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Hadronic Phases and Isospin Amplitudes in $D(B) \to \pi\pi$ and $D(B) \to K\bar{K}$ Decays

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Abstract

Hadronic phases in $\pi\pi$ and $K\bar{K}$ channels are calculated à la Regge. At the $D$ mass one finds $\delta_{\pi\pi} \simeq \frac{\pi}{3}$ and $\delta_{K\bar{K}} \simeq -\frac{\pi}{6}$ in good agreement with the CLEO data while at the $B$ mass these angles are predicted to be, respectively, $11^\circ$ and $-7^\circ$. With the hadronic phase $e^{i\delta_{K\bar{K}}}$ taken into account, a quark diagram decomposition of the isospin invariant amplitudes in $D \to K\bar{K}$ decays fits the data provided the exchange diagram contribution is about $1/3$ of the tree level one.
1 Introduction

To extract information on weak interaction parameters from non leptonic two body decays of the \( D \) and \( B \) mesons, it is crucial to understand the final hadronic effects which are at work in these decays. For \((K\pi)\), \((\pi\pi)\) and \((K\bar{K})\) decay modes, the important hadronic parameter is an angle \( \delta \) which is the difference between \( s \)-wave phase shifts in the appropriate isospin invariant amplitudes.

In a previous paper \[1\] we used a Regge model to determine \( \delta_{K\pi} \) as a function of energy. Good agreement with data at the \( D \) mass \[2\] was obtained and \( \delta_{K\pi} \) is predicted to be around \( 20^\circ \) at the \( B \) mass.

In this letter we extend this Regge analysis to \((\pi\pi)\) and \((K\bar{K})\) channels and determine \( \delta_{\pi\pi}(s) \) and \( \delta_{K\bar{K}}(s) \). At the \( D \) mass, we find \( \delta_{\pi\pi}(m_D^2) \simeq \frac{\pi}{3} \) and \( \delta_{K\bar{K}}(m_D^2) \simeq -\frac{\pi}{6} \) once again in agreement with the data\[2\][4]. At the \( B \) mass these angles are predicted to be of the order of \( 11^\circ \) and \(-7^\circ\) respectively, implying that hadronic effects in \( B \) decays remain important. Details of the derivation of these results are given in Section 2.

With hadronic phases thus determined it becomes interesting to compare a quark diagram decomposition of isospin invariant amplitudes with the data. In Section 3, we argue that for \( D \to K\bar{K} \) decays a fit to the data \[3\] implies that the contribution of exchange quark diagrams be of the order of \( 1/3 \) of the tree-level one.
2 $\delta_{\pi \pi}(s)$ and $\delta_{K \bar{K}}(s)$ in a Regge Model

In $\pi \pi \rightarrow \pi \pi$ scattering, isospin eigenamplitudes ($I = 0, 1, 2$) in the $s, t, u$-channels are related by the crossing matrices

\[
\begin{pmatrix}
A_0^s \\
A_1^s \\
A_2^s
\end{pmatrix} =
\begin{pmatrix}
1/3 & 1 & 5/3 \\
1/3 & 1/2 & -5/6 \\
1/3 & -1/2 & 1/6
\end{pmatrix}
\begin{pmatrix}
A_0^t \\
A_1^t \\
A_2^t
\end{pmatrix} =
\begin{pmatrix}
1/3 & 1 & 5/3 \\
-1/3 & -1/2 & 5/6 \\
1/3 & -1/2 & 1/6
\end{pmatrix}
\begin{pmatrix}
A_0^u \\
A_1^u \\
A_2^u
\end{pmatrix}
\] (1)

while for $K \bar{K} \rightarrow K \bar{K}$ scattering ($I = 0, 1$) the $s - t$ crossing matrix reads

\[
\begin{pmatrix}
\tilde{A}_0^s \\
\tilde{A}_1^s
\end{pmatrix} =
\begin{pmatrix}
1/2 & 3/2 \\
1/2 & -1/2
\end{pmatrix}
\begin{pmatrix}
\tilde{A}_0^t \\
\tilde{A}_1^t
\end{pmatrix}.
\] (2)

