will choose safe sex to protect his partner. If altruism is low or if the probability that the virus was already transmitted in $T_1$, $P_n$, is high, he will choose risky sex.

Hence, the decision to test is strategic and may depend on beliefs about the partner’s testing choice. First, testing may be a dominant strategy if health care is highly valued or if altruism is very high. Second, not testing may be a dominant strategy if both altruism and health care quality are low. Third, testing if the partner does not test may be a Nash equilibrium if health care value is high, if altruism is intermediate, and if the probability that the virus was already transmitted between $i$ and $j$ is high. If these conditions are satisfied, altruism is not high enough to induce anyhow testing: $i$ prefers to bet on the fact that his partner will test and then to test in $T_2$ if he observes that $j$ is infected. Finally, testing may be optimal if and only if the partner tests $T_1$ ("test-test" and "does not test-does not test" are two Nash equilibria). This latter case occurs if altruism is high, if the utility of risky sex is much higher than the utility of safe sex, and if the probability $s_{11}$ that both individuals are infected is high. If these conditions are satisfied, the motivation for $i$ to test is altruism, and not the access to ARV. Because $i$
is altruistic, he prefers to test and to choose safe sex if he is sure that his partner is not infected. However, if he has no information about the status of his partner (because \(j\) does not test in \(T_1\)), testing and opting for safe sex is too costly because he is almost sure that his partner is also infected. The following proposition compares these four possible cases.

**Proposition 4.2.** We assume full observability. The proposition 3.3 applies if altruism is low that is, if:

\[
K_{q=1} \equiv p_i(1 - p_{ij})P_n[\alpha P_N - (U_N - U_S)] < 0. 
\]  

(18)

If altruism is high that is, if \(K_{q=1} > 0\), not testing in \(T_1\) is a dominant strategy for \(i\) iff:

\[
\begin{cases} 
A + \max(K_{q=0}, 0) < 0 \\
A - O_p + K_{q=1} < 0 \text{ or } A + K_{q=1} < 0. 
\end{cases} 
\]  

(19)

If altruism is high, then testing in \(T_1\) is a dominant strategy iff:

\[
\begin{cases} 
A + \max(K_{q=0}, 0) > 0 \\
A - O_p + K_{q=1} > 0. 
\end{cases} 
\]  

(20)

If altruism is high, then testing is optimal if the partner does not test iff:

\[
\begin{cases} 
A + \max(K_{q=0}, 0) > 0 \\
A - O_p + K_{q=1} < 0. 
\end{cases} 
\]  

(21)

If altruism is high, then testing is optimal if the partner tests iff:

\[
\begin{cases} 
A + \max(K_{q=0}, 0) < 0 \\
A - O_p + K_{q=1} > 0 \\
A + K_{q=1} > 0. 
\end{cases} 
\]  

(22)

**Proof** The proof is in appendix. \(\Box\)

The figure A.11 represents the choice of \(i\) as a function of both altruism \(\alpha\) and transmission probability \(P_n\) for the full observability case \((U_N - U_S = 1,\)
$S = 1$). I restricted the domain to the values of $\alpha$ that are high enough to ensure that $K_{q=1} > 0$ (if $K_{q=1} < 0$ that is, if altruism is low, the proposition 3.3 applies). The relation $A + \max(K_{q=0}, 0)$ is represented by two different sections: the plane which is increasing in $P_n$ if $P_n$ is high, and the plane which is sharply decreasing in $P_n$ if $P_n$ is low. It may be interpreted as the condition $A > 0$ for an altruistic individual. The relation $A - O_p + K_{q=1}$ is represented by the plane which is sharply decreasing in $P_n$. This plane may be interpreted as the condition $A - O_p$ for an altruistic individual. It is worth noting that $A + \min(K_{q=0}, 0)$ and $A - O_p - K_{q=1}$ are almost superimposed if $P_n$ is low. The testing cost $C$, not represented, would be a horizontal plane.

Similarly, the figure A.12 shows the relations $A + \min(K_{q=0}, 0)$ and $A - O_p + K_{q=1}$ as a function of $\beta$ and $n$ ($\alpha = 5$, $U_N - U_S = 1$, $S = 1$, $N = 100$). With this calibration, altruism is high enough to ensure that $K_{q=1} > 0$.

The figures A.11 and A.12 may be interpreted as follows. If the testing cost is low, if altruism is high and if the expected probability that the virus was already transmitted ($P_n$) is high, “$i$ tests-$j$ does not test” and “$i$ does not test-$j$ tests” are two Nash equilibria. Hence, if $i$ and $j$ have had some risky encounters before $T_1$, an overestimation of the transmission rate $\beta$ may imply that only one partner tests in the couple. In this case, disclosing a lower transmission rate may favor that both individuals test in the couple. Indeed, if they think that the transmission rate is low, they may want to test to protect their partner if the test result is positive.

There exist no an explicit equivalent of the corollaries 3.4 and 3.6 when altruism is introduced. Indeed, it is not possible to find an explicit formula for the thresholds $\beta^+$ and $\beta^-$. However, assuming that the transmission rate $\beta$ is overestimated ($\beta \approx 1$), it is easy to see on the figure A.12 that a diminution of the transmission rate to some low value $\beta^+$ may favor testing of both partner if $i$ and $j$ are in the case where testing is optimal only if the partner does not test that is, if the condition (21) is satisfied.

5. Discussion

In this section, I first summarize the main objectives and results of the paper. Second, I propose some possible extensions of the model. Finally, I assess the possibility to test empirically the model.
In this paper, I discussed the strong assumptions made in the seminal papers of Philipson and Posner (1994) and Boozer and Philipson (2000). In their models, the access to health care and ARV therapies is not considered. Testing is a signal for people at an intermediate risk of infection to have access to the market of risky sex. Furthermore, they assume that the test result is observable by the partner, and that the transmission rate of HIV is equal to 1. In this paper, I first introduced formally the access to health care and ARV treatment for positively tested individuals. Second, I considered egoistic individuals, as well as altruistic and self-protecting people. Third, I introduced formally the observability of the test result and the per-act expected transmission rate of HIV/AIDS into the analysis.

In my model, an individual may test to have access to ARV treatment, to protect himself if he is uninfected (self-protecting behavior), or to protect his partner if he is altruistic. I showed that ARV treatments, altruism, and self-protecting behavior sharply change the incentives to test. When the observability of partner’s test result is introduced in the model, I showed that the decision to test in a couple is strategic. If observability is expected to be high, an individual may bet on the fact that his partner will test. He will wait and test only if he observes that his partner is positive. In doing so, he may avoid the testing cost, which may be large if the cost includes both objective and subjective costs (social stigma, fear of partner’s reaction, etc.). However, this strategy may be dangerous if, in reality, he is not able to observe the test result, or if his partner plays the same strategy. In these cases, he may falsely infer that his partner is not infected. Furthermore, if he already had some risky encounters with this partner, he will believe that both he and his partner are uninfected. He will then continue to have risky sex. However, this reasoning may be misleading in two cases. On the one side, he may have been infected before his relationship with his current partner, and the virus may not have been transmitted yet. On the other side, the partner may have been infected before their relationship, and the virus may not have been transmitted yet. Using the model, I show that the overestimation of the per-act transmission rate of HIV/AIDS may exacerbate this phenomenon. Disclosing a lower transmission rate may favor that both individuals test in the couple if the relative cost of testing is low.

This paper is a first attempt to include altruism, observability of partner’s test result, and misevaluation of the per-act transmission rate into a formal
analysis of the testing decision. The accuracy of this new framework may be improved in several ways. First, the model may be adapted in a repeated game form. In the current version of the model, the timing of the testing decisions in T1 and T2 is exogenously given. In reality, the decision to attend a health center for testing is repeated almost continuously. Integrating the timing of the decision to test into the model may point out other strategic interactions in the couple. Furthermore, more realistic utility functions, that take into account the recent insights of psychology and behavioral economics, will be more easily introduced in a multi-period framework. For example, the quasi-hyperbolic discounting framework (Laibson, 1997), the temptation models (Gul and Pesendorfer, 2001, 2004) or the dual-self models (Thaler and Shefrin, 1981; Fudenberg and Levine, 2006) may be adapted to explain why people postpone testing. Second, the model may be extended to take into account the choice of the risk level before and after testing. Indeed, in my model, the decision of the level of risk before and after testing is an exogenous process. The choice of risk after testing mainly depends on altruism and self-protection. Introducing the behavioral choice after testing is a difficult task because empirical studies did not permit to show whether altruistic or self-protecting behavior is more important after testing (Thornton, 2008; Delavande and Kohler, 2009; De Paula et al., 2010; Gong, 2010). Fourth, in my analysis, I considered that the probabilities $p_{ii}$ and $p_{ij}$ that $i$ and $j$ are infected prior to their relationship are exogenously given. However, these probabilities are related to behavior prior to the relation between $i$ and $j$. Expressing $p_{ii}$ and $p_{ij}$ as a function of both the number of risky sexual encounters before the relationship and the expected transmission rate $\beta$ may slightly change the analysis of the optimal per-act expected transmission rate for testing to be a dominant strategy. Finally, the definition of observability may be extended in several ways. In this paper, I defined the observability as the probability to observe the test result of the partner if it is positive. This definition may be extended to include the observability of HIV/AIDS symptoms. A new crucial parameter will then be the belief about when ARV are required compared to the first appearance of symptoms. If an individual thinks that ARV treatment is needed only when the disease is symptomatic, then he will test and bear the testing cost only if he feels symptoms or if he observes that his partner is sick.