The basic physical idea of a Regge model is that the high energy behaviour of $s$-channel amplitudes is determined by "exchanges" in the crossed channels. For $\pi \pi$ scattering, the dominant exchanges in the $t$-channel are the Pomeron ($P$) and the exchange degenerate $\rho - f_2$ trajectories while in $K \bar{K}$ scattering one must also add the exchange degenerate $\omega - a_2$ trajectories. The $u$-channel exchanges in $\pi \pi$ scattering are identical to the $t$-channel ones ($P, \rho, f_2$) while in $K \bar{K}$ scattering there are no exchanges in the (exotic) $u$ channel ($KK \rightarrow KK$).

In the energy range ($3 \text{ GeV}^2 \lesssim s \lesssim 35 \text{ GeV}^2$) which is of interest to us, the Pomeron ($P$) contribution to the isoscalar $t$-channel amplitude is phenomenologically well described by the formula

\[A_P = i\beta_P(0)e^{ib_P s}\] (3)

where the residue $\beta_P(0)$ and slope $b_P$ depend on the scattering process considered. The $\rho, f_2, \omega, a_2$ Regge trajectories are all degenerate i.e.

\[\alpha_\rho(t) = \alpha_{f_2}(t) = \alpha_\omega(t) = \alpha_{a_2}(t) = \frac{1}{2} + t\] (4)
The $\rho$ and $\omega$ trajectories have negative signatures while the $f_2$ and $a_2$ trajectories are of positive signature.

The effective $\rho$ trajectory contribution to the isovector $t$-channel amplitude is written as

$$A_{\rho} = \frac{\bar{\beta}_\rho}{\sqrt{\pi}}(1 + ie^{-i\pi t}s^{0.5+t})$$

while the $a_2$ contribution to $\tilde{A}_1^t$ reads

$$A_{a_2} = \frac{\tilde{\bar{\beta}}_{a_2}}{\sqrt{\pi}}(-1 + ie^{-i\pi t}s^{0.5+t}).$$

Similar expressions are used for the effective $\omega$ and $f_2$ trajectory contributions to the isoscalar $t$-channel amplitude.

The residues of these trajectories are related as follows:

a) in $\pi\pi$ scattering

$$\bar{\beta}_{f_2} = \frac{3}{2}\bar{\beta}_\rho$$

b) in $KK$ scattering

$$\tilde{\bar{\beta}}_{f_2} = \tilde{\bar{\beta}}_\rho$$

$$\tilde{\bar{\beta}}_{a_2} = \tilde{\bar{\beta}}_\omega.$$ 

Furthermore, $SU(3)$ symmetry and ideal mixing give the additional relation

$$\tilde{\bar{\beta}}_\rho = \tilde{\bar{\beta}}_\omega.$$ 

Eqs (7)-(9) follow from “duality” : the scattering processes $(\pi^+\pi^+ \rightarrow \pi^+\pi^+)(A^s_{f_2})$ and $(KK \rightarrow KK)$ are purely diffractive, hence the imaginary part of the Regge trajectory contributions to these processes must cancel [4].

Using Eqs (1) (3)-(5) and (7) our Regge model for $\pi\pi$ scattering near the forward direction ($t$ small) reads:

$$A^s_0(s \text{ large, small } t) = \frac{i}{3}\beta_P(0)e^{b_pt}s + \frac{1}{2}\sqrt{\pi}\bar{\beta}_\rho s^{0.5+t} + \frac{3i}{2\sqrt{\pi}}\bar{\beta}_\rho e^{-i\pi t}s^{0.5+t}.$$ 

4
\[ A_2(s \text{ large, small } t) = \frac{i}{3} \beta_P(0)e^{b_P t} s - \frac{\beta_\rho}{\sqrt{\pi}} s^{0.5+t}. \]  

(12)

In the backward direction (\( u \text{ small} \)) exactly the same formulae hold with \( t \) replaced by \( u \).