The model of this paper needs to be empirically tested to be epistemologically valid. I showed that the decision to test in a couple may be strategic if
observability is expected to be high. Hence, the empirical test of the model using standard methods would induce a simultaneity bias: the probability that $i$ tests depend on the probability that $j$ tests and vice versa. The empirical test of the model requires either an experiment or a valid instrument for the partner’s testing decision. Specific information about both the expected observability of the test result and the expected transmission rate is also necessary. To my knowledge, no such data is available.

6. Conclusion

This paper studies the decision to test for HIV/AIDS. It enlarges the framework considered by Philipson and Posner (1994) and Boozer and Philipson (2000) in two directions. First, testing is not anymore a signal to have access to the market of risky sex. On the contrary, an individual chooses to test to have access to treatments, to protect himself if he is uninfected (self-protecting behavior), or to protect his partner if he is altruistic. Second, this paper study formally the joint impact of the expected per-act transmission rate of HIV and the expected observability of the partner’s test result.

I showed that access to ARV treatment, altruism and self-protection are important factors for testing. Furthermore, I showed that the decision to test in a couple is strategic when the test result of the partner is observable. If the observability and the transmission rate are expected to be high, people may not test, but may expect that their partner will test “in place of them”. Disclosing that the transmission rate is lower may favor testing if the cost of testing is low compared to the benefits from the access to health care and ARV therapies.

7. References


Appendix A. Figures

<table>
<thead>
<tr>
<th>Wife</th>
<th>no test</th>
<th>test and no result</th>
<th>test and result (voluntary)</th>
<th>Total</th>
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<td>0.00%</td>
<td>0.22%</td>
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<td>1.98%</td>
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<td>87.04%</td>
<td>2.41%</td>
<td>10.56%</td>
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</table>

Figure A.1: Testing in Lesotho (DHS 2004).

<table>
<thead>
<tr>
<th>Wife</th>
<th>Don’t want to test</th>
<th>Want to test</th>
<th>Don’t know</th>
<th>Total</th>
</tr>
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<tr>
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<td>2.28%</td>
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<tr>
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<td>1.18%</td>
<td>0.93%</td>
<td>0.45%</td>
<td>0.22%</td>
</tr>
<tr>
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<td>87.04%</td>
<td>2.41%</td>
<td>10.56%</td>
<td>100.00%</td>
</tr>
</tbody>
</table>

Figure A.2: Desire to be tested in Lesotho (DHS 2004).
<table>
<thead>
<tr>
<th>Country</th>
<th>Year</th>
<th>Prevalence</th>
<th>Access to ARV (WHO - 2006)</th>
<th>Both don't want to test</th>
<th>Only the woman want to test</th>
<th>Only the man want to test</th>
<th>Both want to test</th>
<th>Both did not test or did not get the result</th>
<th>Only the woman tested</th>
<th>Only the man tested and got the result</th>
<th>Both tested and got the result</th>
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</thead>
<tbody>
<tr>
<td>Cameroon</td>
<td>2004</td>
<td>5.30%</td>
<td>41% (34-51)</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>79.27%</td>
<td>7.68%</td>
<td>9.50%</td>
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<td>1.43%</td>
<td>... (58-86)</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
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<td>0.84%</td>
<td>2.51%</td>
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<tr>
<td>Ghana</td>
<td>2003</td>
<td>2.21%</td>
<td>36% (29-44)</td>
<td>6.79%</td>
<td>16.79%</td>
<td>12.93%</td>
<td>60.89%</td>
<td>87.79%</td>
<td>4.64%</td>
<td>6.68%</td>
<td>0.71%</td>
</tr>
<tr>
<td>Ghana</td>
<td>2008</td>
<td>n/c</td>
<td>36% (29-44)</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
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<td>2005</td>
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<td>n/a</td>
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<tr>
<td>Kenya</td>
<td>2003</td>
<td>6.73%</td>
<td>69% (55-79)</td>
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<td>14.57%</td>
<td>14.74%</td>
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<td>66% (55-79)</td>
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<td>n/a</td>
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<td>43.46%</td>
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<td>20% (15-22)</td>
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<td>n/a</td>
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<td>1.42%</td>
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<td>2001</td>
<td>n/c</td>
<td>65% (53-81)</td>
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<td>19.38%</td>
<td>12.20%</td>
<td>48.20%</td>
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<td>65% (53-81)</td>
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<td>2006</td>
<td>n/c</td>
<td>33% (29-40)</td>
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<td>n/a</td>
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<td>11.7%</td>
<td>3.16%</td>
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<td>2007</td>
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<td>... (20-32)</td>
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<td>n/a</td>
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<td>4.94%</td>
<td>6.24%</td>
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<td>90% (87-95)</td>
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<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
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<td>12.11%</td>
<td>7.54%</td>
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<tr>
<td>Senegal</td>
<td>2005</td>
<td>4.68%</td>
<td>72% (69-92)</td>
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<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
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<td>92.72%</td>
<td>2.36%</td>
<td>4.43%</td>
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<tr>
<td>Sierra Leone</td>
<td>2008</td>
<td>1.47%</td>
<td>29% (23-39)</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>89.79%</td>
<td>5.52%</td>
<td>3.49%</td>
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<tr>
<td>Swaziland</td>
<td>2006</td>
<td>18.89%</td>
<td>86% (72-95)</td>
<td>n/a</td>
<td>n/a</td>
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<td>50.29%</td>
<td>24.82%</td>
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<tr>
<td>Tanzania</td>
<td>2004</td>
<td>5.75%</td>
<td>44% (26-65)</td>
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<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>n/a</td>
<td>77.31%</td>
<td>6.83%</td>
<td>10.05%</td>
</tr>
</tbody>
</table>
Testing (T1)

Testing (T2)

Probability $q$ that the test result is observed by the partner

Beginning of the relationship

$n$ risky encounters

Testing (T1) Testing (T2)

Probability $q$ that the test result is observed by the partner

Beginning of the relationship

$n$ risky encounters

$n$ risky encounters

$N$ risky encounters

Figure A.4: Sequence of the model.

Condition $A > 0$

Testing = dominant strategy

Testing = Nash equilibrium

Condition $A > 0$

Test sequence

Testing (T1) Testing (T2)

Figure A.5: Conditions $A > 0$ and $A > O_p$ if the relative testing price is low.
Figure A.6: Conditions $A > 0$ and $A > O_p$ if the relative testing price is high.

Figure A.7: Conditions $A > 0$ and $A > O_p$ as a function of $\beta$ and $n$. 

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Figure A.8: Conditions $A > 0$ and $A > O_p$ with partial observability ($0 < q < 1$).

Figure A.9: Conditions $A > 0$ and $A + K_{q=0}$ as a function of altruism $\alpha$. 

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Figure A.10: Conditions $A > 0$ and $A + K_{q=0} > 0$ as a function of $\beta$ and $n$.

Figure A.11: Conditions $A + \max(K_{q=0}, 0) > 0$ and $A - 0_p + K_{q=1} > 0$ as a function of altruism $\alpha$. 
Appendix B. Proofs

**Proposition 3.3.** We assume full observability. Testing in $T_1$ is a dominant strategy iff $A > O_p$, where the opportunism $O_p$ is defined as:

$$O_p \equiv [p_{ii} p_{ij} + p_{ii} (1 - p_{ij}) P_n + (1 - p_{ii}) p_{ij} P_n] S - [p_{ij} + p_{ii} (1 - p_{ij}) P_n] C.$$

If the condition $A > 0$ is satisfied and the condition $A > O_p$ is not satisfied, testing in $T_1$ is optimal for $i$ iff his partner does not test.

Not testing in $T_1$ is a dominant strategy for $i$ iff $A < 0$.

**Proof** If $i$ believes that $j$ will not test, the situation is similar to the proposition (3.1): $i$ chooses to test if $A > 0$. Let us review the possible choices if $i$ thinks that $j$ will choose to test.