Similarly, for \( K\bar{K} \rightarrow K\bar{K} \) scattering one obtains from Eq.(2), using the relations Eqs (8)-(10), that

\[ \tilde{A}_0(s \text{ large, small } t) = \frac{i}{2} \tilde{\beta}_P(0)e^{\tilde{\beta}_P t} s + \frac{4i\tilde{\beta}_P}{\sqrt{\pi}} e^{-i\pi t} s^{0.5+t} \]  

(13)

\[ \tilde{A}_1(s \text{ large, small } t) = \frac{i}{2} \tilde{\beta}_P(0)e^{\tilde{\beta}_P t} s. \]  

(14)

From Eqs (11)-(14) we compute the \( l=0 \) partial wave amplitudes and find, up to irrelevant overall real factors

\[ a_0(s) = \frac{i}{3} \beta_P(0) \frac{\beta_\rho s^{1/2}}{b_P} + \frac{3i}{2\sqrt{\pi}} \beta_\rho (\ln s) + i\pi s^{1/2} \]  

(15)

\[ a_2(s) = \frac{i}{3} \beta_P(0) \frac{\beta_\rho s^{1/2}}{\sqrt{\pi} \ln s}. \]  

(16)

\[ \tilde{a}_0(s) = \frac{i}{2} \tilde{\beta}_P(0) \frac{4i\tilde{\beta}_\rho (\ln s) + i\pi}{\sqrt{\pi}} s^{1/2} \]  

(17)

\[ \tilde{a}_1(s) = \frac{i}{2} \tilde{\beta}_P(0) \frac{\tilde{\beta}_\rho}{b_P}. \]  

(18)

The \( u \)-channel contributions in Eqs. (15)-(16) are identical to the \( t \)-channel ones and we have dropped a (common) factor of 2 in these equations.

Clearly the phases \( e^{i\delta_0} \) and \( e^{i\delta_2} \) of \( a_0(s) \) and \( a_2(s) \) depend on the phenomenological parameter

\[ x_{\pi\pi} = \frac{\sqrt{\pi} \beta_P(0) \frac{1}{\beta_\rho(0)}}{b_P}, \]  

(19)

and similarly \( e^{i\tilde{\delta}_0} \) and \( e^{i\tilde{\delta}_1} \) depend on

\[ x_{\bar{K}K} = \frac{\sqrt{\pi} \tilde{\beta}_P(0) \frac{1}{\tilde{\beta}_\rho(0)}}{b_P}. \]  

(20)

From the fits given in references [5] and [6] we extract the values

\[ x_{\pi\pi} = 0.69 \pm 0.10 \]  

(21)

\[ x_{\bar{K}K} = 1.72 \pm 0.30. \]  

(22)
With these values we obtain respectively

\[
\delta_{\pi\pi}(m_D^2) = \delta_2(m_D^2) - \delta_0(m_D^2) = 60^\circ \pm 4^\circ \quad (23)
\]

\[
\delta_{K\bar{K}}(m_D^2) = \tilde{\delta}_1(m_D^2) - \tilde{\delta}_0(m_D^2) = -29^\circ \pm 4^\circ \quad (24)
\]

and predict

\[
\delta_{\pi\pi}(m_B^2) = 11^\circ \pm 2^\circ \quad (25)
\]

\[
\delta_{K\bar{K}}(m_B^2) = -7^\circ \pm 1^\circ. \quad (26)
\]

At the $D$ mass, the experimental values given by the CLEO collaboration \[3\][2] are,

\[
\delta_{\pi\pi}(m_D^2) = 82^\circ \pm 10^\circ \quad (27)
\]

\[
\delta_{K\bar{K}}(m_D^2) = \pm(24^\circ \pm 13^\circ). \quad (28)
\]

Clearly our Regge model calculation of these phases is in good agreement with the data as announced previously.