If $i$ tests in $T_1$, the utility of $i$ is given by:

$$V(T_1 = 1|T_j = 1) = U_n + U_N + s_{i11} (S - d) + s_{i10} (S - d) + s_{i01} (-p_{ij} P_N d) - C.$$
If $i$ tests neither in $T_1$, nor in $T_2$, he does not enjoy health care if he is infected. We assume that he finds a new sexual partner if he observes that $j$ is infected. His utility is then given by:

$$V(T_1 = 0, T_2 = 0 | T_j = 1) = U_n + U_N + (s_{i11} + s_{i10})(-d) + s_{i01}p_{ij}P_N(-d).$$

Similarly, if $i$ does not test in $T_1$, but tests in $T_2$ if $j$ is observed to be positive, the utility of $i$ is given by:

$$V(T_1 = 0, T_2 = 1 | T_j = 1) = U_n + U_N + s_{i11}(S - d - C) + s_{i10}(-d) + s_{i01}(-C - p_{ij}P_Nd).$$

If $j$ does not test, $i$ will test if:

$$A \equiv V(T_1 = 1 | T_j = 0) - V(T_1 = 0 | T_j = 0) = -C + S_{pii} + S(1 - p_{ii})p_{ij}P_n > 0.$$

If $j$ does the test, $i$ will prefer to test in $T_1$ rather than in $T_2$ iff:

$$A - O_p \equiv V(T_1 = 1 | T_j = 1) - V(T_1 = 0, T_2 = 1 | T_j = 1) = (1 - p_{ij})[-C + S_{pii} + p_{ii}P_n(C - S)] > 0,$$

where the opportunism $O_p$ is equal to:

$$O_p = [p_{ii}p_{ij} + p_{ii}(1 - p_{ij})P_n + (1 - p_{ii})p_{ij}P_n]S - [p_{ij} + p_{ii}(1 - p_{ij})P_n]C.$$

After simple algebra, we obtain that:

$$V(T_1 = 1 | T_j = 1) - V(T_1 = 0, T_2 = 0 | T_j = 1) = A,$n
$$V(T_1 = 0, T_2 = 1 | T_j = 1) - V(T_1 = 0, T_2 = 0 | T_j = 1) = O_p,$n
$$A - O_p > 0 \Rightarrow A > A - O_p > 0.$$

The first equality means that if $j$ tests, $i$ prefers to test in $T_1$ rather than not testing at all iff $A > O$. The second equality stands for the choice of testing in $T_2$ if $i$ observes that $j$ is positive. The third equality implies that if testing is prefered when $j$ also tests, then testing is prefered even if $j$ does not test.

Hence, we distinguish 4 cases:
\[ A - O_p > 0 \Rightarrow A > A - O_p > 0 \Rightarrow \text{Testing = dominant strategy}, \]
\[ A > 0 > A - O_p \Rightarrow \text{Testing if j does not test}, \]
\[ 0 > A > A - O_p \Rightarrow O_p > 0 \Rightarrow \text{No testing in T1 but testing in T2}, \]
\[ 0 > A - O_p > A \Rightarrow O_p < 0 \Rightarrow \text{No testing in T1 nor in T2}. \]

\[ \square \]

**Corollary 3.4.** Let us assume that the transmission rate \( \beta \) is overestimated \((\beta \approx 1)\). If \( n = 0 \), testing is a dominant strategy iff \( \frac{C}{S} < p_{ii} \). Testing does not depend on the expected transmission rate \( \beta \).

If \( n \geq 1 \), diminishing the expected transmission rate to the value \( \beta^+ = 1 - \left( \frac{(1-p_{ij})C}{p_{ii}(S-C)} \right)^{1/n} \) would induce that testing becomes a dominant strategy iff \( \frac{C}{S} < p_{ii} \).

If \( \frac{C}{S} > p_{ii} + (1 - p_{ii})p_{ij} \), diminishing the expected transmission rate below \( \beta^- = 1 - \left( \frac{C - Sp_{ii}}{S(1-p_{ii})p_{ij}} \right)^{1/n} \) would induce that not testing becomes a dominant strategy.