In summary, for $(\pi\pi)$ and $(K\bar{K})$ decay channels as well as for $(K\pi)$ channels, hadronic angles are correctly predicted at the $D$ mass by a Regge model and are found to be quite sizeable at the $B$ mass: hadronic effects simply cannot be ignored in $B$ decays.

### 3 Isospin Amplitudes and Quark Diagrams in $(D \rightarrow K\bar{K})$ Decays

Having determined the hadronic phase $\delta_{K\bar{K}}$, we now illustrate the strategy advocated in Ref.\[8\] to analyze the $D \rightarrow K\bar{K}$ data.

In lowest order the weak Hamiltonian responsible for $(D \rightarrow K\bar{K})$ decays contains an isodoublet ($H_W^{1/2}$) and an isoquadruplet ($H_W^{3/2}$) part. With the reduced matrix elements

\[
w_1 = \ll D \mid H_W^{1/2} \mid (K\bar{K})I = 1 \gg \quad (29)
\]
\[ w_0 = \ll D | H_W^{1/2} | (K \bar{K}) I = 0 \gg \]  
\[ v_1 = \ll D | H_W^{3/2} | (K \bar{K}) I = 1 \gg \]

and the hadronic angle \( \delta_{KK} = \delta_1 - \delta_0 \) one readily obtains, up to an overall phase factor

\[
A(D^+ \to K^+ \bar{K}^0) = -\frac{v_1}{2} + w_1
\]
\[
A(D^0 \to K^+ \bar{K}^-) = \frac{v_1}{2} + \frac{w_1}{2} + \frac{w_0}{2}e^{-i\delta_{KK}}
\]
\[
A(D^0 \to K^0 \bar{K}^0) = -\frac{v_1}{2} - \frac{w_1}{2} + \frac{w_0}{2}e^{-i\delta_{KK}}.
\]

We assume again all reduced matrix elements to be real and expressed in terms of quark diagrams classified following their \((1/N\)-inspired) topology.

In this “phenomenological” picture where we keep the explicit \((V-A)\) times \((V-A)\) \(W^\pm\) propagations, the annihilation diagram \((A)\) is helicity-suppressed and the \(b, s\) and \(d\) quarks are exchanged in the penguin diagrams \((P_q)\). On the contrary, in the “formal” language the effects of \(W^\pm\) and \(b\) would be hidden in the short-distance Wilson coefficients of local operators built out of the \(u, d, s\) and \(c\) quarks only.

The contributions from the tree-level \((T)\), annihilation \((A)\), penguins \((\Delta P)\) exchanges with either a \(u \bar{u}\) or a \(d \bar{d}\) pair created \((E)\) and, finally, exchanges with a \(s \bar{s}\) pair created \((E_s)\) lead to the relations

\[
w_0 = T + \Delta P + 2E - E_s
\]
\[
w_1 = T + \Delta P + \frac{1}{3}E_s - \frac{2}{3}A
\]
\[
v_1 = \frac{2}{3}E_s + \frac{2}{3}A.
\]

If one assumes

\[
E = E_s
\]

\(^1\)If we neglect the (multi-) Cabibbo-suppressed \(b\) quark contribution, there are two diagrams to consider with opposite signs: the “chin” of the penguin is either a \(d\) quark or a \(s\) quark. In the limit where \(m_d = m_s\), \(\Delta P \equiv P_s - P_d = 0\)
which is what one expects in the $SU(3)$ limit, then Eqs (35)-(37) imply

$$w_0 = w_1 + v_1$$  \hspace{1cm} (39)

and Eqs (32)-(34) now read

$$A(D^+ \to K^+ \bar{K}^0) = w_0 - \frac{3v_1}{2}$$  \hspace{1cm} (40)

$$A(D^0 \to K^+ K^-) = \frac{w_0}{2}(1 + e^{-i\delta_{KK}})$$  \hspace{1cm} (41)

$$A(D^0 \to K^0 \bar{K}^0) = \frac{-w_0}{2}(1 - e^{-i\delta_{KK}}).$$  \hspace{1cm} (42)

Eqs (41)-(42) are in good agreement with experimental data when Eq.(24) is used.