**Proof** \( P_n \) is increasing in the expected transmission rate \( \beta \). \( A \) is increasing in \( P_n \) and \( A - O_p \) is decreasing in \( P_n \). If \( C/S < p_{ii} \) is satisfied, \( A \) is positive \( \forall P_n \). Hence, decreasing \( \beta \) will decrease \( P_n \) and \( A - O_p \) without compromising \( A > 0 \). We denote \( B(\beta, n) = 0 \) the implicit function \( A - 0_p = 0 \) where \( P_n \) is replaced by its specification in terms of \( \beta \) and \( n \): \( P_n = [1 - (1 - \beta)^n] \). \( \beta^+ \) is the solution of \( B(\beta, n) = 0 \) for \( \beta \). It is the threshold under which the condition \( A - O_p > 0 \) is satisfied and testing becomes a dominant strategy.

If \( C/S < p_{ii} \) is not satisfied, decreasing \( \beta \) may implies \( A > 0 \) not satisfied anymore. We denote \( A(\beta, n) = 0 \) the implicit function \( A = 0 \) where \( P_n \) is replaced by its specification in terms of \( \beta \) and \( n \): \( P_n = [1 - (1 - \beta)^n] \). \( \beta^- \) is the solution of \( A(\beta, n) = 0 \) for \( \beta \). It is the threshold under which the condition \( A < 0 \) is satisfied and not testing becomes a dominant strategy for \( i \). \( \square \)
Proposition 3.5. The individual $i$ observes with a probability $0 < q < 1$ a positive test result of his partner. Within this framework, testing in $T1$ is a dominant strategy iff $A > qO_p$.

If the condition $A > 0$ is satisfied and the condition $A > qO_p$ is not satisfied, testing in $T1$ is optimal for $i$ iff the partner does not test.

Not testing in $T1$ is a dominant strategy for $i$ iff $A < 0$.

Proof If $j$ does not test, observability plays no role and the condition $A > 0$ does not change. If $j$ does the test, the condition $A - O_p > 0$ is modified to take into account that observability is not perfect. The individual $i$ thinks that he will be in the full observability case with a probability $q$, in the no observability case with a probability $(1 - q)$. Hence, if $i$ believes that $j$ will not test, he will choose to test if:

\[ A + (1 - q)O_p = q(1 - p_{ij})[-c + p_{ii}S + p_{ii}P_n(C - S)] + (1 - q)[-S + Sp_{ii} + S(1 - p_{ii})p_{ij}P_n] > 0. \]

It is easy to show that $A - qO_p > 0 \Rightarrow A > 0$. □

Corollary 3.6. Let us assume that the transmission rate $\beta$ is overestimated ($\beta \approx 1$). If $n = 0$, testing is a dominant strategy iff $\frac{C}{S} < p_{ii}$. Testing does not depend on the expected transmission rate $\beta$.

If $n \geq 1$, diminishing the expected transmission rate to the value $\beta^+$ would induce that testing becomes a dominant strategy iff $\frac{C}{S} < p_{ii}$ and $q > \frac{p_{ij}}{p_{ii} + p_{ij}(1 - p_{ii})}$, with $\beta^+$ given by:

\[ \beta^+ = 1 - \left( \frac{C + [p_{ij} + p_{ii}(1 - p_{ij})][qC + (1 - q)S]}{(1 - q)(1 - p_{ii})p_{ij}S - qp_{ii}(1 - p_{ij})(S - C)} \right)^{1/n} \]

If $\frac{C}{S} > p_{ii} + (1 - p_{ii})p_{ij}$, diminishing the expected transmission rate below $\beta^- = 1 - \left( 1 - \frac{C - Sp_{ii}}{S(1 - p_{ii})p_{ij}} \right)^{1/n}$ would induce that not testing becomes a dominant strategy.

Proof $P_n$ is increasing in the expected transmission rate $\beta$. $A$ is increasing in $P_n$. $A - O_p$ is decreasing in $P_n$ iff:
\[ \frac{\partial A - O_p}{\partial P_n} < 0. \]

This condition is satisfied if:

\[ q > \frac{p_{ij}}{p_{ii} + p_{ij}(1 - p_{ii})}. \]

If \( C/S < p_{ii} \) is satisfied, \( A \) is positive \( \forall P_n \). Hence, if the both conditions \( p > p_{ij}/[p_{ii} + p_{ij}(1 - p_{ii})] \) and \( C/S < p_{ii} \) are satisfied, decreasing \( \beta \) will decrease \( P_n \) and \( A - qO_p \) without compromising \( A > 0 \). We denote \( B'(\beta, n) = 0 \) the implicit function \( A - q0_p = 0 \) where \( P_n \) is replaced by is specification in terms of \( \beta \) and \( n: P_n = [1 - (1 - \beta)^n] \). \( \beta^+ \) is the solution of \( B'(\beta, n) = 0 \) for \( \beta \). It is the threshold under which the condition \( A - qO_p > 0 \) is satisfied and testing becomes a dominant strategy.