It is difficult to imagine a more spectacular illustration of final state hadronic effects than Eqs (41)-(42). Furthermore, from the experimental value

$$\frac{\Gamma(D^0 \to K^+ K^-)}{\Gamma(D^+ \to K^+ \bar{K}^0)} \simeq 1.6$$  \hspace{1cm} (43)

one deduces $v_1 \simeq \frac{1}{3}w_0$ or, in terms of quark diagrams,

$$\frac{E + A}{T + \Delta P + E} \simeq \frac{1}{4}. $$  \hspace{1cm} (44)

Since $A$ and $\Delta P$ are expected to be quite small in our phenomenological picture, Eq.(44) entails

$$\frac{E}{T} \simeq \frac{1}{3}. $$  \hspace{1cm} (45)

This result may be at odds with some theoretical prejudices but is required by the data: with $\delta_{KK}$ given by Eq.(24), all the $D \to K \bar{K}$ data are indeed very nicely fitted by Eqs (40)-(42) provided Eq.(44) holds.

The detailed analysis of $D \to K \bar{K}$ decays presented in this section can be repeated for $(D \to \pi \pi)$ and $(D \to K \pi)$ channels. In these channels, sizeable color-suppressed quark diagrams (C) operate and nothing as striking as Eq.(42) or as unexpected as Eq.(45) emerges from such an analysis.
To summarize our earlier work on \((K\pi)\) channels as well as the results of the present paper on \((\pi\pi)\) and \((K\bar{K})\) decays let us insist on the following points:

- at the \(D\) mass, hadronic phases are rather well estimated in the context of a Regge model. Note that the hierarchy

\[
\delta_{K\pi} \simeq \frac{\pi}{2}, \quad \delta_{\pi\pi} \simeq \frac{\pi}{3}, \quad \delta_{K\bar{K}} \simeq -\frac{\pi}{6}
\]  

(46)

follows from the difference in \(u\)-channel exchanges for the corresponding scattering processes combined with different Clebsch Gordan coefficients weighing the relative contributions of the Pomeron and the \(I = 1\) Regge trajectories. In this paper, we have ignored inelastic channels such as \(\{K\eta\} \rightarrow \{K\pi\}\) or \(\{\pi\eta\} \rightarrow \{K\bar{K}\}\). In fact, our Regge analysis shows that they have little effect on phases at least at the \(D\) mass.

- at the \(B\) mass, hadronic phases are predicted to be non negligible in the three channels considered so far:

\[
\delta_{K\pi} \approx 17^\circ, \quad \delta_{\pi\pi} \approx 11^\circ, \quad \delta_{K\bar{K}} \approx -7^\circ.
\]  

(47)

We have assumed that inelastic channels have a small overall effect on these hadronic phases \([1]\). Whether this is true or not is an experimental question. But clearly final state hadronic phases remain large in \(B\) decays and the prospect for CP-asymmetries looks particularly promising in the \(K\pi\) channel.

- the parametrization suggested in Ref \([9]\) works very nicely as exemplified by our analysis of \((D \rightarrow K\bar{K})\) decays. When hadronic phases are important, quark-diagram absorptive parts and inelastic effects on the phases seem to be negligible. These latter conclusions may not hold for decay processes where hadronic phases are quite small.

Acknowledgements

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[10] The sensitivity of the $D^0 \rightarrow K^0 \overline{K^0}$ decay channel to hadronic final state interactions has been first emphasized by H.J. Lipkin, Phys. Rev. Lett. 44 (1980) 710; see also X.Y. Pham, Phys. Lett. B193 (1987) 331.

[11] We thank H. Lipkin for useful e-mail exchanges on this subject.