If the condition \( q > p_{ij}/[p_{ii} + p_{ij}(1 - p_{ii})] \) is not satisfied, diminishing \( \beta \) would decrease the incentive to test both if \( j \) tests and if \( j \) does not test.

If \( C/S < p_{ii} \) is not satisfied, decreasing \( \beta \) may implies \( A > 0 \) not satisfied anymore. As in the proof of the corrolary (3.4), we denote \( A(\beta, n) = 0 \) the implicit function \( A = 0 \) where \( P_n \) is replaced by is specification in terms of \( \beta \) and \( n: P_n = [1 - (1 - \beta)^n] \). \( \beta^- \) is the solution of \( A(\beta, n) = 0 \) for \( \beta \). It is the threshold under which the condition \( A < 0 \) is satisfied and not testing becomes a dominant strategy for \( i \). \( \square \)

**Proposition 4.1.** An altruistic individual \( i \) who believes that the partner’s test result is not observable will test and choose safe sex if he is infected iff:

\[
\begin{cases}
A + K_{q=0} > 0 \\
K_{q=0} > 0.
\end{cases}
\]

The individual \( i \) will test and choose risky sex if he is infected iff:

\[
\begin{cases}
A > 0 \\
K_{q=0} < 0.
\end{cases}
\]

The individual will not test iff:

\[
\begin{cases}
A < 0 \\
A + K_{q=0} < 0.
\end{cases}
\]

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Proof The individual $i$ has 3 possible choices. First, the individual $i$ will prefer to test and to engage in safe sex after a positive test result if $A + K_{q=0}$ is positive (testing and safe sex is preferred to not testing) and if $K_{q=0}$ is positive (safe sex if preferred after testing). Second, $i$ will prefer to test and to engage in risky sex after a positive test result if $A$ is positive (testing and risky sex is preferred to not testing) and if $K_{q=0}$ is negative (risky sex is preferred after testing). Finally, $i$ will choose not to test if both $A$ and $A + K_{q=0}$ are negative. \[\□\]

Proposition 4.2. We assume full observability. The proposition 3.3 applies if altruism is low that is, if:

$$K_{q=1} \equiv p_{ii}(1 - p_{ij})P_n[\alpha P_N - (U_N - U_S)] < 0.$$ 

If altruism is high that is, if $K_{q=1} > 0$, not testing in T1 is a dominant strategy for $i$ iff:

$$\begin{cases} 
A + \max(K_{q=0}, 0) < 0 \\
A - O_p + K_{q=1} < 0 \text{ or } A + K_{q=1} < 0.
\end{cases}$$

If altruism is high, then testing in T1 is a dominant strategy iff:

$$\begin{cases} 
A + \max(K_{q=0}, 0) > 0 \\
A - O_p + K_{q=1} > 0.
\end{cases}$$

If altruism is high, then testing is optimal if the partner does not test iff:

$$\begin{cases} 
A + \max(K_{q=0}, 0) > 0 \\
A - O_p + K_{q=1} < 0.
\end{cases}$$

If altruism is high, then testing is optimal if the partner tests iff:

$$\begin{cases} 
A + \max(K_{q=0}, 0) < 0 \\
A - O_p + K_{q=1} > 0 \\
A + K_{q=1} > 0.
\end{cases}$$

Proof If $i$ believes that $j$ does not test in T1, the analysis is similar to the proof of proposition 4.1.
If \( i \) believes that \( i \) tests, the analysis changes because \( i \) observes \( i \)'s test result if it is positive. The utility of testing and risky sex is given by:

\[
V_{\alpha,R}(T_{1i} = 1|T_{1j} = 1) = U_i + U_N + s_{ii1}(S - d) + s_{i10}(S - d - \alpha P_N) + s_{i01}(-P_N d) - C.
\]

The utility of testing and safe sex if the test result is positive is given by:

\[
V_{\alpha,R}(T_{1i} = 1|T_{1j} = 1) = U_i + U_N + s_{ii1}(U_N + S - d) + s_{i10}(U_S + S - d) + s_{i01}(U_N - P_N d) + s_{i00}(U_N) - C.
\]

If \( i \) does not test in \( T_1 \), but tests in \( T_2 \) if he observes that \( j \) is infected, his utility is given by:

\[
V_{\alpha}(T_{1i} = 0, T_{2j} = 1) = U_i + U_N + s_{ii1}(S - d - C) + s_{i10}(-d - \alpha P_N) + s_{i01}(-p_{ij} P_N d - C).
\]

Finally, if \( i \) does not test in \( T_1 \), nor in \( T_2 \), his utility is given by:

\[
V_{\alpha}(T_{1i} = 0, T_{2j} = 1) = U_i + U_N + s_{ii1}(-d) + s_{i10}(-\alpha P_N) + s_{i01}(-p_{ij} P_N d).
\]

The individuals \( i \) prefers safe sex to risky sex after testing if kindness \( K_{q=1} \) is low that is, if:

\[
K_{q=1} \equiv V_{\alpha,S}(T_{1i} = 1|T_{1j} = 1) - V_{\alpha,R}(T_{1i} = 1|T_{1j} = 1) = p_{ii}(1 - p_{ij}) P_n[\alpha P_N - (U_N - U_S)] > 0.
\]

It is straightforward that \( K_{q=1} > K_{q=0} \). If altruism is low \((K_{q=1} > K_{q=0} > 0)\), the fear of infecting the partner is not high enough to induce a behavioral change, and the conclusions of proposition 3.3 apply.

If altruism is intermediate \((K_{q=1} > 0 > K_{q=0})\), the individual \( i \) will choose safe sex both \( i \) and \( j \) tested and if only \( i \) is infected. He will choose risky sex if his partner does not test. If altruism is high \((0 > K_{q=1} > K_{q=0})\), the individual \( i \) will choose safe sex if he is positively tested.

The difference between testing in \( T_1 \) (and safe sex) and testing in \( T_2 \) if \( j \) is observed to be positive is given by:
\[ V_{\alpha,S}(T1_i = 1|T1_j = 1) - V_{\alpha}(T1_i = 0, T2_i = 1) = A - 0_p + K_{q=1}. \]

The difference between testing in T1 (and safe sex) and not testing at all if \( j \) is observed to be positive is given by:

\[ V_{\alpha,S}(T1_i = 1|T1_j = 1) - V_{\alpha}(T1_i = 0, T2_i = 0) = A + K_{q=1}. \]

We assume that altruism is intermediate or high that is, \( K_{q=1} > 0 \) (otherwise proposition 3.3 applies). If \( i \) believes that \( j \) does not test in T1, the proposition 4.1 applies and \( i \) does not test in T1 if both \( A < 0 \) and \( A + K_{q=0} < 0 \), that is if \( A + \max(K_{q=0}, 0) < 0 \). If \( i \) believes that \( j \) will test in T1, \( i \) does not test in T1 if \( A - 0_p + K_{q=1} < 0 \) or \( A + K_{q=1} < 0 \). Putting these conditions together, we obtain that not testing is a dominant strategy for \( i \) iff:

\[
\begin{align*}
A + \max(K_{q=0}, 0) &< 0 \\
A - 0_p + K_{q=1} &< 0 \text{ or } A + K_{q=1} < 0.
\end{align*}
\]

Similarly, if \( j \) does not test, \( i \) will test if \( A + \max(K_{q=0}, 0) > 0 \). If \( j \) tests, \( i \) will also test if \( A - 0_p + K_{q=1} > 0 \) and \( A + K_{q=1} > 0 \). Putting these conditions together, we obtain that testing is a dominant strategy for \( i \) iff:

\[
\begin{align*}
A + \max(K_{q=0}, 0) &> 0 \\
A - 0_p + K_{q=1} &> 0.
\end{align*}
\]

If \( j \) does not test, testing is optimal for \( i \) if \( A + \max(K_{q=0}, 0) > 0 \). If \( j \) tests in T1, then \( i \) does not want to test if \( A - 0_p + K_{q=1} < 0 \) or \( A + K_{q=1} < 0 \). Putting these conditions together, we obtain that testing is optimal for \( i \) if his partner does not test iff:

\[
\begin{align*}
A + \max(K_{q=0}, 0) &> 0 \\
A - 0_p + K_{q=1} &< 0.
\end{align*}
\]

Finally, \( i \) will test only if \( j \) also tests iff:

\[
\begin{align*}
A + \max(K_{q=0}, 0) &< 0 \\
A - 0_p + K_{q=1} &> 0 \\
A + K_{q=1} &> 0.
\end{align*}
\]

\( \square \)